

**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich



# FeynRules

## Lecture I

Claude Duhr

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# FeynRules

- **Aim of the lecture:** Give an introduction to the Mathematica package FeynRules.
- **Lecture I:** The basics.  
How to implement a model and compute its Feynman rules.
- **Lecture II:** Advanced topics.
  - ➔ SUSY
  - ➔ Computing two-body decays.
  - ➔ Spectrum generation with ASperGe.
  - ➔ Towards NLO.

# Going Beyond SM

- A BSM model can be defined via
  - ➔ The particles appearing in the model.
  - ➔ The values of the parameters ('Benchmark point').
  - ➔ The interactions among the particles, usually dictated by some symmetry group, and quantified in the Lagrangian of the model.
- All this information needs to be implemented into the MC codes, usually in the form of text files that contain the definitions of the particles, the parameters and the vertices.

# Going Beyond SM

- This can be a very tedious exercise.
- Most of these codes have only a very limited amount of models implemented by default ( $\sim$  SM and MSSM).
- However, still these codes do not work at the level of Lagrangians, but need explicit vertices.
- The process of implementing Feynman rules can be particularly tedious and painstaking:
  - ➔ Each code has its own conventions (signs, factors of  $i$ , ...).
  - ➔ Vertices need to be implemented one at the time.
- Most codes can only handle a limited amount of color and / or Lorentz structures ( $\sim$  SM and MSSM)

# Going Beyond SM

- The aim of these lectures is to present a code that automatizes all these steps, and allows to implement the model into Matrix element generators starting directly from the Lagrangian.
- Workflow:
  - ➔ Define your particles and parameters.
  - ➔ Enter your Lagrangian.
  - ➔ Let the code compute the Feynman rules.
  - ➔ Output all the information in the format required by your favorite MC code.

# Plan of the Lecture

- What is FeynRules?
- Getting started:
  - ➔  $\phi^4$  theory.
  - ➔ Adding gauge interactions (scalar QCD).
  - ➔ Adding mixings.
- Extending existing implementations.
- Towards LHC phenomenology: The FeynRules interfaces.

N.B.: Tutorials in the afternoon!

What is FeynRules?

# FeynRules

- FeynRules is a Mathematica package that allows to derive Feynman rules from a Lagrangian.

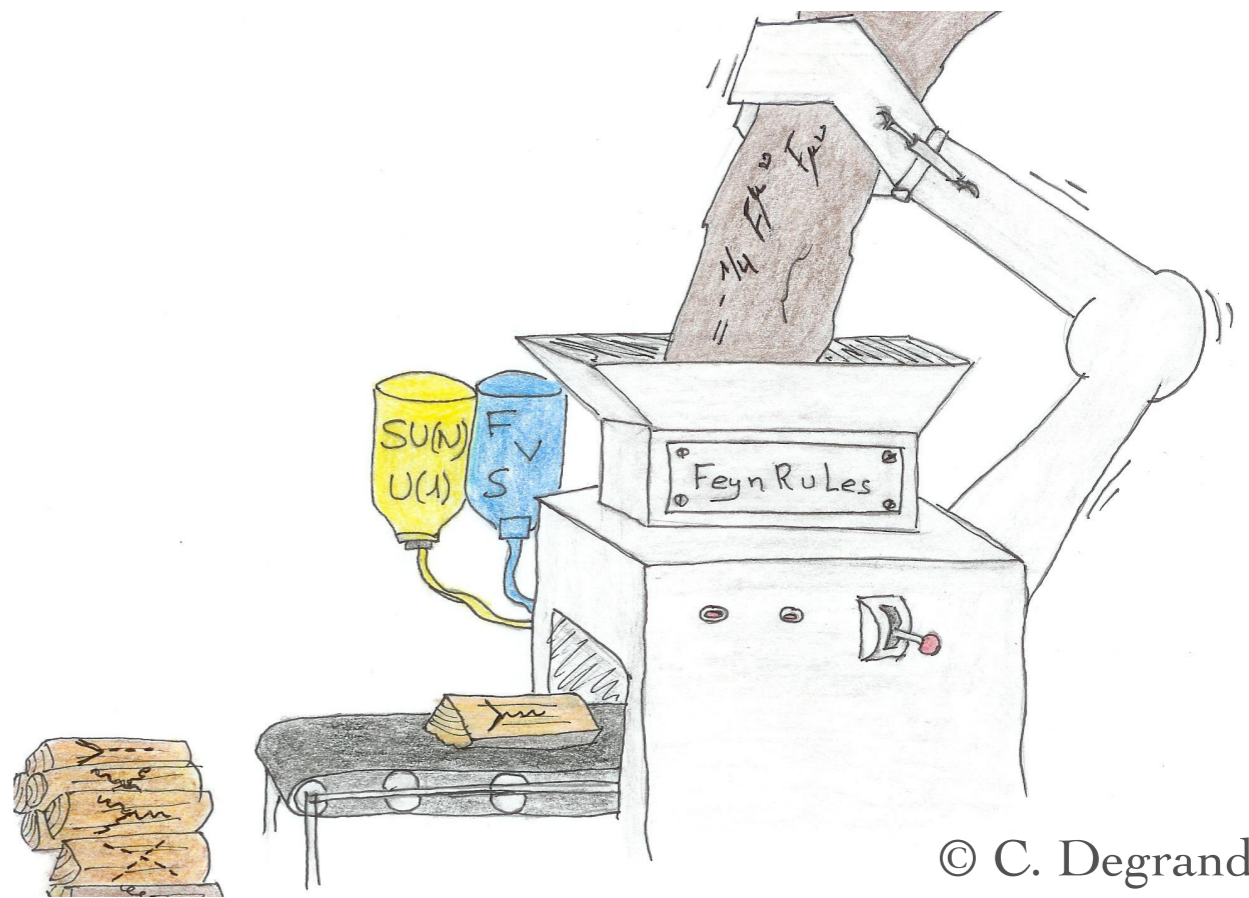
[Alloul, Christensen, Degrande, CD, Fuks]

- The only requirements on the Lagrangian are:
  - ➔ All indices need to be contracted (Lorentz and gauge invariance).
  - ➔ CPT invariance ( $\sim$  'normal' particle/anti-particle relation).
  - ➔ Locality.
  - ➔ Supported field types: spin 0, 1/2, 1, 3/2, 2 & ghosts.



