Aspects of Bottom-up Hidden Sector Models

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Chapter 1

Introduction

The standard model (SM) of particle physics can explain an astounding variety of experimental observations, but it does not present a complete description of nature. Indeed, there are many compelling pieces of evidence for the existence of physics beyond the SM, and in particular, new physics is required to explain the small but finite SM neutrino masses, the origin of dark matter, and the primordial baryon asymmetry in the universe (BAU).

Given that the SM is an incomplete description of reality, it has to be either extended or replaced with some other, more fundamental theory of the universe. In general, this can be done using one of two approaches: First, a SM extension can be constructed "top-down", by postulating a specific set of fundamental principles that determine the shape of the theory. This approach is taken e.g. in super-symmetry or string theory. Alternatively, the SM can be extended "bottom-up", by adding minimal extensions that can help explain specific pieces of evidence for new physics. The main advantage of the bottom-up approach is that it remains largely agnostic about the underlying unified theory. Typically, a large variety of bottom-up SM extensions can explain any specific piece of evidence for new physics, so that one does not have to commit to any specific conception of nature. However, this flexibility is also a major disadvantage, since each of the available SM extensions has to be tested and falsified individually.

In recent years, there has been a push towards obtaining predictions that are valid for wider classes of bottom-up SM extensions rather than just individual models. Within the context of collider phenomenology, the standard model effective theory (SMEFT) has become a popular tool to study generic bottom-up SM extensions without collider-accessible hidden degrees of freedom [3, 4, 5, 6, 7, 8].

In my doctoral studies, I have investigated aspects of bottom-up hidden sector models by following two main lines of research:

In my first line of research, I have constructed portal effective theories (PET), in which the SM couples to generic hidden sectors via a single light, gauge-singlet mediator with spin $S = 0, 1/2, 1$ and mass at or below the electroweak scale. These PETs are fully general, making as few assumptions as possible about the internal structure of the hidden sector, or the types of permissible portal operators. In chapter 2, I present a general framework that I have developed for constructing generic PET Lagrangians. Applying this framework, I construct both electroweak and GeV scale PETs that couple the standard model to a wide range of potential hidden sectors. For the GeV scale PETs, I focus primarily on quark flavour changing transitions, and to account for these transitions, I include higher dimensional portal operators generated by virtual $W^{\pm}$ exchanges. In chapter 3, I apply the resulting GeV scale PETs in order to construct a generic PET chiral perturbation theory (χPT) Lagrangian. This Lagrangian captures general hidden-sector induced meson transitions, such as $\pi^0 \to \gamma\gamma_{\text{hidden}}$ or $K^+ \to \pi^+ S_{\text{hidden}}$, and it can be used to compute the corresponding transition amplitudes for low-energy fixed target experiments such as NA62, or SHiP.

In my second line of research, I have investigated aspects of type-I seesaw models, which are one specific example of a bottom-up SM extension with $n \geq 2$ sterile neutrinos that mix with the SM neutrinos.

In chapter 4 I study the prospect of observing lepton number violation (LNV) in sterile neutrino decays at colliders. The results in this chapter have been published in [2]. In principle, LNV is a generic feature of the type-I seesaw model, but there has been significant debate on whether *observable* LNV at colliders is possible in type-I seesaw models without fine-tuning. Using two distinct notions of fine-tuning, I derive both upper and lower bounds on the relative suppression of LNV in sterile neutrinos at colliders.

Finally, in chapter 5, I study the impact of finite temperature and spectator effects in leptogenesis with heavy sterile neutrinos, where a finite lepton asymmetry is generated via out-of-equilibrium reactions involving the sterile neutrinos. The results in this chapter have been published in [1], and they present a continuation of work that I have begun during my master thesis. To study finite temperature and spectator effects, I consider a minimal model of leptogenesis with two strongly hierarchical sterile neutrinos that couple to the same linear combination of SM lepton flavours. Using methods of non-equilibrium and thermal quantum field theory, I derive a novel set of fully relativistic fluid equations for high-scale leptogenesis and compute the reaction rates that appear in these equations.

Chapter 2

The Portal Effective Theory Framework

2.1 Philosophy and Limitations

In this chapter, I consider a general theory in which the standard model (= SM) is augmented by a generic hidden sector to which it couples via a single hidden mediator with a mass at or below the electroweak scale. The corresponding Lagrangian may be cast as

$$\mathcal{L}_{\text{PET}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{portal}} + \mathcal{L}_{\text{hidden}} \ , \tag{2.1}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, $\mathcal{L}_{\text{hidden}}$ is a generic hidden sector Lagrangian, and $\mathcal{L}_{\text{portal}}$ denotes a portal sector that contains all interactions involving both SM and hidden degrees of freedom.

The Lagrangian (2.1) may be thought of as the low-energy effective theory realization of some more fundamental ultraviolet (=UV) theory. In the most general case, the SM could couple to hidden sectors via multiple mediators with arbitrary quantum numbers and masses. If there are no light mediators with masses at or below the electroweak scale, the standard model effective theory (=SMEFT) Lagrangian can be used to constrain the coupling to hidden sectors in a way that is largely agnostic about the underlying UV theory[3, 4, 5, 6, 7, 8]. However, if there is at least one light mediator, it is not immediately obvious how to constrain the shape of the portal Lagrangian in a way that remains equally agnostic about the shape of the underlying UV theory. My goal is to provide a first step towards a less model dependent description of the coupling to light hidden sectors by constructing a set of generic portal Lagrangian that are intended to cover a wide range of popular hidden sector models.

In principle, there is no physical reason to suppose the SM only couples to a single light mediator rather than a set of multiple different mediators with different quantum numbers. Mainly, I restrict the discussion to a single mediator as a matter of feasibility. Since I would like to make as few assumptions as possible about the full theory, it is difficult to construct a generic portal Lagrangian that captures the physics of an arbitrary number of mediators, even if one specifies the quantum numbers of each mediator. However, restricting myself to a single mediator, it becomes possible to explicitly construct a most general portal Lagrangian by specifying the quantum numbers of the light mediator and the symmetries respected by the underlying full theory. For each mediator type, this generic portal Lagrangian then defines a "portal effective theory", or PET, that is suitable for studying the coupling of the SM to generic classes of hidden sector models in a way that remains largely, albeit not fully, agnostic about the internal structure of the hidden sector.

There is a further complication. At the electroweak scale, the full portal Lagrangian for each mediator has to involve all SM fields. At energies well below the electroweak scale, say for instance at center of mass energies $\sqrt{s} \lesssim 10\,\mathrm{GeV}$, it is necessary to integrate out the heavier SM degrees of freedom in order to compute observables at low energies. In addition, it may be appropriate to work within a SM effective theory, such as chiral perturbation theory [9, 10, 11], heavy quark effective theory [12, 13, 14], (potential) nonrelativistic quantum chromo dynamics (=pNRQCD) [15, 16, 17], or soft collinear effective theory [18, 19, 20]. In this case, the portal Lagrangian should not contain the SM degrees of freedom themselves, but rather the effective degrees of freedom present within each of the SM effective theories. For any given mediator type, this implies the existence of a whole class of PETs, one for each specific setup that is being considered.

The remainder of this chapter in organized as follows: In section 2.2, I introduce my general approach for constructing specific PET and review common techniques used for identifying minimal sets of effective operators without redundancies. In section 2.3, I construct generic electroweak scale portal Lagrangian for a selection of different mediator fields with spins 0, $1/2$, and 1, and in section 2.4, I use these electroweak scale PETs to construct a corresponding set of GeV scale portal Lagrangian that can be used as starting points for the derivation of more specific low energy PETs.

2.2 The Construction of General PET Lagrangians

2.2.1 General Approach

In general, a PET can be constructed by following the same general strategy as the construction of any effective field theory (=EFT). The first step is to identify the relevant degrees of freedom for the problem at hand, as well as the symmetries that need to be respected by the experimental predictions of the theory. Afterwards, the theory is constructed by writing down the most general partition function consistent with the required symmetries and degrees of freedom.

Within the PET framework, I assume that the portal Lagrangian contains only SM degrees of freedom, as well as a single light hidden mediator field. As a matter of practicality, I restrict myself to mediators of spin $s = 0$, $1/2$, and 1. One important subtlety is that a generic mediator field may contain more than a single degree of freedom, so that working with a single hidden mediator *field* may not be the same as working with a single hidden degree of freedom. For instance, adding a single complex scalar field to the theory is equivalent to adding two real-valued scalar fields, which corresponds to adding two hidden degrees of freedom. Similarly, adding a single Dirac fermion is equivalent to adding two Weyl fermions, where each Weyl fermions contains two hidden degrees of freedom. In both cases, the PET Lagrangian actually contains multiple light hidden degrees of freedom.

In terms of symmetries, I require that the most general PET Lagrangian is at least both Lorentz and gauge invariant. To remain fully general, I do not enforce any other a priori symmetries. In particular, I do not require P or CP invariance, as these symmetries are already broken within the SM. For the construction of the generic electroweak scale PETs, I assume that the portal Lagrangian is invariant with respect to the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_I \times \mathrm{U}(1)_Y$ SM gauge group and that the hidden sector is not charged with respect to the SM gauge group. However, for the construction of PETs below the electroweak scale, the strict requirement of invariance with respect to the full SM gauge group has to be relaxed due to the spontaneous $\mathrm{SU}(2)_I \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_V$ breaking of the electroweak symmetry. Indeed, the portal Lagrangians at low energies only have to be invariant with respect to the unbroken $\mathrm{SU}(3)_C \times \mathrm{U}(1)_V$ subgroup.

Another complication is that PET Lagrangians below the electroweak scale do not explicitly contain all SM degrees of freedom, as it is necessary to integrate out the heavier SM fields. This is especially important when working with an unphysical renormalization scheme such as MS or $\overline{MS}$, since heavy particles do not decouple in these schemes unless they are integrated out by hand.

2.2.2 Power Counting and Naive Dimensional Analysis

By construction, an effective theory has to contain all interactions that are consistent with its symmetries. However, the corresponding most general Lagrangian typically consists of an infinite tower of operators that are accompanied by an equally infinite number of free parameters. If the effective theory is meant to be useful in practice, there has to be some way to sort operators into finite sets that are sufficient for computing S-matrix elements at a specified accuracy. The usual way to think of this is that it has to be possible to expand the S-matrix elements in powers of a small parameter ϵ. In this view, each operator is associated with a power of ϵ, and the corresponding power counting scheme is taken to be part of the definition of the effective theory.

For example, consider a generic effective theory that constitutes the low-energy realization of a more fundamental UV theory with an additional high-energy scale Λ. In this case it is natural to expand S-matrix elements in powers of $\epsilon \equiv \sqrt{s}/\Lambda$, which defines a suitable power counting scheme with only a finite number of operators entering at each order in ϵ.

Within the PET framework, the portal interactions must be highly suppressed (e.g. by some small coupling or a large beyond the standard model (=BSM) mass scale) in order to remain consistent with experimental evidence. As in the generic case, I quantify this suppression in terms of a generic order parameter $\epsilon_{\mathrm{Pet}} \ll 1$, and by construction, I take all portal operators to be at least of order ϵ_{Pet} or higher. At the electroweak scale, higher dimensional portal operators should be suppressed by a corresponding UV scale that I take to be $\Lambda_{\mathrm{pet}} \equiv v/\epsilon_{\mathrm{Pet}}$. Thus, the most general portal Lagrangian can only contain operators of dimension $d = 5$ or less.

Portal operators with dimension $d > 5$ may appear at low energies, after the heavy SM fields have been integrated out. Expanding to leading order in $\epsilon_W \sim {}^{s}\!/\!_{M_W^2}$, the most general portal Lagrangian may now contain operators of dimension $d = 7$ or less. In addition to this dimensional suppression, higher dimensional portal operators may also be suppressed by loop factors $\sim {}^{1}\!/\!_{(4\pi)^2}$. The loop factors can be integrated into the power counting using "naive dimensional analysis" (=NDA) [21, 22, 23, 24]. NDA assumes that the low-energy effective theory Lagrangian may be cast as

$$\mathcal{L} = \mathcal{L}^{d\leq 4} + \sum_i \frac{C_i}{\Lambda^{d_i}} \hat{O}_i \ , \tag{2.2}$$

where Λ is the high-energy scale associated with the small momentum expansion in powers of $\epsilon \sim {}^{\sqrt{s}}\!/\!_{\Lambda}$, and the $\hat{O}_i$ denote some effective operators of dimension $d_i > 4$. In the version of [22], $\mathcal{L}^{d\leq 4}$ may contain some generic gauge interactions with coupling constants g_i, some generic Yukawa interactions with coupling constants y_i, some generic ϕ^3 interactions with coupling constants κ_i, and some generic ϕ^4 interactions with coupling constants λ_i. With this setup, the NDA powercounting of [22] stipulates that the Wilson coefficients C_i may be expected to be of order one, or smaller, if the $\hat{O}_i$ are normalized as

$$\begin{aligned} \hat{O}_i \sim \frac{\Lambda^4}{(4\pi)^2} \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{y}{4\pi}\right)^{N_y} \left(\frac{\kappa}{4\pi\Lambda}\right)^{N_\kappa} \\ \times \left(\frac{\lambda}{(4\pi)^2}\right)^{N_\lambda} \left(\frac{p^2}{\Lambda^2}\right)^{N_p} \left(\frac{4\pi\phi}{\Lambda}\right)^{N_\phi} \left(\frac{(4\pi)^2\psi\psi}{\Lambda^3}\right)^{2N_\psi} \ , \end{aligned} \tag{2.3}$$

where ϕ and ψ denote any of the respective bosonic and fermionic fields that are present within the effective theory, and the kinetic part of the Lagrangian is assumed to be canonically normalized. p^2 stands for any light mass scale, i.e. it includes both derivatives $\partial \sim p$ and light masses $m \sim p$.

The NDA power counting is self consistent in the sense that an arbitrary diagram with insertions of higher dimensional operators that are normalized according to (2.3) will also generate an operator with the appropriate normalization. That is, the Wilson coefficients mix as [22]

$$\delta C_i \sim \prod_j C_j \ , \tag{2.4}$$

which implies that the Wilson coefficients should satisfy $C_i \lesssim 1$, even if the underlying UV theory is strongly coupled [24]. However, if the underlying UV theory is weakly coupled, the Wilson coefficients may be much smaller than one, $C_i \ll 1$. In this case, the 4π power counting of naive dimensional analysis may be broken by strongly hierarchical values of the Wilson coefficients, which could potentially satisfy $4\pi C_i \ll C_j$ for certain $i \neq j$.

For the purpose of using NDA to count factors of 4π within PET, the electromagnetic coupling $e \sim \sqrt{4\pi\alpha}$ is the only SM coupling that has to be counted at low energies. Although the QCD coupling g_s formally has to be associated with a factor of $1/4\pi$, this does not generate any additional degrees of smallness due to the nonperturbative nature of QCD, giving $g_S/4\pi \gtrsim 1$. In contrast, the small portal coupling ϵ has to be counted, since it has to be associated with a factor $1/4\pi$ in order to maintain the selfconsistency of the NDA power counting.

2.2.3 Elimination of Redundant Operators

A naive listing of all possible operators available at each order in the power counting often contains a number of redundant operators that can be eliminated from the theory without affecting the prediction of S-Matrix elements. I use a number of standard techniques to identify a minimal set of operators without redundancies:

1. **Partial integration** can be used to reshuffle derivatives within the Lagrangian, assuming that the fields are vanishing at infinity. Since covariant derivatives are defined such that they respect the product rule, partial integration can also be used to reshuffle covariant derivatives.

2. **Algebraic relations** identify sets of seemingly different operators with each other. Bianchi identities relate the covariant derivatives of a given field strength tensor $X^{\mu\nu}$ to each other,

$$D^\alpha X^{\beta\gamma} + D^\beta X^{\gamma\alpha} + D^\gamma X^{\alpha\beta} = 0 \ . \tag{2.5}$$

This relation implies two important corollaries,

$$\sigma_{\alpha\beta}(D^{\gamma}X^{\alpha\beta}+2D^{\alpha}X^{\beta\gamma})=0\ ,\qquad D_{\alpha}\widetilde{X}^{\alpha\beta}=0\ , \tag{2.6}$$

where $\widetilde{X}^{\alpha\beta}\equiv\epsilon^{\alpha\beta\mu\nu}X_{\mu\nu}$. Fierz completeness relations identify products of fermion bilinears with each other. In four component notation, the Fierz identities can be summarized as[25]

$$\begin{aligned}&(\overline{\Psi}_a\Gamma^A\,\Psi_b)(\overline{\Psi}_c\Gamma^B\,\Psi_d)\\ =\ &\frac{1}{4}\,\mathrm{tr}\,[\Gamma^A\Gamma_C\Gamma^B\Gamma_D](\overline{\Psi}_a\Gamma^D\,\Psi_d)(\overline{\Psi}_c\Gamma^C\,\Psi_b)\ ,\end{aligned} \tag{2.7a}$$

where the sum over the latin indices is understood. The matrices Γ^A denote a set of contravariant basis vectors of the $(\frac{1}{2},0)\times(0,\frac{1}{2})$ representation of the Lorentz group, and the Γ_A denote their respective covariant partners. One possible choice is [25]

$$\begin{aligned}\Gamma^A&\in\{\mathrm{P}_L,\mathrm{P}_R,\gamma^\mu\,\mathrm{P}_L,\gamma^\mu\,\mathrm{P}_R,\Sigma^{\mu\nu}\}\ ,\\ \Gamma_A&\in\{\,\mathrm{P}_L,\mathrm{P}_R,\gamma_\mu\,\mathrm{P}_R,\gamma_\mu\,\mathrm{P}_L,\frac{1}{2}\Sigma_{\mu\nu}\}\ ,\end{aligned} \tag{2.7b}$$

where $\mu>\nu$ in order to avoid double counting, and the explicit prefactor $\frac{1}{2}$ is included to enforce the normalization condition $\mathrm{tr}\,[\Gamma^A\Gamma_B]=2\delta^A_B$. In the Weyl-basis, these matrices take the form

$$\gamma^\mu\equiv\begin{pmatrix}0&\sigma^\mu\\ \overline{\sigma}^\mu&0\end{pmatrix}\ ,\quad \gamma_5\equiv\begin{pmatrix}\mathbb{1}&0\\0&-\mathbb{1}\end{pmatrix}\ ,\quad \mathrm{P}_{R/L}\equiv\frac{1}{2}\,(\mathbb{1}\pm\gamma_5)\ , \tag{2.8a}$$

and

$$\Sigma^{\mu\nu}=\frac{\mathrm{i}}{2}\,[\gamma^\mu,\gamma^\nu]=\begin{pmatrix}\overline{\sigma}^{\mu\nu}&0\\0&\sigma^{\mu\nu}\end{pmatrix}\ , \tag{2.8b}$$

where the complex, Hermitian 2×2 matrices

$$\overline{\sigma}^{\mu\nu}\equiv\frac{\mathrm{i}}{2}\,(\sigma^\mu\overline{\sigma}^\nu-\sigma^\nu\overline{\sigma}^\mu)\ ,\qquad \overline{\sigma}^\mu\equiv(\mathbb{1},-\sigma^i)\ , \tag{2.8c}$$

$$\sigma^{\mu\nu}\equiv\frac{\mathrm{i}}{2}\,(\overline{\sigma}^\mu\sigma^\nu-\overline{\sigma}^\nu\sigma^\mu)\ ,\qquad \sigma^\mu\equiv(\mathbb{1},\sigma^i) \tag{2.8d}$$

are defined in terms of the standard Pauli matrices σ^i.

Throughout this work, I follow the two-component notation of [26], in which the Fierz completeness relations take a different form. For scalar-valued contractions of two fermion bilinears, one obtains the explicit identities

$$
\begin{aligned}
(\psi_a\psi_b)(\psi_c\psi_d) &= \frac{1}{2}(\psi_a\psi_d)(\psi_c\psi_b) \\
&\quad + \frac{1}{4}(\psi_a\overline{\sigma}_{\mu\nu}\psi_d)(\psi_c\overline{\sigma}^{\mu\nu}\psi_b)\ , \qquad (2.9a)\\
(\psi_a^\dagger\psi_b^\dagger)(\psi_c^\dagger\psi_d^\dagger) &= \frac{1}{2}(\psi_a^\dagger\psi_d^\dagger)(\psi_c^\dagger\psi_b^\dagger) \\
&\quad + \frac{1}{4}(\psi_a^\dagger\sigma_{\mu\nu}\psi_d^\dagger)(\psi_c^\dagger\sigma^{\mu\nu}\psi_b^\dagger)\ , \qquad (2.9b)
\end{aligned}
$$

and

$$
\begin{aligned}
(\psi_a^\dagger\overline{\sigma}_\mu\psi_b)(\psi_c\sigma^\mu\psi_d^\dagger) &= 2(\psi_a^\dagger\psi_d^\dagger)(\psi_c\psi_b)\ , \qquad (2.9c)\\
(\psi_a^\dagger\overline{\sigma}_\mu\psi_b)(\psi_c^\dagger\overline{\sigma}^\mu\psi_d) &= -(\psi_a^\dagger\overline{\sigma}_\mu\psi_d)(\psi_c^\dagger\overline{\sigma}^\mu\psi_b)\ , \qquad (2.9d)\\
(\psi_a\sigma_\mu\psi_b^\dagger)(\psi_c\sigma^\mu\psi_d^\dagger) &= -(\psi_a\sigma_\mu\psi_d^\dagger)(\psi_c\sigma^\mu\psi_b^\dagger)\ . \qquad (2.9e)
\end{aligned}
$$

Using the master formula (2.7a), one can also show that the only remaining scalar-valued contraction

$$
(\psi_a\overline{\sigma}_{\mu\nu}\psi_d)(\psi_c^\dagger\sigma^{\mu\nu}\psi_b^\dagger) = 0 \qquad (2.10)
$$

is vanishing. For vector-valued contractions of two fermion bilinears, one obtains

$$
\begin{aligned}
(\psi_a\psi_b)(\psi_c^\dagger\overline{\sigma}^\mu\psi_d) &= \frac{1}{2}(\psi_a\psi_d)(\psi_c^\dagger\overline{\sigma}^\mu\psi_b) \\
&\quad - \mathrm{i}\,\frac{3}{8}(\psi_a\overline{\sigma}^{\mu\alpha}\psi_d)(\psi_c^\dagger\overline{\sigma}_\alpha\psi_b)\ , \qquad (2.11a)\\
(\psi_a^\dagger\psi_b^\dagger)(\psi_c^\dagger\overline{\sigma}^\mu\psi_d) &= \frac{1}{2}(\psi_a^\dagger\overline{\sigma}^\mu\psi_d)(\psi_c^\dagger\psi_b^\dagger) \\
&\quad - \mathrm{i}\,\frac{3}{8}(\psi_a^\dagger\overline{\sigma}_\alpha\psi_d)(\psi_c^\dagger\sigma^{\mu\alpha}\psi_b^\dagger)\ . \qquad (2.11b)
\end{aligned}
$$

3. **Local field redefinitions** of the form $\phi \to \varphi \equiv F[\phi]$ leave on-shell S-matrix elements unchanged, provided that the fields ϕ and φ have the same quantum number and create the same single particle states. More precisely, on-shell S-matrix elements computed using the generating functional

$$Z[J] = \int \mathcal{D}\phi \, e^{iS[F[\phi]] - \mathrm{i}(J,\phi)} \tag{2.12}$$

are the same as those computed using the generating functional

$$Z[J] = \int \mathcal{D}\varphi \, e^{iS[\varphi] - \mathrm{i}(J,\varphi)} \ , \tag{2.13}$$

provided that both ϕ and φ create and annihilate the same single particle states. This statement is known as either the "S matrix equivalent theorem" or the "representation independence theorem", and several proofs can be found in [27, 28, 24]. In particular, the equivalence theorem applies to field redefinitions that can be written as

$$F[\phi] = \phi + \epsilon^n f[\phi(x)] \ , \tag{2.14}$$

where ϵ is a small parameter and f is a finite polynomial that depends only on the field operator and its derivatives evaluated at x.

Within the context of effective field theories, field redefinition are often used to eliminate terms that are proportional to the equations of motion for the effective degrees of freedom, see e.g. [4, 9, 24]. Consider a general effective theory with an action $\mathcal{S}$ that can be expanded in some small parameter ϵ,

$$\mathcal{S} = \mathcal{S}_0[\phi] + \epsilon \mathcal{S}_1[\phi] + \epsilon^2 \mathcal{S}_2[\phi] + \epsilon^3 \mathcal{S}_3[\phi] + \dots \ , \tag{2.15}$$

where ϕ is a shorthand for any type of quantum field contained within the theory. Assume that $\mathcal{S}$ contains a term of the shape

$$\epsilon^n \int \mathrm{d}^4x \, f[\phi(x)] \frac{\delta \mathcal{S}_0}{\delta \phi} \ , \tag{2.16}$$

where $n > 0$ and f according to Eq. (2.14). At the level of the classical action, this term can be eliminated to order $O(\epsilon^n)$ by using the field redefinition

$$\phi \to \phi - \epsilon^n f[\phi] \ . \tag{2.17}$$

The S-matrix equivalence theorem now ensures that one does not have to worry about new terms being generated by either the path integral measure or the source terms involving the external currents. In practice, this means that operators of the shape (2.16) can always be eliminated in favour of operators that are of higher order in ϵ, which is especially useful when operators of order $\mathcal{O}(\epsilon^{n+1})$ or higher are negligible. Since the condition

$$\frac{\delta \mathcal{S}_0}{\delta \phi} \stackrel{!}{=} 0 \tag{2.18}$$

corresponds to imposing the zeroth order classical equations of motion, this procedure is usually thought of as "using" the equations of motion to simplify the effective Lagrangian.

Within the PET framework, the zeroth order equations of motion are just the SM equations of motion, which can be used to simplify the PET Lagrangian. A rigorous proof for the representation independence theorem that focuses specifically on the standard model equations of motion is given in [28]. In principle, the number of independent portal operators can be further reduced by using the equations of motion of the hidden fields, but this requires knowledge about the shape of the hidden field equations of motion.

2.2.4 Should one Diagonalize Quadratic Portal Interactions?

In general, the portal Lagrangian will contain quadratic interactions that mix the SM and hidden sector fields. Assuming the quadratic part of the hidden Lagrangian has been internally diagonalized, it is possible to rotate away the quadratic portal interactions. However, the trade-off is that this field redefinition may result in qualitatively new types of portal interactions. For instance, consider a case in which the SM couples to the hidden sector only via a single scalar field that mixes with the SM Brout-Higgs-Englert boson ("Higgs boson"). Remaining fully agnostic about the internal structure of the hidden sector, the scalar mediator may couple to additional hidden fermions via a hidden Yukawa interaction. In this case, the rotation used to eliminate a potential mixing between the hidden scalar and the Higgs boson will result in new portal couplings between the Higgs boson and the hidden fermions.