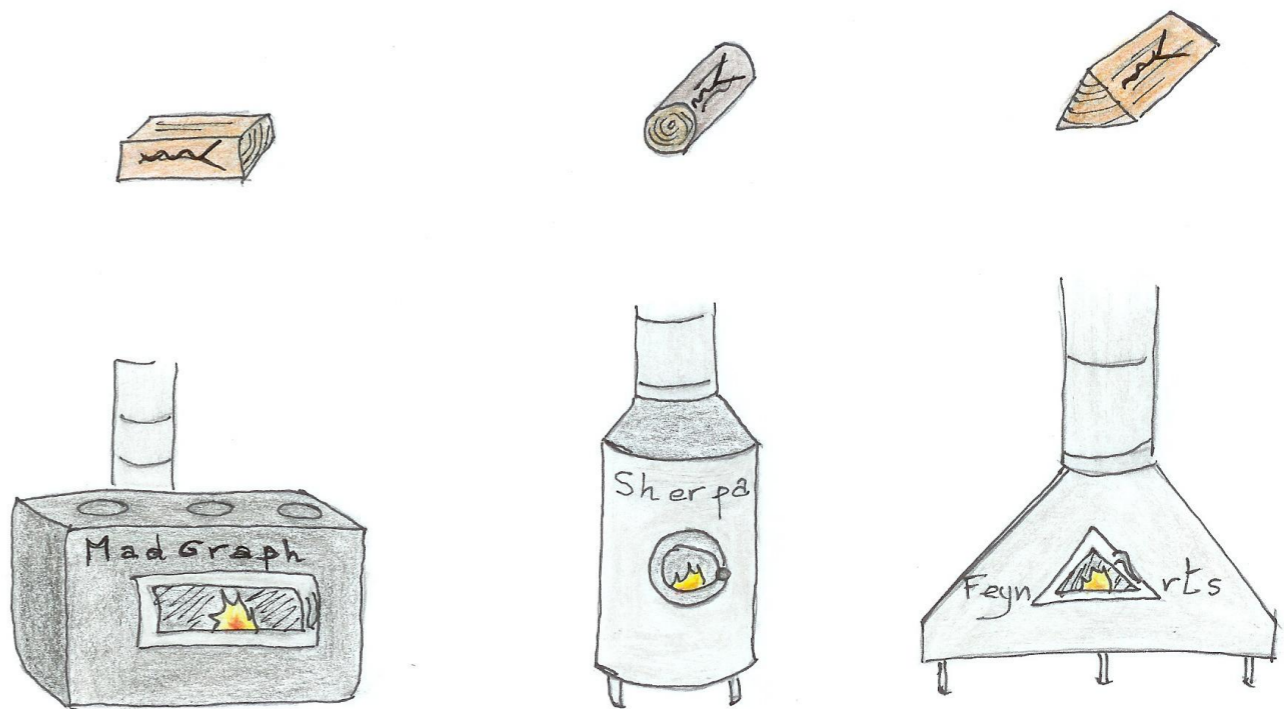
# FeynRules

- FeynRules comes with a set of interfaces, that allow to export the Feynman rules to various matrix element generators.
- Interfaces coming with current public version
  - ➔ CalcHep / CompHep
  - ➔ FeynArts / FormCalc
  - ➔ MadGraph
  - ➔ Sherpa
  - ➔ Whizard / Omega



# FeynRules

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  - ➔ Whizard / Omega



# FeynRules

- The input requested from the user is twofold.

- **The Model File:**

Definitions of particles and parameters (e.g., a quark)

F[1] ==

```
{ClassName      -> q,  
SelfConjugate  -> False,  
Indices        -> {Index[Colour]},  
Mass           -> {MQ, 200},  
Width          -> {WQ, 5} }
```

- **The Lagrangian:**

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + i\bar{q} \gamma^\mu D_\mu q - M_q \bar{q} q$$

L =

```
-1/4 FS[G,mu,nu,a] FS[G,mu,nu,a]  
+ I qbar.Ga[mu].DC[q,mu]  
- MQ qbar.q
```

# FeynRules

- Once this information has been provided, FeynRules can be used to compute the Feynman rules for the model:

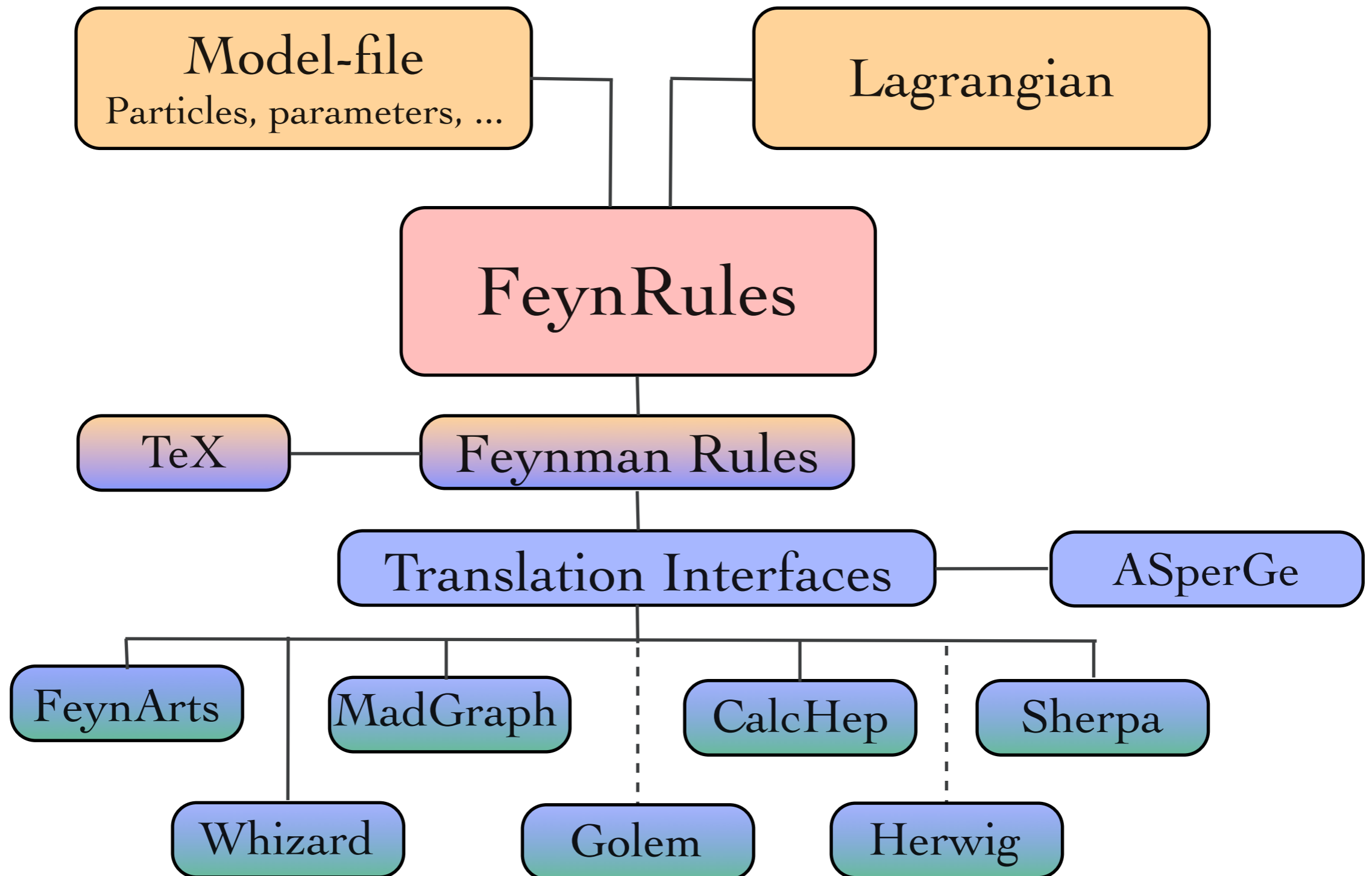
`FeynmanRules[ L ]`

- Equivalently, we can export the Feynman rules to a matrix element generator, e.g., for MadGraph 5,

`WriteUFO[ L ]`

- This produces a set of files that can be directly used in the matrix element generator (“plug ‘n’ play”).

# FeynRules: a quick overview



# References

- The FeynRules website: <http://feynrules.phys.ucl.ac.be>
- The FeynRules manual:  
N. D. Christensen, CD, CPC 180 (2009) 1614-1641,  
[arXiv:0806.4194]
- The FeynRules superspace module:  
CD, B. Fuks, CPC 181 (2011) 2404-2426, [arXiv:1102.4191]
- The UFO format:  
C. Degrande, CD, B. Fuks, D. Grellscheid, O. Mattelaer, T. Reiter,  
CPC 183 (2012) 1201-1214, [arXiv:1108.2040]
- ASperGe:  
A. Alloul, J d'Hondt, K. de Causmaecker, B. Fuks, M. Rausch de  
Traubenberg, Eur. Phys. J. C73 (2013) 2325, [arXiv:1301.5932]

# Getting Started: phi4 theory

# Phi 4 theory

- Let us consider a model consisting of two complex scalar fields, interacting with each other:

$$\mathcal{L} = \partial_\mu \phi_i^\dagger \partial^\mu \phi_i - m^2 \phi_i^\dagger \phi_i + \lambda (\phi_i^\dagger \phi_i)^2$$

- We need to implement into a FeynRules model file
  - ➔ The two fields  $\phi_1$  and  $\phi_2$ , or rather one field carrying an index.
  - ➔ The two new parameters  $m$  and  $\lambda$ .
- In a second step, we need to implement the Lagrangian into Mathematica.



# How to write a model file

- A model file is simply a text file (with extension *.fr*).
- The syntax is *Mathematica*.
- General structure:

## Preamble

(Author info, model info, index definitions, ... )

## Particle Declarations

(Particle class definitions, spins, quantum numbers, ...)

## Parameter Declarations

(Numerical Values, ...)

# Preamble of the model file

- The preamble allows to ‘personalize’ the model file, and define all the indices that are carried by the fields
  - ➔ In our case we have one index, taking the values 1 or 2.

```
M$ModelName = "Phi_4_Theory";
```

```
M$Information = {Authors -> {"C. Duhr"},  
                 Version -> "1.0",  
                 Date -> "09. 09. 2011"};
```

```
IndexRange[ Index[Scalar] ] = Range[2];  
IndexStyle[ Scalar, i];
```

# Preamble of the model file

- Sometimes it is useful to introduce auxiliary indices to obtain compact Lagrangians, but these indices should always be expanded.
  - ➔ **Example:** Weak isospin indices.
- There is a way to instruct FeynRules at run time to expand certain indices (see later).

```
IndexRange[ Index[Scalar] ] = Range[2];  
IndexStyle[ Scalar, i];
```

# Preamble of the model file

- Sometimes it is useful to introduce auxiliary indices to obtain compact Lagrangians, but these indices should always be expanded.
  - ➔ **Example:** Weak isospin indices.
- There is a way to instruct FeynRules at run time to expand certain indices (see later).
- In addition, one can specify in the model file if a certain type of indices should **always** be expanded:

```
IndexRange[ Index[Scalar] ] = Range[2];  
IndexStyle[ Scalar, i];
```

# Preamble of the model file

- Sometimes it is useful to introduce auxiliary indices to obtain compact Lagrangians, but these indices should always be expanded.
  - ➔ **Example:** Weak isospin indices.
- There is a way to instruct FeynRules at run time to expand certain indices (see later).
- In addition, one can specify in the model file if a certain type of indices should **always** be expanded:

```
IndexRange[ Index[Scalar] ] = Unfold[ Range[2] ];  
IndexStyle[ Scalar, i];
```

# Particle Declaration

- Particles are defined as 'classes', grouping together particles with similar quantum numbers, but different masses (~multiplet).

```
M$ClassesDescription = {  
  S[1] == {  
    ClassName -> phi,  
    ClassMembers -> {phi1,phi2},  
    SelfConjugate -> False,  
    Indices -> {Index[Scalar]},  
    FlavorIndex -> Scalar,  
    Mass -> {MS, 100}  
  }  
};
```

# Particle Declaration

- Particles are defined as 'classes', grouping together particles with similar quantum numbers, but different masses (~multiplet).

```
M$ClassesDescription = {  
  S[1] == { Spin (S, F, V, U, T)  
    ClassName -> phi,  
    ClassMembers -> {phi1,phi2},  
    SelfConjugate -> False,  
    Indices -> {Index[Scalar]},  
    FlavorIndex -> Scalar,  
    Mass -> {MS, 100}  
  }  
};
```

# Particle Declaration

- Particles are defined as 'classes', grouping together particles with similar quantum numbers, but different masses (~multiplet).

```
M$ClassesDescription = {
```

```
  S[1] == {
```

```
    ClassName -> phi,
```

```
    ClassMembers -> {phi1,phi2},
```

```
    SelfConjugate -> False,
```

```
    Indices -> {Index[Scalar]},
```

```
    FlavorIndex -> Scalar,
```

```
    Mass -> {MS, 100}
```

```
  }
```

```
};
```

Symbol used for the  
particle in the Lagrangian.

Antiparticle called  
phibar.



# Particle Declaration

- Particles are defined as 'classes', grouping together particles with similar quantum numbers, but different masses (~multiplet).

```
M$ClassesDescription = {  
  S[1] == {  
    ClassName -> phi,  
    ClassMembers -> {phi1,phi2},  
    SelfConjugate -> False,  
    Indices -> {Index[Scalar]},  
    FlavorIndex -> Scalar,  
    Mass -> {MS, 100}  
  }  
};
```

The field is complex, i.e.,  
there is an antiparticle.

# Particle Declaration

- Particles are defined as 'classes', grouping together particles with similar quantum numbers, but different masses (~multiplet).

```
M$ClassesDescription = {
```

```
  S[1] == {
```

```
    ClassName -> phi,
```

```
    ClassMembers -> {phi1,phi2},
```

```
    SelfConjugate -> False,
```

```
    Indices -> {Index[Scalar]},
```

```
    FlavorIndex -> Scalar,
```

```
    Mass -> {MS, 100}
```

```
  }
```

```
};
```

Symbol for the mass  
used in the Lagrangian,  
+ numerical value in GeV.

# Particle Declaration

- There are many more (optional) properties for particle classes:
  - ➔ **Width**: Total width of the particle. 0 if stable.
  - ➔ **QuantumNumbers**: U(1) charges carried by the field.
  - ➔ **PDG**: PDG code of the particle (if existent).
  - ➔ **ParticleName/AntiParticleName**: A string, by which the particle will be referred to in the MC code.
  - ➔ **Unphysical**: If True, then the particle is tagged as not a mass eigenstate, and will not be output to the MC code.
  - ➔ Many more. See the FeynRules manual.

# Parameter Declaration

- Parameter classes are defined in a similar way to the particle classes.
  - ➔ In our case, we have two parameters, the mass  $m$  and the coupling  $\lambda$ .
  - ➔ The mass was already defined with the particle, no need to define it a second time.

```
M$Parameters = {  
  lam == {  
    Value -> 0.1  
  }  
};
```

# Parameter Declaration

- Parameters belong to two different classes, specified by the option **ParameterType**:

- ➔ **External**: Numerical input parameters of the model.  
The **Value** must be a **real** floating point number.

Example:

$$\alpha_s = 0.118$$

- ➔ **Internal**: Dependent on other external and/or internal parameters. The **Value** can be a floating point number or an algebraic expression (in Mathematica syntax).

Example:

$$g_s = \sqrt{4\pi\alpha_s}$$

- By default every new parameter is **External**.

# Parameter Declaration

- By default, all parameters are defined as real. It can be made complex by setting the `ComplexParameter` option to `True`.

# Parameter Declaration

- By default, all parameters are defined as real. It can be made complex by setting the `ComplexParameter` option to `True`.
- Just like particles, parameters can carry `Indices`, i.e., they can be matrices
- It is possible to specify that a matrix is hermitian, etc.
  - ➔ `Hermitian`: `True/False`.
  - ➔ `Orthogonal`: `True/False`.
  - ➔ `Unitary`: `True/False`.

# The *Mathematica* session

- We now run `FeynRules` to obtain the Feynman rules of the model
  - ➔ This is done in a *Mathematica* notebook.
- **Step 1:** Load `FeynRules` into *Mathematica*

```
In[1]:= $FeynRulesPath = SetDirectory["~/FeynRules-SVN/feynrules-current"];
```

```
In[2]:= << FeynRules`
```



# The *Mathematica* session

- We now run `FeynRules` to obtain the Feynman rules of the model
  - ➔ This is done in a *Mathematica* notebook.
- **Step 1:** Load `FeynRules` into *Mathematica*

```
In[1]:= $FeynRulesPath = SetDirectory["~/FeynRules-SVN/feynrules-current"];
```

```
In[2]:= << FeynRules`
```

```
– FeynRules –
```

```
Authors: C. Duhr, N. Christensen, B. Fuks
```

```
Please cite: Comput.Phys.Commun.180:1614–1641,2009 (arXiv:0806.4194).
```

```
http://feynrules.phys.ucl.ac.be
```

# The *Mathematica* session

- Step 2: Load the model file

```
In[3]:= SetDirectory["~/FeynRules-SVN/trunk/models/Phi_4_Theory"];
```

```
In[4]:= LoadModel["Phi_4_Theory.fr"]
```

# The *Mathematica* session

- Step 2: Load the model file

```
In[3]:= SetDirectory["~/FeynRules-SVN/trunk/models/Phi_4_Theory"];
```

```
In[4]:= LoadModel["Phi_4_Theory.fr"]
```

This model implementation was created by

C. Duhr

Model Version: 1.0

For more information, type `ModelInformation[]`.

# The Mathematica session

- Step 3: Enter the Lagrangian

$$\mathcal{L} = \partial_{\mu} \phi_i^{\dagger} \partial^{\mu} \phi_i - m^2 \phi_i^{\dagger} \phi_i + \lambda (\phi_i^{\dagger} \phi_i)^2$$

```
In[5]:= L = del [phibar[i], mu] del [phi[i], mu] - MS ^ 2 phibar[i] phi[i] +  
lam (phibar[i] phi[i]) (phibar[j] phi[j])
```

```
Out[5]= lam phi_i phi_j phi_i^{\dagger} phi_j^{\dagger} + MS^2 (-phi_i) phi_i^{\dagger} + \partial_{\mu}(phi_i) \partial_{\mu}(phi_i^{\dagger})
```

# The *Mathematica* session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

# The *Mathematica* session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
Starting Feynman rule calculation.
```

```
Collecting the different structures that enter the vertex...
```

```
Found 1 possible non zero vertices.
```

```
Start calculating vertices...
```



```
1 vertex obtained.
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(* * * * * *)
```

```
Vertex 1
```

```
Particle 1 : Scalar , phi
```

```
Particle 2 : Scalar , phi
```

```
Particle 3 : Scalar , phi†
```

```
Particle 4 : Scalar , phi†
```

```
Vertex:
```

$$2 i \text{ lam } \delta_{i_1, i_4} \delta_{i_2, i_3} + 2 i \text{ lam } \delta_{i_1, i_3} \delta_{i_2, i_4}$$

```
(* * * * * *)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(* * * * * *)
```

```
Vertex 1
```

```
Particle 1 : Scalar , phi
```

```
Particle 2 : Scalar , phi
```

```
Particle 3 : Scalar , phi†
```

```
Particle 4 : Scalar , phi†
```

```
Vertex:
```

$$2 i \text{ lam } \delta_{i_1, i_4} \delta_{i_2, i_3} + 2 i \text{ lam } \delta_{i_1, i_3} \delta_{i_2, i_4}$$

```
(* * * * * *)
```

Feynman rule for  
the particle class!



# The *Mathematica* session

- Step 4: Computing the Feynman rules

```
In[7]:= FeynmanRules [L, FlavorExpand → True]
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[7]:= FeynmanRules[L, FlavorExpand → True]
```