This is problematic because the PET framework is build on the assumption that the SM couples only to a single hidden degree of freedom. It is not possible to use the PETs after the diagonalization, if this diagonalization results in the SM actually coupling to multiple light hidden mediators. On the other hand, it is extremely restrictive to assume that the SM couples only to a single hidden degree of freedom *after* diagonalizing the Lagrangian, since it is equivalent to making a number of additional assumptions about the internal structure of the hidden sector. For instance, the aforementioned model with a single scalar mediator mixing with the Higgs boson can only be captured within a PET if there are no hidden Yukawa interactions that involve the scalar mediator. Thus, in order to remain as general as possible, the best course of action is not to diagonalize the quadratic portal interactions.

Of course, not diagonalizing the portal Lagrangian also comes with its own set of problems. For example, consider a type-I seesaw model in which the standard mode is augmented by a single heavy neutral lepton. After electroweak symmetry breaking, the mixing between the heavy neutral lepton and the SM neutrinos generates a mass for one of the SM neutrinos, which in turn induces oscillations between the different standard model neutrino flavours. These oscillations typically occur on macroscopic length scales, for instance with a wave length of a several kilometers for neutrinos produced in nuclear reactors [29].

In order to capture these oscillations, one has to resum the mass insertions within the light neutrino propagators. However, this resummation is nontrivial if the hidden sector mixing is not diagonalized, so an accurate description of neutrino oscillations becomes more challenging. On the other hand, a perturbative treatment of the neutrino masses is perfectly adequate for the description of individual microscopic scattering processes, in which the impact of neutrino oscillations is negligible.

2.3 General Electroweak Scale PETs

In this section, I discuss general electroweak scale portal Lagrangians for mediators of spin 0, $1/2$, and 1. General electroweak scale PETs are of interest for model independent hidden particle searches at colliders, and as a starting point for the subsequent construction of more specific PETs at the electroweak scale and at low energies. At leading order in ϵ, the portal Lagrangians discussed in this section exhibit a number of accidental symmetries that can be used to constrain the general shape of the effective Lagrangians.

2.3.1 Constructing a Minimal Basis of Portal Operators

I begin by constructing a minimal basis of portal operators for each mediator spin. My general strategy is to proceed in two steps: First, I classify the various possible types of candidate portal operator by their field content, and secondly, I use the general techniques discussed in section 2.2.3 to eliminate the redundant operators. Before discussing the individual portal operators for each mediator type, it is convenient to make a few general statements that apply to mediators of all three spin categories.

General Considerations

At leading order in ϵ_{Pet}, portal operators can be of dimension $d = 5$ or less. In terms of field content, each portal operator can only contain either zero or two fermion fields. For operators with fermions, I follow the two-component notation of [26].

Portal operators with zero fermions can only appear for integer spin mediators. In addition to the hidden fields, they can contain either only Higgs field operators or only SM field strength tensors. By gauge invariance, an operator with both Higgs field operators and SM field strength tensors would need to involve at least two Higgs bosons, one field strength tensor, and two more derivatives or vector field operators in order to contract the Lorentz indices, which gives a dimension of at least $d = 6$.

Portal operators with two fermions can appear for both integer spin and half-integer spin mediators. They contain either a scalar type fermion bilinear of the shape $\psi\psi$, a vector type fermion bilinear of the shape $\psi^\dagger\overline{\sigma}^\mu\psi$, or a tensor type fermion bilinear of the shape $\psi\overline{\sigma}^{\mu\nu}\psi$. Portal operators with a tensor-type fermion bilinear cannot contain any SM fermions, since the corresponding operators would have to involve at least one Higgs boson in order to be gauge invariant, and two more derivatives or vector field operators in order to contract the Lorentz indices, which gives a dimension of at least $d = 6$. Similarly, any portal operator with a scalar-type fermion bilinear that contains at least one SM fermion also has to contain at least one Higgs field operator in order to be gauge invariant. Such an operator cannot involve any derivatives or SM field strength tensors, as this would increase the dimension of the operator to at least $d = 7$. Finally, operators with a vector-type fermion bilinear have to involve either a derivative or vector field in order to be Lorentz invariant. If they involve two SM fermions, they cannot involve any further SM Higgs bosons or field strength tensors, as this would increase the dimension to at least $d = 6$. On the other hand, if they involve exactly one SM fermion, they also have to involve exactly one Higgs boson in order to be gauge invariant. This way, it is possible to sort the electroweak scale portal operators into four types, based on their field content:

1. Higgs portal operators containing only hidden fields and the SM Higgs boson,
2. Gauge portal operators containing only hidden fields and SM field strength tensors,
3. Yukawa portal operators containing a fermion bilinear and a Higgs field operator but no other SM bosons,
4. Fermion current portal operators involving a fermion bilinear and no SM bosons.

After using the above classification, I obtain a naive list of all possible portal operators to which I apply the techniques of section 2.2.3 in order to identify a minimal basis of operators. In particular, I use the SM equations of motion

$$\mathrm{i}\,\not{D} l = \Gamma_e \overline{e}^\dagger \phi \,, \qquad \mathrm{i}\,\not{D} \overline{e}^\dagger = \Gamma_e^\dagger \phi^\dagger l \tag{2.19a}$$

and

$$\mathrm{i}\,\not{D}q = \Gamma_u \overline{u}^\dagger \widetilde{\phi} + \Gamma_d \overline{d}^\dagger \phi \ , \quad \mathrm{i}\,\not{D}\overline{u}^\dagger = \Gamma_u^\dagger \widetilde{\phi}^\dagger q \ , \quad \mathrm{i}\,\not{D}\overline{d}^\dagger = \Gamma_d^\dagger \phi^\dagger q \ , \tag{2.19b}$$

and

$$(D^2\phi)^I = \mu_\phi^2 \phi^I - \lambda \left|\phi\right|^2 \phi^I - \overline{e}\Gamma_e^\dagger l^I - \overline{d}\Gamma_d^\dagger q^I - q_J^\dagger \Gamma_u \overline{u}^\dagger \epsilon^{IJ} \ , \tag{2.20a}$$

$$(D_\mu G^{\mu\nu})^x = q_a^\dagger \overline{\sigma}^\nu T^x q_a + \overline{u}_a \sigma^\nu T^x \overline{u}_a^\dagger + \overline{d}_a \sigma^\nu T^x \overline{d}_a^\dagger \ , \tag{2.20b}$$

$$(D_\mu W^{\mu\nu})^I = \phi^\dagger \,\mathrm{i}\, \overleftrightarrow{D}^{\nu I} \phi + l_b^\dagger \overline{\sigma}^\nu T^I l_b + q_b^\dagger \overline{\sigma}^\nu T^I q_b \ , \tag{2.20c}$$

$$\partial_\mu B^{\mu\nu} = Y_\phi (\mathrm{i}\,\phi^\dagger \overleftrightarrow{D}_\mu \phi) + \sum_{\psi \in \mathrm{SM}} Y_\psi \overline{\psi} \gamma_\mu \psi \ , \tag{2.20d}$$

where the Γ_f denote the SM Yukawa matrices coupling the SM fermions to the Higgs boson. $m_H^2 \equiv \lambda v^2$ is the Higgs boson mass, and λ is the Higgs selfcoupling. Using the above equations of motion, I eliminate all operators involving the building blocks $\not{D}\psi$, $\not{D}\psi^\dagger$, $D^2\phi$, and $D_\mu X^{\mu\nu}$, where $X^{\mu\nu}$ denotes any SM field strength tensor.

Spin 0 Mediators

I start by considering the available portal operators for spin 0 mediators. Spin 0 mediators can be either real scalar fields, real pseudoscalar fields, or complex scalar fields. Since I do not enforce parity conservation, a real pseudoscalar mediator will couple to the SM in the same way as a real scalar mediator, and a complex scalar will couple to the SM in the same way as a combination of two real scalar mediators. As a result, it is possible to restrict the discussion to portal Lagrangians with two real hidden scalar fields S_i with $i = 1, 2$ without loss of generality.

1) Higgs Type Operators: Higgs type portal operators contain either two or four Higgs field operators. Operators with four Higgs bosons cannot involve any derivatives, while operators with two Higgs bosons can involve either zero or two derivatives. For the operators without derivatives, a complete basis can be constructed from the 4 operators

$$\begin{aligned} &S_i\,\phi^\dagger\phi \ , && S_i S_j\,\phi^\dagger\phi \ , && S_i S_j S_k\,\phi^\dagger\phi \ , \\ &S_i\,(\phi^\dagger\phi)^2 \ . \end{aligned} \tag{2.21}$$

Up to dimension $d = 5$, the operators with two Higgs bosons and two derivatives can only involve a single hidden field operator, and it is possible to reshuffle the derivatives such that they do not act on the hidden field. In addition, I eliminate operators in which both derivatives act on the same Higgs boson by using the SM equations of motion. The only remaining operator with two derivatives is

$$S_i D_\mu \phi^\dagger D^\mu \phi \; . \tag{2.22}$$

2) Gauge Type Operators: Gauge type portal operators have to contain two field strength tensors. Operators with one field strength tensor would need to involve at least two derivatives in order to be Lorentz invariant. Up to $d = 5$, these derivatives could be reshuffled such that both of them act on the field strength tensor, which implies that the corresponding operators vanish due to the antisymmetry of the field strength tensor. Operators with two field strength tensors cannot contain any additional derivatives, but the Lorentz indices can be contracted using either the metric tensor $g^{\mu\nu}$ or the Levi-Civita symbol $\epsilon^{\mu\nu\alpha\beta}$. Hence, the only available gauge-type operators are the 6 operators of the type

$$S_i \, X_{\mu\nu} X^{\mu\nu} \; , \qquad\qquad S_i \, \widetilde{X}_{\mu\nu} X^{\mu\nu} \; , \tag{2.23}$$

where $X_{\mu\nu} \in \{G_{\mu\nu}, W_{\mu\nu}, B_{\mu\nu}\}$, and the trace over gauge indices is implied.

3) YukawaType Operators: Yukawa type portal operators have to involve two SM fermions, a Higgs field operator, and a hidden field operator, so that they cannot contain any derivatives. The only gauge invariant combinations are the 3 operators

$$S_i \, \overline{e}_a l_b \phi^\dagger \; , \qquad S_i \, \overline{d}_a q_b \phi^\dagger \; , \qquad S_i \, \overline{u}_a q_b \widetilde{\phi}^\dagger \; . \tag{2.24}$$

4) Fermion Current Type Operators: There are no fermion current type portal operators. These operators would have to involve a vector type fermion bilinear $\psi^\dagger \overline{\sigma}^\mu \psi$ and at leas one derivative in order to contract the Lorentz index. However, up to dimension $d = 5$, the derivative can be reshuffled such that it acts on either of the two fermion fields, so that the fermion type portal operators can be eliminated using the SM equations of motion.

Spin 1/2 Mediators

Spin 1/2 mediators can be either Dirac, Majorana, or Weyl fermions. A Majorana fermion can be rewritten in terms of a single left-handed Weyl fermion, while a Dirac fermion can be decomposed into two left-handed Weyl fermions. Therefore, it is possible to restrict the discussion to portal Lagrangians with hidden left-handed Weyl fermions ξ_i with $i = 1, 2$ without loss of generality.

1) Higgs Type Operators: Higgs type portal operators have to contain two hidden fermion fields and two Higgs field operators, and they cannot contain any derivatives. The only available operator is

$$\xi_i \xi_j \phi^\dagger \phi + \text{h.c.} \; . \tag{2.25}$$

2) Gauge Type Portal Operators: Gauge type portal operators have to contain two hidden fermion fields and a single SM field strength tensor. The only available operator is

$$\xi_i \overline{\sigma}^{\mu\nu} \xi_j B_{\mu\nu} + \text{h.c.} \; . \tag{2.26}$$

3) Yukawa Type Operators: Yukawa type portal operators have to contain one hidden fermion, one SM fermion, and a Higgs field operator. They can contain either no derivatives or one derivative. By using partial integration and the SM equations of motion, the derivative can be reshuffled such that it acts on the Higgs field operator. Thus, the only available operators are

$$\xi_i l_a \widetilde{\phi}^\dagger \; , \qquad\qquad \xi_i^\dagger \overline{\sigma}^\mu l_a D_\mu \widetilde{\phi}^\dagger \; . \tag{2.27}$$

4) Fermion Current Type Operators: As in the case of scalar mediators, there are no fermion current type portal operators. These operators would have involve one hidden fermion and a single SM fermion field. However, none of the SM fermions is gauge invariant by itself.[1]

[1]Recall that the SM neutrinos only appear as part of the left-handed $SU(2)_I$ doublets.

Spin 1 Mediators

Finally, I consider the available portal operators for spin 1 mediators. Spin 1 mediators can be either vector or pseudovector fields. Since I do not enforce parity conservation, a pseudovector field will couple to the SM in the same way as vector field. As before, this implies that the discussion can be restricted to portal Lagrangians with a single vector fermion at no loss of generality. There are no gauge type portal operators and no Yukawa type portal operators.