```
(*****)
```

```
Vertex 1
```

```
Particle 1 : Scalar , phi1
```

```
Particle 2 : Scalar , phi1
```

```
Particle 3 : Scalar , phi1†
```

```
Particle 4 : Scalar , phi1†
```

```
Vertex:
```

```
4 i lam
```

```
(*****)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[7]:= FeynmanRules[L, FlavorExpand → True]
```

```
(* * * * * *)
```

```
Vertex 2
```

```
Particle 1 : Scalar , phi1
```

```
Particle 2 : Scalar , phi1†
```

```
Particle 3 : Scalar , phi2
```

```
Particle 4 : Scalar , phi2†
```

```
Vertex:
```

```
2 i lam
```

```
(* * * * * *)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[7]:= FeynmanRules [L, FlavorExpand → True]  
      (* * * * * *)  
Vertex 3  
Particle 1 : Scalar , phi2  
Particle 2 : Scalar , phi2  
Particle 3 : Scalar , phi2†  
Particle 4 : Scalar , phi2†  
Vertex:  
4 i lam  
      (* * * * * *)
```

# The *Mathematica* session

- A selection of options for the `FeynmanRules` function:
  - ➔ **FlavorExpand**: List of all flavor indices that should be expanded. If `True`, then all flavor indices are expanded.
  - ➔ **ScreenOutput**: If `False`, the vertices are not printed on screen (useful for big models with 100's of vertices).
  - ➔ **SelectParticles**: Allows to only compute certain specific vertices.
  - ➔ **MaxParticles/MinParticles**: an integer, specifying the maximal/minimal number of particles that should appear in a vertex.
  - ➔ **Exclude4Scalars**: If `True`, rejects all four-scalar vertices (useful for big models with a plethora of phenomenologically irrelevant four-scalar interactions).

# Getting Started: Gauging our model

# Gauging phi<sup>4</sup> theory

- Let us gauge our model, say the scalar is in the adjoint of SU(3) (QCD octet).
- The change in the Lagrangian is very minor:
  - ➔ add field strength tensor
  - ➔ replace derivative by covariant derivative.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + D_\mu\phi_i^\dagger D^\mu\phi_i - m^2\phi_i^\dagger\phi_i + \lambda(\phi_i^\dagger\phi_i)^2$$

$$D_\mu = \partial_\mu - ig_s T^a G_\mu^a$$

- Technically speaking, we just added two new objects to our model:
  - ➔ a new particle: the gluon  $G$ .
  - ➔ a new parameter: the gauge coupling  $g_s$ .

# Preamble of the model file

- The fields now carry an index in the adjoint index.
  - ➔ Need to define this new index in the preamble.

```
M$ModelName = "Phi_4_Theory_Octet";
```

```
M$Information = {Authors -> {"C. Duhr"},  
                 Version -> "1.0",  
                 Date -> "09. 09. 2011"};
```

```
IndexRange[ Index[Scalar] ] = Range[2];
```

```
IndexStyle[ Scalar, i];
```

```
IndexRange[ Index[Gluon] ] = Range[8];
```

```
IndexStyle[ Gluon, a];
```



# Particle Declaration

- The scalar is now an octet.

```
M$ClassesDescription = {  
  S[1] == {  
    ClassName -> phi,  
    ClassMembers -> {phi1,phi2},  
    SelfConjugate -> False,  
    Indices -> {Index[Scalar], Index[Gluon]},  
    FlavorIndex -> Scalar,  
    Mass -> {MS, 100}  
  }  
};
```

# Particle Declaration

- We also need to define the gluon field.

```
M$ClassesDescription = {  
  S[1] == {...},  
  
  V[1] == {  
    ClassName -> G,  
    SelfConjugate -> True,  
    Indices -> {Index[Gluon]},  
    Mass -> 0  
  }  
};
```

# Parameter Declaration

- We also need to define the gauge coupling.

```
M$Parameters = {  
  lam == {  
    Value -> 0.1  
  },  
  
  gs == {  
    Value -> 1.22  
  }  
};
```

# Gauge groups

- We have now defined the gauge coupling and the gauge boson.
- To gauge the theory we need however more:
  - ➔ Structure constants.
  - ➔ Representation matrices.
  - ➔ ...
- FeynRules allows to define gauge group classes in a similar way to particle and parameter classes.

# Gauge groups

- FeynRules allows to define gauge group classes in a similar way to particle and parameter classes.

```
M$GaugeGroups = {  
  
  SU3C == {  
    Abelian -> False,  
    GaugeBoson -> G,  
    StructureConstant -> f,  
    CouplingConstant -> gs  
  }  
}
```

- Could add other representations via  
Representation -> {T, Colour}

# The Mathematica session

- Step 1: Load FeynRules into Mathematica
- Step 2: Load the model file
- Step 3: Enter the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + D_\mu \phi_i^\dagger D^\mu \phi_i - m^2 \phi_i^\dagger \phi_i + \lambda (\phi_i^\dagger \phi_i)^2$$

```
In[9]:= L = -1 / 4 FS[G, mu, nu, a] FS[G, mu, nu, a] +
          DC[phibar[i, a], mu] DC[phi[i, a], mu] - MS^2 phibar[i, a] phi[i, a] +
          lam (phibar[i, a] phi[i, a]) (phibar[j, b] phi[j, b])
```

```
Out[9]= (∂mu(phii,a) - i gs Gmu,a phii,i FSU3Ca,ia$979) (∂mu(phii,a†) + i gs Gmu,a FSU3Ci$978,aa$978 phii,i†) +
          lam phii,a phij,b phii,a† phij,b† - 1/4 (gs Gmu,bb$976 Gnu,cc$976 fa,bb$976,cc$976 - ∂nu(Gmu,a) + ∂mu(Gnu,a))
          (gs Gmu,bb$977 Gnu,cc$977 fa,bb$977,cc$977 - ∂nu(Gmu,a) + ∂mu(Gnu,a)) + MS^2 (-phii,a) phii,a†
```

# The *Mathematica* session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(* ***)
```

```
Vertex 1
```

```
Particle 1 : Vector , G
```

```
Particle 2 : Vector , G
```

```
Particle 3 : Vector , G
```

```
Vertex:
```

$$g_s p_1^{\mu_3} f_{a_1, a_2, a_3} \eta_{\mu_1, \mu_2} - g_s p_2^{\mu_3} f_{a_1, a_2, a_3} \eta_{\mu_1, \mu_2} - g_s p_1^{\mu_2} f_{a_1, a_2, a_3} \eta_{\mu_1, \mu_3} + \\ g_s p_3^{\mu_2} f_{a_1, a_2, a_3} \eta_{\mu_1, \mu_3} + g_s p_2^{\mu_1} f_{a_1, a_2, a_3} \eta_{\mu_2, \mu_3} - g_s p_3^{\mu_1} f_{a_1, a_2, a_3} \eta_{\mu_2, \mu_3}$$

```
(* ***)
```



# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(*****)
```

```
Vertex 2
```

```
Particle 1 : Vector , G
```

```
Particle 2 : Vector , G
```

```
Particle 3 : Vector , G
```

```
Particle 4 : Vector , G
```

```
Vertex:
```

$$\begin{aligned} & i g s^2 \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} f_{a_1, a_3, a_1} f_{a_2, a_4, a_1} + i g s^2 \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} f_{a_1, a_2, a_1} f_{a_3, a_4, a_1} + \\ & i g s^2 \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} f_{a_1, a_4, a_1} f_{a_2, a_3, a_1} - i g s^2 \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} f_{a_1, a_2, a_1} f_{a_3, a_4, a_1} - \\ & i g s^2 \eta_{\mu_1, \mu_2} \eta_{\mu_3, \mu_4} f_{a_1, a_4, a_1} f_{a_2, a_3, a_1} - i g s^2 \eta_{\mu_1, \mu_2} \eta_{\mu_3, \mu_4} f_{a_1, a_3, a_1} f_{a_2, a_4, a_1} \end{aligned}$$

```
(*****)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(* * * * * *)
```

```
Vertex 3
```

```
Particle 1 : Vector , G
```

```
Particle 2 : Scalar , phi
```

```
Particle 3 : Scalar , phi†
```

```
Vertex:
```

$$g_S p_3^{\mu_1} f_{a_3, a_1, a_2} \delta_{i_2, i_3} - g_S p_2^{\mu_1} f_{a_3, a_1, a_2} \delta_{i_2, i_3}$$

```
(* * * * * *)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(*****)
```

```
Vertex 4
```

```
Particle 1 : Vector , G
```

```
Particle 2 : Vector , G
```

```
Particle 3 : Scalar , phi
```

```
Particle 4 : Scalar , phi†
```

```
Vertex:
```

$$i g s^2 \eta_{\mu_1, \mu_2} \delta_{i_3, i_4} f_{a_1, a_4, a_1} f_{a_2, a_3, a_1} + i g s^2 \eta_{\mu_1, \mu_2} \delta_{i_3, i_4} f_{a_1, a_3, a_1} f_{a_2, a_4, a_1}$$

```
(*****)
```

# The Mathematica session

- Step 4: Computing the Feynman rules

```
In[6]:= FeynmanRules [L]
```

```
(*****  
Vertex 5  
Particle 1 : Scalar , phi  
Particle 2 : Scalar , phi  
Particle 3 : Scalar , phi†  
Particle 4 : Scalar , phi†  
Vertex:  
2 i lam δa1,a4 δa2,a3 δi1,i4 δi2,i3 + 2 i lam δa1,a3 δa2,a4 δi1,i3 δi2,i4  
*****)
```

# Getting Started: Mixings

# Mixings

- So far our model has the following form:

$$\mathcal{L} = D_\mu \phi_i^\dagger D^\mu \phi_i - m^2 \phi_i^\dagger \phi_i + \lambda (\phi_i^\dagger \phi_i)^2$$

# Mixings

- So far our model has the following form:

$$\mathcal{L} = D_\mu \phi_i^\dagger D^\mu \phi_i - m^2 \phi_i^\dagger \phi_i + \lambda (\phi_i^\dagger \phi_i)^2$$

- In many BSM models the new fields are not mass eigenstates, but they mix, e.g.

$$\mathcal{L} = D_\mu \phi_i^\dagger D^\mu \phi_i - m^2 \phi_i^\dagger \phi_i - m_{12}^2 (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) + \lambda (\phi_i^\dagger \phi_i)^2$$

- The gauge and mass eigenstates are then related via some unitary rotation,

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

# Mixings

- FeynRules offers the possibility to write the Lagrangian in terms of the gauge eigenstates, and let Mathematica perform the rotation.
- **N.B.:** There is a way to let FeynRules diagonalize the mass matrices.
  - ➔ More on this in tomorrow's lecture!
- For small mixing matrices, this can simply be done in Mathematica.
- For larger matrices, need to use some external numerical code.



# Mixings

- The mixing matrix is declared as a parameter:

```
M$Parameter = {  
  ...  
  
  UU == {  
    ComplexParameter -> True,  
    Unitary -> True  
    Indices -> {Index[Scalar], Index[Scalar]},  
    Value -> { UU[1,1] -> ...,  
              UU[1,2] -> ...,  
              ...}  
  }  
  ...  
};
```

# Mixings

- The mass eigenstates are declared as normal particles

```
M$ClassesDescription = {  
  ....  
  S[11] == {  
    ClassName      -> PP,  
    ClassMembers  -> {PP1,PP2},  
    SelfConjugate  -> False,  
    Indices        -> {Index[Scalar], Index[Gluon]},  
    FlavorIndex    -> Scalar,  
    Mass           -> {{MP1, ...}, {MP2, ...}}  
  }  
  ...  
};
```

# Mixings

- The gauge eigenstates are declared in a similar way