1) Higgs Type Operators: Higgs type portal operators have to contain two Higgs fields, since operators with four Higgs bosons would be at least of dimension $d = 6$. To be Lorentz invariant, the operators have to contain either two hidden field operators or one hidden field operator and one derivative. A complete basis can be constructed from the 3 operators

$$V_\mu V^\mu \phi^\dagger \phi \ , \qquad \partial_\mu V^\mu \phi^\dagger \phi \ , \qquad V^\mu \, \mathrm{i} \, \phi^\dagger \overleftrightarrow{D}_\mu \phi \ . \tag{2.28}$$

2) Gauge Type Operators: There are no gauge type portal operators. Such operators would have to contain only one field strength tensor, since operators with two field strength tensors would be at least of dimension $d = 6$. In order to be Lorentz invariant, they would have to contain either two hidden field operators or one hidden field operator and one derivative. In the first case, the operators vanish due to the antisymmetry of the field strength tensor, and in the second case it is possible to eliminate the operator using either the SM equations of motion or the Bianchi identity $D_\mu \widetilde{B}^{\mu\nu} = 0$.

3) Yukawa Type Operators: There are no Yukawa type portal operators. Such operators would have to contain a scalar type SM fermion bilinear, a Higgs boson, and either two hidden vector field operators or one hidden vector field operator and one derivative. In either case, the operator would be at least of dimension $d = 6$.

4) Fermion Current Type Operators: fermion current type portal operators have to contain a vector type fermion bilinear and a single vector field operator, since portal operators with three vector fields would be at last of dimension $d = 6$. A complete basis can be constructed from the 5 operators of the type

$$V_\mu \, \psi^\dagger \overline{\sigma}^\mu \psi \ , \tag{2.29}$$

where the possible gauge singlets can be

$$(\psi^\dagger \psi) \in \{(l_a^\dagger l_b), (\overline{e}_a^\dagger \overline{e}_b), (q_a^\dagger q_b), (\overline{u}_a^\dagger \overline{u}_b), (\overline{d}_a^\dagger \overline{d}_b)\} \ . \tag{2.30}$$

2.3.2 The Full Portal Lagrangian

Having constructed a minimal basis of portal operators for each mediator, it is possible to make a number of general statements about the corresponding most general portal Lagrangians.

The final set of portal operators for each mediator type is summarized in table 2.1. Spin 0 mediators couple to the SM via a total of 14 operators, spin $1/2$ mediators couple to the SM via a total of 4 operators, and spin 1 mediators couples to the SM via a total of 8 operators. Notice that the electroweak portal Lagrangians for each mediator exhibit a number of accidental symmetries that can be used to constrain the shape of the GeV scale portal Lagrangians at leading order in ϵ_{Pet}. All three portal Lagrangians conserve baryon number, and the ones for integer spin mediators also conserve lepton number. The vector portal Lagrangian respects chiral symmetry for all SM fermions, and the spin $1/2$ portal Lagrangian respects chiral symmetry for the SM quark fields.

Aside from exact accidental symmetries, the general electroweak scale PETs restrict low energy PETs in a number of ways. Although the electroweak scale spin $1/2$ PET is lepton number violating, low energy portal Lagrangians cannot contain interactions with $|\Delta L| > 1$, since the electroweak scale PET only contains interactions with $|\Delta L| = 0, 1$. Furthermore, the electroweak scale spin $1/2$ PET does not contain any righthanded charged lepton fields. Finally, the electroweak scale spin 1 PET does not contain operators of dimension $d = 5$, which implies that the corresponding low energy PETs cannot contain portal operators of dimension $d = 7$. It is also worth noting that the operator $\xi_i \sigma^{\mu\nu} \xi_j B_{\mu\nu}$ vanishes if the hidden mediator can be written as a single Weyl spinor, so that it only contributes if the spin $1/2$ mediator is a Dirac fermion.

	d	Higgs	Yukawa +h.c.	Fermion	Gauge +h.c.
$S_{1,2}$	3	$S_i\,\phi^\dagger\phi$			
	4	$S_iS_j\,\phi^\dagger\phi$			
	5	$S_iS_jS_k\,\phi^\dagger\phi$ $S_i\,D^\mu\phi^\dagger D_\mu\phi$ $S_i\,(\phi^\dagger\phi)^2$	$S_i\,\overline{u}_a q_b\,\widetilde{\phi}^\dagger$ $S_i\,\overline{d}_a q_b\,\phi^\dagger$ $S_i\,\overline{e}_a \ell_b\,\phi^\dagger$		$S_i\,G^a_{\mu\nu}G^{\mu\nu}_a$ $S_i\,W^a_{\mu\nu}W^{\mu\nu}_a$ $S_i\,B_{\mu\nu}B^{\mu\nu}$ $S_i\,\widetilde{G}^a_{\mu\nu}G^{\mu\nu}_a$ $S_i\,\widetilde{W}^a_{\mu\nu}W^{\mu\nu}_a$ $S_i\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$
$\xi_{1,2}$	4		$\xi_i\,\ell_a\,\widetilde{\phi}^\dagger$		
	5	$\xi_i\xi_j\,\phi^\dagger\phi$	$\xi_i^\dagger\overline{\sigma}^\mu\,\ell_a D_\mu\widetilde{\phi}^\dagger$		$\xi_i\,\overline{\sigma}^{\mu\nu}\xi_j B_{\mu\nu}$
V_μ	4	$V_\mu V^\mu\,\phi^\dagger\phi$ $(\partial_\mu V^\mu)\,\phi^\dagger\phi$ $V^\mu\,(\phi^\dagger \overleftrightarrow{D}_\mu\phi)$		$V^\mu\,q_a^\dagger\overline{\sigma}_\mu q_b$ $V^\mu\,\ell_a^\dagger\overline{\sigma}_\mu\ell_b$ $V^\mu\,\overline{u}_a^\dagger\overline{\sigma}_\mu\overline{u}_b$ $V^\mu\,\overline{d}_a^\dagger\overline{\sigma}_\mu\overline{d}_b$ $V^\mu\,\overline{e}_a^\dagger\overline{\sigma}_\mu\overline{e}_b$	

Table 2.1: Electroweak scale Portal operators for mediators with Spin $S \leq 1$.

Finally, the results in this section are readily generalized to models with multiple mediators of equal spin. For spin 0 and spin 1/2 mediators, it is sufficient to let the index i run over all potential mediator fields. For spin 1 mediators, it is likewise sufficient to introduce an analogous index that also runs over all potential mediator fields.

It is convenient to combine the portal operators for the individual electroweak scale PETs inside a single, general PET Lagrangian,

$$\mathcal{L}_{\text{portal}} \equiv \delta\mathcal{L}_{\text{Quark}} + \delta\mathcal{L}_{\text{Lepton}} + \delta\mathcal{L}_{\text{Higgs}} + \delta\mathcal{L}_{\text{Gauge}} + \delta\mathcal{L}_{\text{Yukawa}}\ , \quad (2.31)$$

where

$$\delta\mathcal{L}_{\text{Quark}} = q_a^\dagger\overline{\sigma}_\mu q_b\; l^\mu_{q,ab} + \overline{u}_a^\dagger\sigma_\mu\overline{u}_b\, r^\mu_{\overline{u},ab} + d_a^\dagger\overline{\sigma}_\mu\overline{d}_b\; r^\mu_{\overline{d},ab}\ , \tag{2.32a}$$

$$\delta\mathcal{L}_{\text{Lepton}} = l_a^\dagger\overline{\sigma}_\mu l_b\; l^\mu_{l,ab} + \overline{e}_a^\dagger\overline{\sigma}_\mu\overline{e}_b\; r^\mu_{\overline{e},ab}\ , \tag{2.32b}$$

$$\delta\mathcal{L}_{\text{Higgs}} = D_\mu\phi^\dagger D^\mu\phi\, h + \mathrm{i}(\phi^\dagger\overleftrightarrow{D}_\mu\phi)\, v^\mu + \phi^\dagger\phi\;\widetilde{\mu} + \frac{1}{2}(\phi^\dagger\phi)^2\,\widetilde{\lambda}\ , \tag{2.32c}$$

$$\begin{aligned}\delta\mathcal{L}_{\text{Gauge}} &= G_{\mu\nu}G^{\mu\nu}\, a_G + W_{\mu\nu}W^{\mu\nu}\, a_W + B_{\mu\nu}B^{\mu\nu}\, a_B \\ &\quad + \widetilde{G}_{\mu\nu}G^{\mu\nu}\,\widetilde{a}_G + \widetilde{W}_{\mu\nu}W^{\mu\nu}\,\widetilde{a}_W + \widetilde{B}_{\mu\nu}B^{\mu\nu}\,\widetilde{a}_B + B^{\mu\nu}\, t_{\mu\nu}\ ,\end{aligned} \tag{2.32d}$$

$$\begin{aligned}\delta\mathcal{L}_{\text{Yukawa}} &= \overline{e}_a l_b\phi^\dagger\, s_{\overline{e},ab} + \overline{d}_a q_b\phi^\dagger\, s_{\overline{d},ab} \\ &\quad + \overline{u}_a q_b\widetilde{\phi}^\dagger\, s_{\overline{u},ab} + \Xi_a\, l_a\widetilde{\phi}^\dagger + \Xi_{a\mu}\, l_a\,\mathrm{i}\, D^\mu\widetilde{\phi}^\dagger + \text{h.c.}\ .\end{aligned} \tag{2.32e}$$

In terms of the hidden mediator fields, the BSM currents are

$$\begin{aligned} a_X &= \frac{\epsilon_{\text{Pet}}}{M_W}C_{a_X i}\, S_i\ , \qquad \widetilde{a}_X = \frac{\epsilon_{\text{Pet}}}{M_W}\widetilde{C}_{a_X}\, S_i\ , \\ s_x &= \frac{\epsilon_{\text{Pet}}}{M_W}C_{s_x i}\, S_i\ , \qquad \widetilde{\lambda} = \frac{\epsilon_{\text{Pet}}}{M_W}C_{\lambda\, i}\, S_i\ , \qquad h = \frac{\epsilon_{\text{Pet}}}{M_W}C_{h\, i}S_i \\ l^\mu_x &= \epsilon_{\text{Pet}}\, C_{l_x}\, V^\mu\ , \qquad r^\mu_x = \epsilon_{\text{Pet}}\, C_{r_x}\, V^\mu\ , \qquad v^\mu = \epsilon_{\text{Pet}}\, C_v\, V^\mu\ ,\end{aligned} \tag{2.33a}$$

$$\Xi_a = \epsilon_{\text{Pet}}\, C_{\Xi_a\,\xi\, i}\,\xi_i\ , \qquad \Xi_{a\mu} = \frac{\epsilon_{\text{Pet}}}{M_W}C_{\Xi_a\,\partial\xi\, i}\,\xi_i^\dagger\overline{\sigma}^\mu\ , \tag{2.33b}$$

and

$$\begin{aligned}\widetilde{\mu} &= \frac{v^2\epsilon_{\text{Pet}}}{M_W}C_{\mu\, i}\, S_i + \epsilon_{\text{Pet}}\, C_{\mu\, ij}\, S_iS_j + \frac{\epsilon_{\text{Pet}}}{M_W}C_{\mu\, ijk}\, S_iS_jS_k \\ &\quad + \frac{\epsilon_{\text{Pet}}}{M_W}C_{\mu\,\partial^2\, i}\cdot\partial^2 S_i + \frac{\epsilon_{\text{Pet}}}{M_W}(C_{\mu\,\xi\xi\, ij}\,\xi_i\xi_j + \text{h.c.}) \\ &\qquad + \epsilon_{\text{Pet}}\, C_{\mu,vv}\cdot V^\mu V_\mu + \epsilon_{\text{Pet}}\, C_{\mu,\partial v}\cdot\partial^\mu V_\mu\ , \\ t_{\mu\nu} &= \frac{\epsilon_{\text{Pet}}}{M_W}C_{t\, ij}\,\xi_i\sigma^{\mu\nu}\xi_j + \text{h.c.}\ .\end{aligned} \tag{2.33c}$$

The l^μ_x, r^μ_x, s_x currents, and by extension their Wilson coefficients, are Hermitian 3x3 matrices in flavour space. Similarly, Ξ and Ξ_μ as well as their coefficients are vectors in flavour space. The remaining currents and Wilson coefficients are flavour singlets, and the operator normalization has been chosen such that the Wilson coefficients are dimensionless. As before, traces over gauge indices are implied. Each portal operator is explicitly suppressed by a factor of ϵ_{Pet}. The higher dimensional operators are associated with a factor of ${}^1\!/\Lambda = {}^\epsilon\!/M_W$, so that all degrees of smallness associated with physics beyond the SM are collected within the explicit ϵ_{Pet} prefactor.

2.3.3 Modifications after Electroweak Symmetry Breaking

To study the modifications after electroweak symmetry breaking, I fix the standard model gauge. At tree level, it is most convenient to work in unitary gauge, but this is not possible when loop corrections are taken into account, since the SM in unitary gauge cannot be renormalized using standard techniques. In the interest of generality, and in order to facilitate loop computations that may be performed on the basis of the portal Lagrangians, I work in the R_ξ gauge. The Higgs field ϕ is taken to be of the shape

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H - \mathrm{i}\,\phi_0) \end{pmatrix} , \qquad \widetilde{\phi} = \begin{pmatrix} -\frac{1}{\sqrt{2}}(v + H + \mathrm{i}\,\phi_0) \\ \phi^- \end{pmatrix} , \tag{2.34}$$

where $\phi^- \equiv (\phi^+)^\dagger$. As before, $\widetilde{\phi} = -\,\mathrm{i}\,\sigma^2\phi^\dagger$ is the $\mathrm{U}(1)_Y$ charge conjugated Higgs boson. The expressions involving the Higgs field have to be replaced as follows:

$$\begin{aligned}
\phi^\dagger\phi &\xrightarrow{\text{EWSB}} \frac{1}{2}\,(v+H)^2 + \frac{1}{2}\phi_0^2 + \phi^+\phi^- \ , && (2.35\text{a}) \\
l_a\phi^\dagger &\xrightarrow{\text{EWSB}} \frac{1}{\sqrt{2}}\,(v + H + \mathrm{i}\,\phi_0)\,e_a + \phi^-\nu_a \ , \\
q_a\phi^\dagger &\xrightarrow{\text{EWSB}} \frac{1}{\sqrt{2}}\,(v + H + \mathrm{i}\,\phi_0)\,d_a + \phi^-u_a \ , \\
l_a\widetilde{\phi}^\dagger &\xrightarrow{\text{EWSB}} \frac{-1}{\sqrt{2}}\,(v + H - \mathrm{i}\,\phi_0)\,\nu_a + \phi^+e_a \ , \\
q_a\widetilde{\phi}^\dagger &\xrightarrow{\text{EWSB}} \frac{-1}{\sqrt{2}}\,(v + H - \mathrm{i}\,\phi_0)\,u_a + \phi^+d_a \ ,
\end{aligned}$$

and

$$\mathrm{i}\,\phi^\dagger \overleftrightarrow{D}_\mu \phi \xrightarrow{\text{EWSB}} \frac{1}{2}\,(v+H)^2\;Z_\mu + \frac{1}{2}\phi_0^2\,Z_\mu \tag{2.36a}$$
$$- \big[(v+H-\mathrm{i}\,\phi_0)W_\mu^+\phi^- + \text{h.c.}\big]$$
$$+\,\phi^-\big[\,\mathrm{i}\,\overleftrightarrow{\partial}_\mu - 2A_\mu - (1-2s_W^2)Z_\mu\big]\phi^+\;,$$

$$\mathrm{i}\,l_a D_\mu \widetilde{\phi}^\dagger \xrightarrow{\text{EWSB}} -\,\mathrm{i}\,\nu_a \frac{1}{\sqrt{2}}\partial_\mu(H-\mathrm{i}\,\phi_0) + \mathrm{i}\,e_a\partial_\mu\phi^+ \tag{2.36b}$$
$$+\,\frac{1}{2}Z_\mu\big[\nu_a\frac{1}{\sqrt{2}}(v+H-\mathrm{i}\,\phi_0) - (1-2s_W^2)e_a\phi^+\big]$$
$$+\,\frac{1}{\sqrt{2}}\big[W_\mu^-\nu_a\phi^+ - W^+ e_a\frac{1}{\sqrt{2}}(v+H-\mathrm{i}\,\phi_0)\big]\;,$$

$$D_\mu\phi^\dagger D^\mu\phi \xrightarrow{\text{EWSB}} \frac{1}{2}\partial_\mu H\partial^\mu H + \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 + \partial_\mu\phi^+\partial^\mu\phi^- \tag{2.36c}$$
$$+\,\frac{1}{4}\big[\,(v+H)^2+\phi_0^2\big](W_\mu^+W^{-\mu} + \frac{1}{2}Z^\mu Z_\mu)$$
$$+\,\frac{1}{2}\big[W_\mu^+W^{-\mu} + (A_\mu + \frac{1}{2}(1-2s_W^2)Z_\mu)^2\big]\phi^+\phi^-$$
$$-\,\frac{1}{2}\,\mathrm{i}\,\phi^-\big[\overleftarrow{\partial}_\mu(2A^\mu + (1-2s_W^2)Z^\mu)$$
$$-\,(2A^\mu + (1-2s_W^2)Z^\mu)\overrightarrow{\partial}_\mu\big]\phi^+\;.$$

For the portal Lagrangian, this prescription yields the modifications

$$\delta\mathcal{L}_{\text{Higgs}} \xrightarrow{\text{EWSB}} \Big[\frac{1}{2}\partial_\mu H\partial^\mu H + \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 + \partial_\mu\phi^+\partial^\mu\phi^-\Big]\,h \tag{2.37a}$$
$$-\,\phi^-\Big[v^\mu(v+H-\mathrm{i}\,\phi_0)W_\mu^+ - v^\mu\,\mathrm{i}\,\partial_\mu\phi^+$$
$$(A^\mu + \frac{1}{2}(1-2s_W^2)Z^\mu)\,h\,\mathrm{i}\,\partial_\mu\phi^+ + \text{h.c.}\Big]$$
$$+\,\frac{1}{4}\,\Big[(v+H)^2+\phi_0^2+2\phi^+\phi^-\Big]\,\Big(W_\mu^+W^{-\mu}\,h + 2\widetilde{\mu}\Big)$$
$$+\,\frac{1}{4}\,\Big[(v+H)^2+\phi_0^2\Big]\,\Big(\frac{1}{2}Z^\mu Z_\mu\,h + 2Z_\mu\,v^\mu\Big)$$
$$+\,\frac{1}{2}\phi^+\phi^-\Big(\frac{1}{4}(1-2s_W^2)^2 Z_\mu Z^\mu\,h - 2(1-2s_W^2)Z_\mu\,v^\mu\Big)$$

$$
\begin{aligned}
&+ \frac{1}{2}\phi^+\phi^-\Big(A_\mu A^\mu\, h + (1-2s_W^2)A_\mu Z^\mu\, h - 4A_\mu\, v^\mu\Big)\\
&+ \frac{1}{8}\Big((v+H)^2 + \phi_0^2 + 2\phi^+\phi^-\Big)^2\, \widetilde{\lambda}\ ,
\end{aligned}
$$

$$
\begin{aligned}
\delta\mathcal{L}_{\rm Gauge} \xrightarrow{\rm EWSB}\ & G_{\mu\nu}G^{\mu\nu}\, a_G + 2\overline{W}^+_{\mu\nu}\overline{W}^{\mu\nu}_-\, a_W + \overline{Z}_{\mu\nu}F^{\mu\nu}\, a_{FZ} \\
&+ \widetilde{G}_{\mu\nu}G^{\mu\nu}\,\widetilde{a}_G + 2\widetilde{\overline{W}}{}^+_{\mu\nu}\overline{W}^{\mu\nu}_-\,\widetilde{a}_W + \widetilde{\overline{Z}}_{\mu\nu}F^{\mu\nu}\,\widetilde{a}_{FZ}\\
&+ F_{\mu\nu}F^{\mu\nu}\, a_F + \widetilde{F}_{\mu\nu}F^{\mu\nu}\,\widetilde{a}_F + F^{\mu\nu}\, t_{F\mu\nu}\\
&+ \widetilde{\overline{Z}}_{\mu\nu}\overline{Z}^{\mu\nu}\,\widetilde{a}_Z + \overline{Z}_{\mu\nu}\overline{Z}^{\mu\nu}\, a_Z + \overline{Z}^{\mu\nu}\, t_{Z\mu\nu}\\
&+ 4\,\mathrm{i}\,\epsilon^{\mu\nu\alpha\beta}\partial_\mu(W^3_\nu W^+_\alpha W^-_\beta)\,\widetilde{a}_W\\
&+ 8g^{\mu[\nu}g^{\alpha]\beta}(W^3_\mu W^3_\nu + \frac{1}{2}W^+_\mu W^-_\nu)W^+_\alpha W^-_\beta\, a_W\\
&- 2\,\mathrm{i}\,\Big[g^{\alpha\beta}\,(\partial_+ - \partial_-)^\mu + g^{\beta\mu}\,(\partial_- - \partial_3)^\alpha\\
&\qquad + g^{\mu\alpha}\,(\partial_3 - \partial_+)^\beta\Big]W^3_\mu W^+_\alpha W^-_\beta\, a_W
\end{aligned}
\tag{2.37b}
$$

$$
\begin{aligned}
\delta\mathcal{L}_{\rm Yukawa} \xrightarrow{\rm EWSB}\ & \frac{1}{\sqrt{2}}(v + H + \mathrm{i}\,\phi_0)(\overline{e}_i e_j\, s_{\overline{e}e,ij} + \overline{d}_i d_j\, s_{\overline{d}d,ij} - \overline{u}_i u_j\, s_{\overline{u}u,ij})\\
&- \frac{1}{\sqrt{2}}(v + H - \mathrm{i}\,\phi_0)\Big[\Xi_i\nu_i - \Xi^\mu_i(\frac{1}{2}Z_\mu\nu_i - \frac{1}{\sqrt{2}}W^+_\mu e_i)\Big]\\
&+ \phi^-(\overline{e}_i\nu_j\, s_{\overline{e}\nu,ij} + \overline{d}_i u_j\, s_{\overline{d}u,ij}) + \phi^+\overline{u}_i d_j\, s_{\overline{u}d,ij}\\
&+ \phi^+\Big[\Xi_i e_i - \Xi^\mu_i(\frac{1}{2}Z_\mu(1-2s_W^2)e_i - \frac{1}{\sqrt{2}}W^-_\mu\nu_i)\Big]\\
&+ \Xi^\mu_i\big[e_i\,\mathrm{i}\,\partial_\mu\phi^+ - \nu_i\frac{1}{\sqrt{2}}\,\mathrm{i}\,\partial_\mu(H - \mathrm{i}\,\phi_0)\big]\\
&+ \text{h.c.}\ .
\end{aligned}
\tag{2.37c}
$$

$$
\begin{aligned}
\delta\mathcal{L}_{\rm Quark} \xrightarrow{\rm EWSB}\ & u^\dagger_i\overline{\sigma}_\mu u_j\, l^\mu_{u,ij} + d^\dagger_i\overline{\sigma}_\mu d_j\, l^\mu_{d,ij}\\
&\ + \overline{u}^\dagger_i\overline{\sigma}_\mu\overline{u}_j\, r^\mu_{\overline{u},ij} + \overline{d}^\dagger_i\overline{\sigma}_\mu\overline{d}_j\, r^\mu_{\overline{d},ij}\ ,
\end{aligned}
\tag{2.37d}
$$

$$
\delta\mathcal{L}_{\rm Lepton} \xrightarrow{\rm EWSB} \nu_i\overline{\sigma}_\mu\nu_j\, l^\mu_{\nu,ij} + e_i\overline{\sigma}_\mu e_j\, l^\mu_{e,ij} + \overline{e}^\dagger_i\overline{\sigma}_\mu\overline{e}_j\, r^\mu_{e,ij}\ , \tag{2.37e}
$$

where the newly defined broken BSM currents are

$$
a_F = s_W^2 a_W + c_W^2 a_B\ , \qquad\qquad a_Z = c_W^2 a_W + s_W^2 a_B\ , \tag{2.38a}
$$

$$
\begin{aligned}
\widetilde{a}_F &= s_W^2 \widetilde{a}_W + c_W^2 \widetilde{a}_B \ , & \widetilde{a}_Z &= c_W^2 \widetilde{a}_W + s_W^2 \widetilde{a}_B \ , \\
a_{FZ} &= 2 c_W s_W (a_W - a_B) \ , & t_{F\mu\nu} &= c_W \, t_{\mu\nu} \ , \\
\widetilde{a}_{FZ} &= 2 c_W s_W (\widetilde{a}_W - \widetilde{a}_B) \ , & t_{Z\mu\nu} &= -s_W \, t_{\mu\nu} \ ,
\end{aligned}
$$

and the antisymmetrization bracket is defined as $T^{[\mu\nu]} \equiv {}^{(T^{\mu\nu} - T^{\nu\mu})}/_{2}$. The mass-diagonalized currents interacting with the SM fermions are defined as

$$
\begin{aligned}
&r_{x,ij} \equiv V^\dagger_{x,ia} r_{x,ab} V_{x,bj} \ , \quad \Xi_i \equiv \Xi_a V_{e,ai} \ , \qquad\qquad (2.39\text{a}) \\
&l_{x,ij} \equiv U^\dagger_{x,ia} l_{x,ab} U_{x,bj} \ , \quad \Xi^\mu_i \equiv X^\mu_a V_{e,ai} \ , \quad s_{xy,ij} \equiv U^\dagger_{x,ia} s_{x,ab} V_{y,bj} \ ,
\end{aligned}
$$

where U_x, V_y are unitary matrices determined by the singular value decomposition of the corresponding SM Yukawa coupling matrix, $\Gamma_{x,ab} = U_{x,ai} m_{x,i} V^\dagger_{x,ib}$. In the absence of a SM neutrino mass term, the CKM matrix is the only combination of U_x and V_x fixed by SM observations. The "diagonalized" neutrino fields ν_i correspond to the conventional gauge-eigenstates within the standard model; if there is a SM neutrino mass term, they have to diagonalized as well, leading to an explicit appearance of the PNMS matrix.

2.4 General GeV Scale PETs

In the previous section, I constructed the general electroweak scale PETs for mediators of Spin $S = 0, 1/2$, and 1. At energies $\sqrt{s} \sim 1\,\text{GeV}$, it is appropriate to integrate out the heavy SM particles with masses at or above the charm quark mass $m_c \approx 1.5\,\text{GeV}$. This necessitates the construction of additional low energy portal Lagrangians for each mediator spin, which I do in this section. Among other applications, these Lagrangians can be used as the starting point for the derivation of more specific PETs, e.g. for chiral perturbation theory.