```
M$ClassesDescription = {  
  S[1] == {  
    ClassName      -> phi,  
    ClassMembers  -> {phi1,phi2},  
    SelfConjugate  -> False,  
    Indices        -> {Index[Scalar], Index[Gluon]},  
    FlavorIndex    -> Scalar,  
    Mass          -> {MS, 100}  
    Unphysical     -> True,  
    Definitions    -> {phi[i_, a_] :=> Module[{j}, UU[i,j] PP[j,a]]}  
  }  
};
```

# Towards LHC phenomenology: The FeynRules interfaces

# The Interfaces

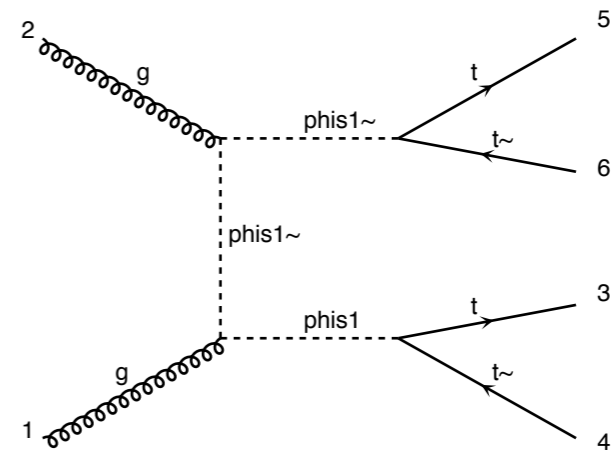
- So far we have only discussed how to implement a model into FeynRules and how to obtain the vertices.
- Next we want to do phenomenology!
- FeynRules contains interfaces to the following codes:
  - ➔ CalcHep / CompHep
  - ➔ FeynArts / FormCalc
  - ➔ MadGraph
  - ➔ Sherpa
  - ➔ Whizard / Omega
- Each interface produces a set of text files that can be read into the existing generators.

# Running Interfaces

- The interfaces are called via the Mathematica commands

<code>WriteCHOutput[ LSM, L ];</code>	<code>(* CalcHep *)</code>
<code>WriteFeynArtsOutput[ LSM, L ];</code>	<code>(* FeynArts/FormCalc *)</code>
<code>WriteMGOutput[ LSM, L ];</code>	<code>(* MadGraph 4 *)</code>
<code>WriteUFO[ LSM, L ];</code>	<code>(* UFO / MadGraph 5 *)</code>
<code>WriteSHOutput[ LSM, L ];</code>	<code>(* Sherpa *)</code>
<code>WriteWOOOutput[ LSM, L ];</code>	<code>(* Whizard / Omega *)</code>

- The files produced by FeynRules can then be processed by the matrix element generators.



# Running Interfaces

- **Important:** although FeynRules can obtain the vertices of very large classes of models, not every model can be output to every MC generator!
- Some interfaces to some generators have the color and / or Lorentz structures hardwired.

	Spins	Lorentz	Color
CalcHep	0,1/2,1,2	~all	1,3,8 (limited)
FeynArts	0,1/2,1	all	all
MadGraph	0,1/2,1,3/2,2	~all	1,3,6,8
Sherpa	0,1/2,1	SM - like	1,3,8
Whizard	0,1/2,1,2	MSSM - like	1,3,8

**N.B.:** These limitations apply to the FeynRules interfaces. Some generators allow for more general structures that are however not implemented into the interface.

# The UFO



UFO = Universal FeynRules Output

- **Idea:** Create Python modules that can be linked to other codes and contain all the information on a given model.
- The UFO is a self-contained Python code, and not tied to a specific matrix element generator.
- The content of the FR model files, together with the vertices, is translated into a library of Python objects, that can be linked to other codes.
- By design, the UFO does not make any assumptions on Lorentz/color structures, or the number of particles.
- GoSam and MadGraph 5 use the UFO as the default model format for BSM, Herwig++ will use it in the future.



# The UFO & ALOHA

- A neat application...

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- A neat application... in supergravity!

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$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + \lambda f^{abc} \text{Tr}(F_{\mu\nu}^a F_b^{\nu\rho} F_{\rho\mu}^c)$$

- Broedel and Dixon had derived a CSW construction for the color-ordered helicity amplitudes, but had no way to check the validity of the construction.

# The UFO & ALOHA

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- Broedel and Dixon had derived a CSW construction for the color-ordered helicity amplitudes, but had no way to check the validity of the construction.
- **Solution:**
  - ➔ Put it into FeynRules, and let it run for a long time...
  - ➔ Get the UFO, and put it into MadGraph 5.
  - ➔ Hack matrix.f to read out the color-ordered helicity amplitudes for individual phase space points.



# The UFO & ALOHA

```
WWW42 = Lorentz(name = 'VVVV42',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,5)*Metric(1,3)*Metric(2,5) - P(1,5)*Metric(2,5)*Metric(3,4) - P(4,5)*Metric(1,2)*Metric(3,5) + P(1,5)*Metric(2,4)*Metric(3,5)')
WWW43 = Lorentz(name = 'VVVV43',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,1)*Metric(1,4)*Metric(2,3) - P(3,1)*Metric(1,4)*Metric(2,5) - P(5,1)*Metric(1,2)*Metric(3,4) + P(3,1)*Metric(1,2)*Metric(4,5)')
WWW44 = Lorentz(name = 'VVVV44',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,1)*Metric(1,5)*Metric(2,3) - P(3,1)*Metric(1,5)*Metric(2,4) - P(4,1)*Metric(1,2)*Metric(3,5) + P(3,1)*Metric(1,2)*Metric(4,5)')
WWW45 = Lorentz(name = 'VVVV45',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,2)*Metric(1,3)*Metric(2,4) - P(3,2)*Metric(1,5)*Metric(2,4) - P(5,2)*Metric(1,2)*Metric(3,4) + P(3,2)*Metric(1,2)*Metric(4,5)')
WWW46 = Lorentz(name = 'VVVV46',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,2)*Metric(1,3)*Metric(2,5) - P(3,2)*Metric(1,4)*Metric(2,5) - P(4,2)*Metric(1,2)*Metric(3,5) + P(3,2)*Metric(1,2)*Metric(4,5)')
WWW47 = Lorentz(name = 'VVVV47',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,1)*Metric(1,5)*Metric(2,3) - P(4,1)*Metric(1,3)*Metric(2,5) - P(2,1)*Metric(1,5)*Metric(3,4) + P(2,1)*Metric(1,3)*Metric(4,5)')
WWW48 = Lorentz(name = 'VVVV48',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,1)*Metric(1,4)*Metric(2,3) - P(5,1)*Metric(1,3)*Metric(2,4) - P(2,1)*Metric(1,4)*Metric(3,5) + P(2,1)*Metric(1,3)*Metric(4,5)')
WWW49 = Lorentz(name = 'VVVV49',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,3)*Metric(1,3)*Metric(2,4) - P(5,3)*Metric(1,2)*Metric(3,4) + P(2,3)*Metric(1,5)*Metric(3,4) - P(2,3)*Metric(1,3)*Metric(4,5)')
WWW50 = Lorentz(name = 'VVVV50',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,3)*Metric(1,3)*Metric(2,5) - P(4,3)*Metric(1,2)*Metric(3,5) + P(2,3)*Metric(1,4)*Metric(3,5) - P(2,3)*Metric(1,3)*Metric(4,5)')
WWW51 = Lorentz(name = 'VVVV51',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(3,4)*Metric(1,4)*Metric(2,5) - P(2,4)*Metric(1,4)*Metric(3,5) - P(3,4)*Metric(1,2)*Metric(4,5) + P(2,4)*Metric(1,3)*Metric(4,5)')
```

# The UFO & ALOHA