2.4.1 General Considerations

For the construction of each portal Lagrangian, I follow the same two step procedure as in the previous section. However, there are a few important modifications.

Most importantly, the low energy portal Lagrangians can contain higher dimensional portal operators of dimensions $d = 6$ and $d = 7$. In terms of power counting, I include portal interactions that are suppressed by at most a factor of $\epsilon_{\text{Pet}}/M_W^3$. At low energies, the SM contains only the light degrees of freedom: the massless gauge bosons, the light u_1, d_1, and d_2 quark fields, the e and μ charged lepton fields, and the standard model neutrinos. The portal Lagrangians have to be invariant under the unbroken $\text{U}(1)_V \times \text{SU}(3)_C$ gauge symmetry, but the full SM gauge symmetry is spontaneously broken, and therefore hidden at lower energies. As a result, operators that appear to violate the full $\text{SU}(3)_C \times \text{SU}(2)_I \times \text{U}(1)_Y$ SM gauge symmetry, such as e.g. the Fermi type vertex

$$(u_a^\dagger \overline{\sigma}_\mu d_b)(\overline{\xi}_i^\dagger \overline{\sigma}^\mu e_c) \ , \tag{2.40}$$

can still contribute to the portal Lagrangian. On the other hand, the low energy PETs to respect the accidental symmetries exhibited by the corresponding electroweak scale PETs. In particular, the portal interactions have to conserve baryon number, and portal interactions containing the integer spin mediators have to conserve lepton number. Portal interactions with a spin $1/2$ mediator are forbidden if they would mediate transitions with $|\Delta L| > 1$.

Chirality further restricts the number of available portal operators. For spin $1/2$ and spin 1 mediators, the electroweak scale portal operators do not mix charged fermions of different chirality. As a result, low energy portal operators that induce chirality flips of charged fermions have to involve at least a SM Yukawa vertex insertion. Consequently, these operators have to be suppressed by an additional factor of m_ψ/M_W, where m_ψ is the mass of the light fermion in question. For spin 0 mediators, the Yukawa type electroweak scale portal operators can induce a single chirality flip. The corresponding GeV scale portal operators have to be suppressed by the factor m_ψ/M_W if they involve at least two charged fermion chirality flips.

As before, I start by classifying the portal operators in terms of their field content. Up to dimension $d = 7$, the portal operators can contain either zero, two, or four fermions. Portal operators with zero fermions can only appear for integer spin mediators. Up to $d \leq 7$, these operators can contain either one, two, or three SM field strength tensors. By gauge invariance, operators with a single SM field strength tensor have to contain a photon tensor rather then a gluon tensor. For integer spin mediators, they can contain either zero or one SM field strength tensors. For spin $1/2$ mediators, they can also contain two SM field strength tensors. Operators with four fermion fields cannot contain any SM field strength tensors, as such operators would be at least of dimension $d = 8$. This way, I sort the portal operators into four types:

1. Photon type portal operators that contain one photon field strength tensor and no fermions,

2. Gauge type portal operators that contain either two or three standard model field strength tensors and no fermions,

3. Two fermion type operators that contain two fermions,

4. Four fermion type portal operators that contain four fermions.

Shorthand Notation

Fermion current type operators can contain either a scalar-type fermion bilinear of the shape $\psi\psi + \text{h.c.}$, a vector-type fermion bilinear of the shape $\psi^\dagger\overline{\sigma}^\mu\psi$, or a tensor-type fermion bilinear of the shape $\psi\overline{\sigma}^{\mu\nu}\psi$. The corresponding operators can only be gauge invariant if the fermion pair is a gauge singlet. It is convenient to define the shorthand notations

$$(\psi\psi)_0 \in \{(\nu_a\nu_b), (\overline{e}_a e_b), (\overline{u}_a u_b), (\overline{d}_a d_b)\}\ , \tag{2.41a}$$

and

$$\begin{aligned}(\psi^\dagger\psi)_- &\in \{(\nu_a^\dagger e_b), (u_a^\dagger d_b), (\overline{u}_a^\dagger \overline{d}_b)\}\ , \\ (\psi^\dagger\psi)_+ &\in \{(e_a^\dagger \nu_b), (d_a^\dagger u_b), (\overline{d}_a^\dagger \overline{u}_b)\}\ , \\ (\psi^\dagger\psi)_0 &\in \{(\nu_a^\dagger\nu_b), (e_a^\dagger e_b), (\overline{e}_a^\dagger\overline{e}_b), (u_a^\dagger u_b), (\overline{u}_a^\dagger\overline{u}_b), (d_a^\dagger d_b), (\overline{d}_a^\dagger\overline{d}_b)\}\ .\end{aligned} \tag{2.41b}$$

in order to denote generic SM fermion pairs. The subscript denotes the total electromagnetic charge of each fermion bilinear. For the four fermion operators, it is convenient to define the generic bilinears build only from charged fermions,

$$\begin{aligned}(qq)_+ &\in \{(\overline{u}_a d_b), (\overline{d}_a u_b)\}\ , \qquad (q^\dagger q^\dagger)_- \in \{(\overline{u}_a^\dagger d_b^\dagger), (\overline{d}_a^\dagger u_b^\dagger)\}\ , \\ (qq)_0 &\in \{(\overline{d}_a d_b), (\overline{u}_a u_b)\}\ , \qquad (q^\dagger q^\dagger)_0 \in \{(\overline{d}_a^\dagger d_b^\dagger), (\overline{u}_a^\dagger u_b^\dagger)\}\ .\end{aligned} \tag{2.42a}$$

$$(\widetilde{\psi}\widetilde{\psi})_0 \in \{(\overline{e}_a e_b), (\overline{d}_a d_b), (\overline{u}_a u_b)\}\ , \tag{2.42b}$$

$$(\widetilde{\psi}^\dagger\widetilde{\psi})_0 \in \{(e_a^\dagger e_b), (\overline{e}_a^\dagger\overline{e}_b), (d_a^\dagger d_b), (\overline{d}_a^\dagger\overline{d}_b), (u_a^\dagger u_b), (\overline{u}_a^\dagger\overline{u}_b)\}\ . \tag{2.42c}$$

For the field strength tensors, it is convenient to define

$$X^{\mu\nu} \in \{F^{\mu\nu}, G^{\mu\nu}\} \tag{2.43}$$

such that $X^{\mu\nu}$ denotes a generic SM field strength tensor. For gauge type portal operators, the convention is that $X^{\mu\nu}$ always has to denote the same type of field strength tensor within each operator. That is, $X_{\mu\nu}X^{\mu\nu}$ may stand for $F_{\mu\nu}F^{\mu\nu}$ or $G_{\mu\nu}G^{\mu\nu}$, but **not** for $F_{\mu\nu}G^{\mu\nu}$. As before, traces over gauge indices are implied. For fermion current type operators, it is understood that $X^{\mu\nu}$ has to be chosen such that the operator is gauge invariant. That is, $\psi\overline{\sigma}_{\mu\nu}X^{\mu\nu}\psi$ may denote e.g. $\overline{e}_a\overline{\sigma}_{\mu\nu}F^{\mu\nu}e_b$ or $\overline{u}_a\overline{\sigma}_{\mu\nu}G^{\mu\nu}u_b$, but **not** $\overline{e}_a\overline{\sigma}_{\mu\nu}G^{\mu\nu}e_b$.

SM Equation of Motion

I also use the SM equations of motion to eliminate redundant operators. At low energies, the SM equations of motion become

$$
\begin{aligned}
&\mathrm{i}\,\not{D} e_i = m_{e\,i}\,\overline{e}_i^\dagger\ , && \mathrm{i}\,\not{D}\overline{e}_i = m_{e\,i}\,e_i^\dagger\ , && \mathrm{i}\,\not{D}\nu_i = 0\ , \\
&\mathrm{i}\,\not{D} d_i = m_{d\,i}\,\overline{d}_i^\dagger\ , && \mathrm{i}\,\not{D}\overline{d}_i = m_{d\,i}\,d_i^\dagger\ , \\
&\mathrm{i}\,\not{D} u_i = m_{d\,i}\,\overline{u}_i^\dagger\ , && \mathrm{i}\,\not{D}\overline{u}_i = m_{d\,i}\,u_i^\dagger\ ,
\end{aligned}
\tag{2.44a}
$$

and

$$
\begin{aligned}
(D_\nu G^{\nu\mu})^x &= \frac{1}{2}(u_a^\dagger\overline{\sigma}^\mu\lambda^x u_a + \overline{u}_a^\dagger\overline{\sigma}^\mu\lambda^x\overline{u}_a \\
&\qquad + \overline{d}_a^\dagger\overline{\sigma}^\mu\lambda^x\overline{d}_a + d_a^\dagger\overline{\sigma}^\mu\lambda^x d_a)\ ,
\end{aligned}
\tag{2.44b}
$$

$$
\partial_\nu A^{\nu\mu} = \sum_{f\text{ is light}} q_f\,\psi_f^\dagger\overline{\sigma}^\mu\psi_f\ . \tag{2.44c}
$$

It is possible to commute covariant derivatives acting on the SM fermion fields by using the identity $[D_\mu, D_\nu]\psi_a = \mathrm{i}\,q_a X_{\mu\nu}\psi_a$. Combining the GeV scale equations of motion and the commutation relation for the covariant derivatives, one can eliminate a large number of operators involving covariant derivatives that act on the fermionic SM degrees of freedom, even if they are not of the shape $\not{D}\psi$. Explicitly, one has

$$
\begin{aligned}
D_\alpha D^\alpha\psi_a &= (\not{D}\not{D} - q_a\frac{1}{2}\overline{\sigma}_{\mu\nu}X^{\mu\nu})\psi_a \\
&\xrightarrow{\text{EoM}} -(m_a^2 + q_a\frac{1}{2}\overline{\sigma}_{\mu\nu}X^{\mu\nu})\psi_a\ ,
\end{aligned}
\tag{2.45a}
$$

as well as

$$
\begin{aligned}
\mathcal{O}^\mu\,(\psi_a\overleftrightarrow{D}_\mu\psi_b) &= \frac{1}{2}\mathcal{O}^\mu\,(\psi_a[\overleftrightarrow{\not{D}}\overline{\sigma}^\mu + \sigma^\mu\overleftrightarrow{\not{D}}]\psi_b) \\
&\xrightarrow{\text{P.I.}} -\mathcal{O}^\mu\,(\psi_a[\overleftarrow{\not{D}}\overline{\sigma}^\mu - \sigma^\mu\overrightarrow{\not{D}}]\psi_b) \\
&\qquad + (\mathrm{i}\,D^\nu\mathcal{O}^\mu)\,(\psi_a\overline{\sigma}_{\nu\mu}\psi_b) \\
&\xrightarrow{\text{EoM}} -\mathrm{i}(m_b + m_a)\mathcal{O}^\mu\,(\overline{\psi}_a^\dagger\overline{\sigma}^\mu\psi_b + \psi_a\sigma^\mu\overline{\psi}_b^\dagger) \\
&\qquad + (\mathrm{i}\,D^\nu\mathcal{O}^\mu)\,(\psi_a\overline{\sigma}_{\nu\mu}\psi_b)\ ,
\end{aligned}
\tag{2.45b}
$$

$$
\begin{aligned}
\mathcal{O}_\mu\,(\psi_a\overline{\sigma}^{\mu\nu}\overleftrightarrow{D}_\nu\psi_b) =&\ \mathrm{i}\,\mathcal{O}_\mu\,(\psi_a[\sigma^\mu \not{D} + \overleftarrow{\not{D}}\overline{\sigma}^\mu - D^\mu - \overleftarrow{D}^\mu]\psi_b) \\
\xrightarrow{\text{EoM}}&\ (m_b + m_a)\,\mathcal{O}_\mu\,(\overline{\psi}_a^\dagger\overline{\sigma}^\mu\psi_b + \psi_a\sigma^\mu\overline{\psi}_b^\dagger) \\
&+ (\mathrm{i}\,D^\mu\mathcal{O}_\mu)\,(\psi_a\psi_b)\ ,
\end{aligned}
\tag{2.45c}
$$

$$
\begin{aligned}
\mathcal{O}\,(D_\alpha\psi_a D^\alpha\psi_b) =&\ \frac{1}{2}\mathcal{O}\,(D_\alpha\psi_a)D^\alpha\psi_b + \frac{1}{2}\mathcal{O}\,(D_\alpha\psi_a)D^\alpha\psi_b \\
\xrightarrow{\text{P.I.}}&\ -\frac{1}{2}O\,(D^2\psi_a)\psi_b - \frac{1}{2}O\,\psi_a(D^2\psi_b) \\
&- \frac{1}{2}(D_\alpha O)\,\partial_\alpha(\psi_a\psi_b) \\
\xrightarrow{\text{EoM}}&\ \frac{1}{2}(D^2 O)\,\psi_a\psi_b + \frac{1}{2}(m_a^2 + m_b^2)O\,\psi_a\psi_b\ ,
\end{aligned}
\tag{2.45d}
$$