```
WWW42 = Lorentz(name = 'VVVV42',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,5)*Metric(1,3)*Metric(2,5) - P(1,5)*Metric(2,5)*Metric(3,4) - P(4,5)*Metric(1,2)*Metric(3,5) + P(1,5)*Metric(2,4)*Metric(3,5)')
WWW43 = Lorentz(name = 'VVVV43',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,1)*Metric(1,4)*Metric(2,3) - P(3,1)*Metric(1,4)*Metric(2,5) - P(5,1)*Metric(1,2)*Metric(3,4) + P(3,1)*Metric(1,2)*Metric(4,5)')
WWW44 = Lorentz(name = 'VVVV44',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,1)*Metric(1,5)*Metric(2,3) - P(3,1)*Metric(1,5)*Metric(2,4) - P(4,1)*Metric(1,2)*Metric(3,5) + P(3,1)*Metric(1,2)*Metric(4,5)')
WWW45 = Lorentz(name = 'VVVV45',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,2)*Metric(1,3)*Metric(2,4) - P(3,2)*Metric(1,5)*Metric(2,4) - P(5,2)*Metric(1,2)*Metric(3,4) + P(3,2)*Metric(1,2)*Metric(4,5)')
WWW46 = Lorentz(name = 'VVVV46',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,2)*Metric(1,3)*Metric(2,5) - P(3,2)*Metric(1,4)*Metric(2,5) - P(4,2)*Metric(1,2)*Metric(3,5) + P(3,2)*Metric(1,2)*Metric(4,5)')
WWW47 = Lorentz(name = 'VVVV47',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,3)*Metric(1,5)*Metric(2,3) - P(4,3)*Metric(1,3)*Metric(2,5) - P(2,1)*Metric(1,5)*Metric(3,4) + P(2,1)*Metric(1,3)*Metric(4,5)')
WWW48 = Lorentz(name = 'VVVV48',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,1)*Metric(1,4)*Metric(2,3) - P(3,2)*Metric(1,3)*Metric(2,4) - P(2,1)*Metric(1,4)*Metric(3,5) + P(2,1)*Metric(1,3)*Metric(4,5)')
WWW49 = Lorentz(name = 'VVVV49',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(5,3)*Metric(1,3)*Metric(2,4) - P(5,3)*Metric(1,2)*Metric(3,4) + P(2,3)*Metric(1,5)*Metric(3,4) - P(2,3)*Metric(1,3)*Metric(4,5)')
WWW50 = Lorentz(name = 'VVVV50',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(4,3)*Metric(1,3)*Metric(2,5) - P(4,3)*Metric(1,2)*Metric(3,5) + P(2,3)*Metric(1,4)*Metric(3,5) - P(2,3)*Metric(1,3)*Metric(4,5)')
WWW51 = Lorentz(name = 'VVVV51',
                spins = [ 3, 3, 3, 3, 3 ],
                structure = 'P(3,4)*Metric(1,4)*Metric(2,5) - P(2,4)*Metric(1,4)*Metric(3,5) - P(3,4)*Metric(1,2)*Metric(4,5) + P(2,4)*Metric(1,3)*Metric(4,5)')
```

All Helicity amplitudes  
agreed out of the box with the  
CSW construction!

# FeynRules MC conventions

- FeynRules itself does not make any assumption on the model, but its core is completely agnostic of any structure, like QCD, QED, etc.
- In order for the MC generator to function properly, they must be able to identify in each new model some standard information, like for example
  - ➔ Color and electric charges of particles.
  - ➔ Color structures of vertices.
  - ➔ Strong and weak coupling constant.
  - ➔ etc.
- Roughly speaking, each MC code needs the information on the SM parameters to be provided in a specific format.



# FeynRules MC conventions

- As a consequence, even though the FeynRules core is completely agnostic, the SM parameters must be entered following specific conventions.

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- The SM gauge groups must be defined in the same way as in the SM implementation, e.g., for QCD,
  - ➔ Fundamental representation matrices:  $T$
  - ➔ Structure constants:  $f$
  - ➔ Strong coupling:  $g_S$

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  - ➔ Fundamental representation matrices:  $T$
  - ➔ Structure constants:  $f$
  - ➔ Strong coupling:  $g_s$
- The SM input parameters should correspond to the SMINPUTS of the SUSY Les Houches Accord:

$$M_Z, \alpha_s, \alpha_{EW}^{-1}, G_F$$

# Interaction orders

- MadGraphs 'tags' coupling constants and counts how many coupling constants of a given type enter a diagram.
- This allows to select certain types of diagrams, e.g., for  $p p \rightarrow t \bar{t}$ 
  - ➔ purely QCD production,
  - ➔ purely EW production,
  - ➔ both QCD and EW production (including interference).
- This requires that each coupling constant has a 'counter' (= interaction order) defined.

# Interaction orders

- This can be done at the FeynRules level using the InteractionOrder property.

```
M$Parameters = {  
  lam == {  
    Value -> 0.1,  
    InteractionOrder -> {YUK, 1}  
  },  
  
  gs == {  
    Value -> 1.22,  
    InteractionOrder -> {QCD, 1}  
  }  
};
```

# Extending existing implementations

# Extending the SM

- So far we have only considered our model standalone.
- For LHC phenomenology, one usually wants a BSM model that is an extension of the SM.
- FeynRules offers the possibility to start from the SM model, and to add/change/remove particles and operators.
- For this, it is enough to load our new model together with the SM implementation:

```
LoadModel[ "SM.fr", "Phi_4_Gauged" ];
```

- Note that the 'parent model' should always be loaded first in order to ensure that everything is set up correctly.

**N.B.:** In the SM implementation, the gluon and the QCD gauge group are already defined, so no need to redefine them.

# Other available models

- The same procedure can be used to extend any other models.
- Many models can be downloaded from the FeynRules web page, and can serve as a start to implement new models (<http://feynrules.irmp.ucl.ac.be/>).
  - ➔ SM (+ extensions: 4th generation, diquarks, See-saw...).
  - ➔ MSSM, NMSSM, RPV-MSSM, MRSSM.
  - ➔ Extra dimensions: UED, LED, Higgsless, HEIDI.
  - ➔ Minimal walking Technicolor.



# Model database

**We encourage model builders writing order to make them useful to a comm FeynRules model database, please see**

- [✉ claude.duhr@durham.ac.uk](mailto:claude.duhr@durham.ac.uk)
- [✉ neil@hep.wisc.edu](mailto:neil@hep.wisc.edu)
- [✉ fuks@cern.ch](mailto:fuks@cern.ch)

## Available models

---

[Standard Model](#)

---

[Simple extensions of the SM \(9\)](#)

---

[Supersymmetric Models \(4\)](#)

---

[Extra-dimensional Models \(4\)](#)

---

[Strongly coupled and effective field theories \(4\)](#)

---

[Miscellaneous \(0\)](#)

---

# Summary

- Implementing a New Physics into a matrix element generator can be a tedious and error-prone task.
- FeynRules tries to remedy this situation by providing a Mathematica framework where a new model can be implemented starting directly from the Lagrangian.
- There are no restrictions on the model, except
  - ➔ Lorentz and gauge invariance
  - ➔ Locality
  - ➔ Spins: 0, 1/2, 1, 3/2, 2, ghosts
- Try it out on your favorite model!  
<http://feynrules.irmp.ucl.ac.be/>