$$
\begin{aligned}
\mathcal{O}\,(D_\mu\psi_a\sigma^{\mu\nu}D_\nu\psi_b) =&\ \mathrm{i}\,\frac{1}{2}\mathcal{O}\,(\psi_a[\overleftarrow{\not{D}}\overrightarrow{\not{D}} - \overleftarrow{D}_\mu\gamma^\nu\gamma^\mu\overrightarrow{D}_\nu]\psi_b) \\
=&\ \mathrm{i}\,\mathcal{O}\,(\psi_a[\overleftarrow{\not{D}}\overrightarrow{\not{D}} - \overleftarrow{D}^\alpha\overrightarrow{D}_\alpha]\psi_b) \\
\xrightarrow{\text{EoM}}&\ \mathrm{i}\,m_a m_b\mathcal{O}\,\overline{\psi}_a^\dagger\overline{\psi}_b^\dagger - \mathrm{i}\,\mathcal{O}\,(D^\alpha\psi_a D_\alpha\psi_b)\ ,
\end{aligned}
\tag{2.45e}
$$

and finally

$$
\begin{aligned}
\mathcal{O}^{[\mu\nu]}\,(\psi_a^\dagger\overline{\sigma}_\nu\overleftrightarrow{D}_\mu\psi_b) =&\ \mathcal{O}^{[\mu\nu]}\ \mathrm{i}\,\frac{1}{2}(\psi_a^\dagger[\sigma_{\mu\nu}\overleftrightarrow{\not{D}} - \overleftrightarrow{\not{D}}\overline{\sigma}_{\mu\nu}]\psi_b) \\
\xrightarrow{\text{P.I.}}&\ \mathcal{O}^{[\mu\nu]}\ \mathrm{i}(\psi_a^\dagger[\sigma_{\mu\nu}\overrightarrow{\not{D}} + \overleftarrow{\not{D}}\overline{\sigma}_{\mu\nu}]\psi_b) \\
&+ D^\alpha\mathcal{O}^{\mu\nu}\,\mathrm{i}\,\frac{1}{2}(\psi_a^\dagger[\overline{\sigma}_\alpha\overline{\sigma}_{\mu\nu} + \sigma_{\mu\nu}\overline{\sigma}_\alpha]\psi_b) \\
\xrightarrow{\text{EoM}}&\ \mathcal{O}^{[\mu\nu]}\,(m_b\psi_a^\dagger\sigma_{\mu\nu}\overline{\psi}_b^\dagger - m_a\overline{\psi}_a\sigma_{\mu\nu}\psi_b) \\
&+ \mathrm{i}\,\frac{1}{2}(\psi_a^\dagger\overline{\sigma}_\beta\psi_b)D_\alpha\widetilde{\mathcal{O}}^{\alpha\beta}\ ,
\end{aligned}
\tag{2.45f}
$$

where ψ_a denotes any SM Weyl fermion, and $\overline{\psi}_a$ denotes its opposite chirality counterpart. For the SM neutrinos, the terms involving $\overline{\psi}_a$ have to be set to zero. The local operator $\mathcal{O}$ is a stand-in that may depend on both hidden sector and SM degrees of freedom.

2.4.2 Scalar Portal Operators

At order ${}^{\epsilon}/_{M_W^3}$, a scalar mediator may couple to the SM via operators of dimension $d \leq 7$. These interactions can involve either no, two, or four SM fermions. As before, it is possible to restrict the discussion to portal Lagrangians with two real hidden scalar fields S_i with $i = 1, 2$ without loss of generality.

1) Photon type operators: Photon type portal operators have to contain either two or four derivatives in order to be Lorentz invariant. In either case, the Lorentz indices of the photon tensor have to be contracted with two of the derivatives.

- There are no operators with four derivatives. Such operators could only contain a single hidden scalar field S_i, so that all derivatives could be made to act on the photon tensor, which implies that these operators vanish due to the antisymmetry of the field strength tensor.
- For operators with two derivatives, the SM equations of motion and the Bianchi identity can be used to eliminate all operators with at least one derivative acting on the photon tensor. Additionally, the antisymmetry of the photon tensor implies that an operator vanishes if the two derivatives act on the same field. Accounting for these restrictions, a complete basis only contains the 2 operators

$$S_i(\partial_\nu S_j)(\partial_\mu S_k)\, F^{\mu\nu} \ , \qquad S_i(\partial_\nu S_j)(\partial_\mu S_k)\, \widetilde{F}^{\mu\nu} \ . \tag{2.46}$$

2) Gauge type operators: Gauge type portal operators can contain either two or three SM field strength tensors, and the operators with two field strength tensors can contain either two photon or two gluon tensors, and either zero or two derivatives.

- Operators with two field strength tensors and without derivatives can contain either one, two, or three hidden fields. A complete basis can be constructed from the 12 operators

$$\begin{aligned} &S_i\, X_{\mu\nu}X^{\mu\nu} \ , \qquad S_iS_j\, X_{\mu\nu}X^{\mu\nu} \ , \qquad S_iS_jS_k\, X_{\mu\nu}X^{\mu\nu} \ , \\ &S_i\, X_{\mu\nu}\widetilde{X}^{\mu\nu} \ , \qquad S_iS_j\, X_{\mu\nu}\widetilde{X}^{\mu\nu} \ , \qquad S_iS_jS_k\, X_{\mu\nu}\widetilde{X}^{\mu\nu} \ . \end{aligned} \tag{2.47}$$

- Operators with two field strength tensors and two derivatives can only contain a single hidden field, so it is possible to reshuffle the derivatives such that both of them act on the field strength tensors. If both of the derivatives act on the same field strength tensor, at least one of them has be contracted with the Lorentz indices of the field strength tensor that it acts on. Hence, the operator can be eliminated using the standard model equations of motion or one of the Bianchi identities. If the two derivatives act on different field strength tensors, one can use the first identity in (2.6) to reshuffle the Lorentz indices such the indices of the derivatives are contracted with each other. A complete basis can be constructed from the 4 operators

$$S_i \, D_\alpha X_{\mu\nu} D^\alpha X^{\mu\nu} \ , \qquad\qquad S_i \, D_\alpha X_{\mu\nu} D^\alpha \widetilde{X}^{\mu\nu} \ . \tag{2.48}$$

- Portal operators with three field strength tensors have to be of the shape $S_i \, X^\mu{}_\nu Y^\nu{}_\alpha Z^\alpha{}_\mu$, where X, Y, and Z denote either a SM field strength tensor or its dual. Any operator with at least one photon tensor is forbidden since the photon tensor commutes with the other field strength tensors, which causes the trace to vanish due to the antisymmetry of the field strength tensors and the cyclicity of the trace. Hence, a complete basis can be constructed from the 2 operators

$$S_i \, G^\nu{}_\mu G^\mu{}_\alpha G^\alpha{}_\nu \ , \qquad\qquad S_i \, \widetilde{G}^\nu{}_\mu G^\mu{}_\alpha G^\alpha{}_\nu \ . \tag{2.49}$$

3) Two fermion type operators: fermion current type can contain either a scalar-type fermion bilinear, a vector-type fermion bilinear, or a tensor-type fermion bilinear.

- Portal operators with a scalar-type fermion bilinear can contain either zero or two derivatives, but they cannot involve a field strength tensors. An operator with a field strength tensor would have to contain at least two additional derivatives in order to contract the Lorentz indices, which would increase the dimension of the operator to at least $d = 8$. Operators with either one of the two derivatives acting on one of the fermion fields can be eliminated using identities (2.45a), (2.45b), and (2.45d). Thus, a complete basis can be constructed from operators of the types

$$\begin{aligned} &S_i\,(\psi\psi)_0 + \text{h.c.}\ , && S_iS_jS_k\,(\psi\psi)_0 + \text{h.c.}\ , \\ &S_iS_j\,(\psi\psi)_0 + \text{h.c.}\ , && S_iS_jS_kS_l\,(\psi\psi)_0 + \text{h.c.}\ , \\ &\partial^2 S_i\,(\psi\psi)_0 + \text{h.c.}\ , && \partial_\mu S_i \partial^\mu S_j\,(\psi\psi)_0 + \text{h.c.}\ , \\ &S_i\partial^2 S_j\,(\psi\psi)_0 + \text{h.c.}\ . && \end{aligned} \tag{2.50}$$

- Portal operators with a single vector-type fermion bilinear can contain either no field strength tensors or one field strength tensor. In either case, the operators have to involve exactly one derivative in order to contract their Lorentz indices. Operators in which the derivative acts on one of the fermion fields can be eliminated using either the SM equations of motion or identity (2.45f), while operators in which the derivative acts one the field strength tensor can be eliminated using the SM equations of motion or the Bianchi identity $D_\mu \widetilde{X}^{\mu\nu} = 0$. Therefore, a complete basis can be constructed from operators in which the derivative acts on one of the hidden fields,

$$\begin{aligned} &(S_i \overleftrightarrow{\partial}_\mu S_j)\,(\psi^\dagger \overline{\sigma}^\mu \psi)_0\ , && S_i(S_j \overleftrightarrow{\partial}_\mu S_k)\,(\psi^\dagger \overline{\sigma}^\mu \psi)_0\ , \\ &(\partial_\nu S_i)\,(\psi^\dagger \overline{\sigma}_\mu X^{\mu\nu}\psi)_0\ , && (\partial_\nu S_i)\,(\psi^\dagger \overline{\sigma}_\mu \widetilde{X}^{\mu\nu}\psi)_0\ . \end{aligned} \tag{2.51}$$

- Portal operators with a single tensor-type fermion bilinear have to contain a field strength tensor. Operators without a field strength tensor would have to contain two derivatives in order to be Lorentz invariant. If both derivatives act on the same hidden field, the corresponding operator would vanish due to the antisymmetry of $\overline{\sigma}^{\mu\nu}$. As a result, it is always possible to reshuffle the derivatives such that at least one of them acts on the SM fermion fields. Operators with only one derivative acting on the SM fermions can then be eliminated using identity (2.45c), while operators in which both derivatives act on the SM fermions can be eliminated using identities (2.45a) and (2.45e). The allowed operators with a field strength tensor cannot contain any additional derivatives, since the would increase the dimension of the operator to at least $d = 8$. Thus, the only available operators are of the types

$$S_i\,(\psi \overline{\sigma}_{\mu\nu} X^{\mu\nu} \psi)_0 + \text{h.c.}\ , \quad S_iS_j\,(\psi \overline{\sigma}_{\mu\nu} X^{\mu\nu}\psi)_0 + \text{h.c.}\ . \tag{2.52}$$

4) Four fermion type operators: Portal operators with four fermions cannot contain any field strength tensors or derivatives, as the corresponding operators would be at least of dimension $d = 8$. In terms of Lorentz structure, the only available operator are of the type

$$\begin{aligned} S\,(\psi_a^\dagger \overline{\sigma}^\mu \psi_b)(\psi_c^\dagger \overline{\sigma}_\mu \psi_d) &= -S\,(\psi_a^\dagger \overline{\sigma}^\mu \psi_b)(\psi_d \sigma_\mu \psi_c^\dagger) \\ &= -2\,S\,(\psi_a^\dagger \psi_c^\dagger)(\psi_d \psi_b)\ . \end{aligned} \tag{2.53a}$$

Operators of the type

$$S\,(\psi_a \overline{\sigma}^{\mu\nu} \psi_b)(\psi_c^\dagger \sigma_{\mu\nu} \psi_d^\dagger) = 0\ , \tag{2.53b}$$

are vanishing, while operators of the types

$$S\,(\psi_a \psi_b)(\psi_c \psi_d) + \text{h.c.}\ , \qquad S\,(\psi_a \overline{\sigma}^{\mu\nu} \psi_b)(\psi_c \overline{\sigma}_{\mu\nu} \psi_d) + \text{h.c.} \tag{2.53c}$$

have to mix fermions of different chirality in order to be gauge invariant and lepton number conserving. As a result, they have to be suppressed by at least a factor ${}^{m_\psi}/_{M_W}$ involving a light fermion mass m_ψ, which do not contribute at order ${}^{\epsilon}/_{M_W^3}$.

In terms of field content, the four fermion portal operators can contain zero, two, or four quark fields. Operators with three quark fields, as the trilinear contraction $\epsilon_{abc}\, q^a q^b q^c$ with colour indices a, b, c are forbidden because they would violate baryon number conservation. The Fierz identity (2.53a) can be used to reshuffle the fermions within the available four fermion operators. Since there are no baryon number violating operators, it is possible to construct a complete basis from products of $\mathrm{SU}(3)_C$ colour singlet fermion bilinears. For operators with two or four quark fields, this convention fully eliminates the redundancy associated with the Fierz identities. For purely leptonic operators, the remaining freedom can be used to construct a complete basis from products of electromagnetically neutral fermion bilinears.

Putting all of these restrictions together, a complete basis can be constructed from operators of the types

$$\begin{aligned} &S\,(\psi^\dagger \overline{\sigma}^\mu \psi)_0 (\psi^\dagger \overline{\sigma}_\mu \psi)_0\ , && S\,(\psi^\dagger \overline{\sigma}^\mu \psi)_+ (\psi^\dagger \overline{\sigma}_\mu \psi)_- + \text{h.c.}\ , \\ &S\,(qq)_0 (q^\dagger q^\dagger)_0 + \text{h.c.}\ , && S\,(qq)_+ (q^\dagger q^\dagger)_- + \text{h.c.}\ , \end{aligned} \tag{2.54a}$$

2.4.3 Fermion Portal Operators

At order ${}^{\epsilon}/_{M_W^3}$, a fermionic mediator can couple to the SM via operators of dimension $d \leq 7$. These interactions can involve either two or four model fermion fields, so there are no gauge type and no photon type portal operators. As before, it is possible to restrict the discussion to portal Lagrangians with hidden left-handed Weyl fermions ξ_i with $i = 1, 2$ at no loss of generality.

1) Two fermion type operators: Portal operators with two fermions can involve either two hidden fields operators or one hidden field and one SM neutrino. They can contain either a scalar-type fermion bilinear of the shape $\psi\psi$, a vector-type fermion bilinear of the shape $\psi^\dagger\overline{\sigma}^\mu\psi$, or a tensor-type fermion bilinear of the shape $\psi\overline{\sigma}^{\mu\nu}\psi$.

- Operators with a scalar-type fermion bilinear and zero SM field strength tensors have to contain a SM neutrino, and they cannot contain any derivatives, since operators with derivatives can always be eliminated using identity (2.45a). The only remaining available operator is

$$\nu_a\xi_i + \text{h.c.} \ . \tag{2.55}$$

- There are no portal operators with a scalar-type fermion bilinear and one SM fields strength tensor, since these operators would have to contain two further derivatives in order to be Lorentz invariant. However, If both derivatives act on the same field, the operator vanishes due to the antisymmetry of the field strength tensor. Therefore, it is always possible to reshuffle the derivatives such that at least one of them acts on the SM field strength tensor, which implies that the operator can be eliminated using either the SM equations of motion or the Bianchi identitiy $D_\mu\widetilde{X}^{\mu\nu}$.

- Operators with a scalar-type fermion bilinear and two SM field strength tensors cannot contain any derivatives either, as they would increase the dimension of the operator to at least $d = 9$. The Lorentz indices of the field strength tensors can be contracted using either the metric tensor $g^{\mu\nu}$ or the Levi-Civita symbol $\epsilon^{\mu\nu\alpha\beta}$, so a complete basis can be constructed from operators of the types

$$\begin{aligned} &\nu_a \xi_i\, X^{\mu\nu} X_{\mu\nu} + \text{h.c.}\ , \qquad && \xi_i \xi_j\, X^{\mu\nu} X_{\mu\nu} + \text{h.c.}\ , \\ &\nu_a \xi_i\, X^{\mu\nu} \widetilde{X}_{\mu\nu} + \text{h.c.}\ , \qquad && \xi_i \xi_j\, X^{\mu\nu} \widetilde{X}_{\mu\nu} + \text{h.c.}\ , \end{aligned} \tag{2.56}$$

- Operators with a vector-type fermion bilinear have to contain at least one derivative in order to be Lorentz invariant, and they have to contain exactly one SM field strength tensor. Operators with no field strength tensor would have to contain a standard model neutrino field operator and either one or three derivatives. These derivatives could always be reshuffled such that all of them act on the neutrino, and so the operators can be eliminated using the SM equations of motion. Operators with two field strength tensors are forbidden, because they would be at least of dimension $d = 8$ due to the one necessary derivative. Similarly, the available operators with a single field strength tensor can only contain one derivative, since operators with three derivatives would also be at least of dimension $d = 8$. If this derivative acts on the field strength tensor, the corresponding operator can be eliminated using either the SM equations of motion or the Bianchi identity $D_\mu \widetilde{X}^{\mu\nu}$. As a result, the only available operators are

$$\begin{aligned} &\nu_a^\dagger \overline{\sigma}_\mu D_\nu \xi_i\, F^{\mu\nu} + \text{h.c.}\ , \qquad && \xi_i^\dagger \overline{\sigma}_\mu D_\nu \xi_j\, F^{\mu\nu} + \text{h.c.}\ , \\ &\nu_a^\dagger \overline{\sigma}_\mu D_\nu \xi_i\, \widetilde{F}^{\mu\nu} + \text{h.c.}\ , \qquad && \xi_i^\dagger \overline{\sigma}_\mu D_\nu \xi_j\, \widetilde{F}^{\mu\nu} + \text{h.c.}\ . \end{aligned} \tag{2.57}$$

- There are no operators with a tensor-type fermion bilinear and no SM field strength tensors. These operators would have to contain least two derivatives, which can always be reshuffled such that they act on the same fermion, which implies that the operators vanish due to the antisymmetry of $\overline{\sigma}^{\mu\nu}$.

- Portal operators with a tensor-type fermion bilinear and a single field strength tensor can contain either zero or two derivatives. Operators with zero derivatives contain a total of four Lorentz indices, which have to be contracted using two metric tensors. Operators involving the Levi-Civita tensor can always be eliminated by using the identity $\epsilon^{\mu\nu\alpha\beta}\overline{\sigma}_{\alpha\beta} \propto \overline{\sigma}^{\mu\nu}$. As a result, there are only two available operators,

$$\nu_a \overline{\sigma}_{\mu\nu} \xi_i F^{\mu\nu} + \text{h.c.} \ , \qquad \xi_i \overline{\sigma}_{\mu\nu} \xi_j \, F^{\mu\nu} + \text{h.c.} \ . \tag{2.58}$$

Operators with two derivatives involve a total of six Lorentz indices, which can be contracted using either three metric tensors, or one Levi-Civita symbol and one metric tensor. The indices of $\overline{\sigma}^{\mu\nu}$ and $X^{\mu\nu}$ cannot be contracted with themselves due to their antisymmetry. For operators involving the Levi-Civita tensor, this implies that at least one the two indices of $\overline{\sigma}^{\mu\nu}$ and $X^{\mu\nu}$ has to be contracted using the Levi-Civita tensor. Furthermore, the object $\overline{\sigma}^{\mu\nu}$ fulfills the identity $\epsilon^{\mu\nu\alpha\beta}\overline{\sigma}_{\alpha\beta} \propto \overline{\sigma}^{\mu\nu}$, which implies that at least one the two indices of $\overline{\sigma}^{\mu\nu}$ has to be contracted using the metric tensor. If both derivatives act on the field strength tensor, the corresponding operators can be eliminated using the SM equations of motion and Bianchi identities. The same is true for operators with one derivative acting on the field strength tensor, provided that the derivative index is contracted either with $X^{\mu\nu}$ or $\widetilde{X}^{\mu\nu}$. As a result, the two derivatives can always be reshuffled such that they act on the same fermion field. However, if both derivatives act on a SM neutrino, the corresponding operator can be eliminated using the SM equations of motion. Therefore, the only available operators are

$$\begin{aligned}
&\xi_i \overline{\sigma}_{\mu\nu} D^2 \xi_j \, F^{\mu\nu} + \text{h.c.} \ , && \nu_a \overline{\sigma}_{\mu\nu} D^2 \xi_i \, F^{\mu\nu} + \text{h.c.} \ , \\
&\xi_i \overline{\sigma}_{\mu\alpha} D_\nu D^\mu \xi_j \, F^{\nu\alpha} + \text{h.c.} \ , && \nu_a \overline{\sigma}_{\mu\alpha} D_\nu D^\mu \xi_i \, F^{\nu\alpha} + \text{h.c.} \ , \\
&\xi_i \overline{\sigma}_{\mu\alpha} D_\nu D^\mu \xi_j \, \widetilde{F}^{\nu\alpha} + \text{h.c.} \ , && \nu_a \overline{\sigma}_{\mu\alpha} D_\nu D^\mu \xi_i \, \widetilde{F}^{\nu\alpha} + \text{h.c.} \ ,
\end{aligned} \tag{2.59}$$

- Operators with a tensor type fermion bilinear and two field strength tensors cannot contain any derivatives. The Lorentz indices of the field strength tensors can be contracted using either the metric tensor or the Levi-Civita symbol, so the only available operators are

$$\begin{aligned}
&\nu_a \sigma_{\mu\nu} \xi_i \, X_\alpha{}^\mu X^{\alpha\nu} + \text{h.c.} \ , && \xi_i \sigma_{\mu\nu} \xi_j \, X_\alpha{}^\mu X^{\alpha\nu} + \text{h.c.} \ , \\
&\nu_a \sigma_{\mu\nu} \xi_i \, X_\alpha{}^\mu \widetilde{X}^{\alpha\nu} + \text{h.c.} \ , && \xi_i \sigma_{\nu\nu} \xi_j \, X_\alpha{}^\mu \widetilde{X}^{\alpha\nu} + \text{h.c.} \ ,
\end{aligned} \tag{2.60}$$

2) Four fermion type operators: Portal operators with four fermions can contain either zero or one derivative.

- Operators without derivatives can be of the density-density types

$$S\,(\psi_a\psi_b)(\psi_c\psi_d) + \text{h.c.}\;, \quad S\,(\psi_a\overline{\sigma}^{\mu\nu}\psi_b)(\psi_c\overline{\sigma}_{\mu\nu}\psi_d) + \text{h.c.}\;, \tag{2.61a}$$

  or of the current-current type

$$\begin{aligned} S\,(\psi_a^\dagger\overline{\sigma}^\mu\psi_b)(\psi_c^\dagger\overline{\sigma}_\mu\psi_d) &= S\,(\psi_a^\dagger\overline{\sigma}^\mu\psi_b)(\psi_d\sigma_\mu\psi_c^\dagger) \\ &= 2\,S\,(\psi_a^\dagger\psi_c^\dagger)(\psi_d\psi_b)\;. \end{aligned} \tag{2.61b}$$

  As before, operators of the type

$$S\,(\psi_a\overline{\sigma}^{\mu\nu}\psi_b)(\psi_c^\dagger\sigma_{\mu\nu}\psi_d^\dagger) = 0 \tag{2.61c}$$

  are vanishing. Operators of the two density-density types in (2.61a) are related to each other via the fierz identity (2.11a). If they involve charged fermions, they have involve at least one chirality flip in order to be gauge invariant. Operators with two or more chirality flips are forbidden, since they would be suppressed by an additional factor of m_ψ^2/M_W^2.

- Operators with one derivative have to contract a vector type fermion bilinear of the shape $\psi^\dagger\overline{\sigma}^\mu\psi$ with either a scalar-type fermion bilinear of the shape $\psi\psi$ or a tensor-type fermion bilinear of the shape $\psi\overline{\sigma}^{\mu\nu}\psi$. The operators cannot involve any charged fermion chirality flips due to associated the light fermion mass suppression. In conjunction with the Fierz identities, this implies that up to two standard model fermions can always be collected within the vector-type bilinear. Furthermore, operators in which the derivative acts on one of SM fermions collected in vector type bilinear can be eliminated in SM equations of motion.

In terms of field content, the portal operators can contain either zero, two, or three charged fermions. In each case, operators with at least one lepton can be lepton number violating, but there are no lepton number violating operators with $|\Delta L| > 1$.

- There are no operators with a single charged fermion, since these operators could not be gauge invariant, whereas operators with four charged fermions are forbidden since could not contain a hidden fermion.

- Operators with no charged fermions can contain either one, two, or three neutrinos.

- Operators with two charged fermions have to contain two either two up-type quarks, two down-type quarks, or two charged leptons. Using the fierz identities (2.9a) to (2.11b), the two charged fermions can always be reshuffled into a single fermion bilinear.

- The only gauge invariant operators with three charged fermions contain an up-type quark, a down-type quark, and a charged lepton. Such operators are suppressed by a factor of ${}^{m_\psi}/_{M_W}$ for each rightchiral charged fermion spinor they contain. Using the fierz identities (2.9a) to (2.11b), it is always to reshuffle the two quark fields into a single fermion bilinear.

Putting all of the above restrictions together, one finds the following list of portal operators:

- The density-density type operators with no derivatives and without charged fermions are

$$(\nu_a\xi)(\xi_i\xi_j) + \text{h.c.} \ . \tag{2.62}$$

- The density-density type operators with no derivatives and with two charged fermions are

$$\begin{aligned} &(\psi\psi)_0(\nu_a\xi_i) + \text{h.c.} \ , \qquad (\psi\overline{\sigma}_{\mu\nu}\psi)_0(\nu_a\overline{\sigma}^{\mu\nu}\xi_i) + \text{h.c.} \ , \\ &(\psi\psi)_0(\xi_i\xi_j) + \text{h.c.} \ , \qquad (\psi\overline{\sigma}_{\mu\nu}\psi)_0(\xi_i\overline{\sigma}^{\mu\nu}\xi_j) + \text{h.c.} \ , \end{aligned} \tag{2.63}$$

  where the charged fermion bilinears $(\psi\psi)_0$ have to be chosen according to Eq. (2.42b).

- The density-density type operators with no derivatives and with three charged fermions have to involve at least one charged lepton.

There are no operators with a righthanded charged lepton, since such an operator would have to involve at least two chirality flips. A complete basis can be constructed from the operators

$$(\overline{d}_a u_b)(e_c \xi_i) + \text{h.c.}\ . \tag{2.64}$$

- The current-current type operators with no derivatives and without charged fermions are

$$\begin{aligned}&(\nu_a^\dagger \overline{\sigma}^\mu \xi_i)(\xi_j^\dagger \overline{\sigma}_\mu \xi_k) + \text{h.c.}\ , \qquad (\nu_a^\dagger \overline{\sigma}^\mu \nu_b)(\xi_i^\dagger \overline{\sigma}_\mu \xi_j) + \text{h.c.}\ , \\ &(\nu_a^\dagger \overline{\sigma}^\mu \nu_b)(\nu_c^\dagger \overline{\sigma}_\mu \xi_i) + \text{h.c.}\ .\end{aligned} \tag{2.65}$$

- The current-current type operators with no derivatives and with two charged fermions are

$$\begin{aligned}&(\widetilde{\psi}^\dagger \overline{\sigma}_\mu \widetilde{\psi})_0 (\nu_a^\dagger \overline{\sigma}^\mu \xi_i) + \text{h.c.}\ , \qquad (\widetilde{\psi}^\dagger \overline{\sigma}_\mu \widetilde{\psi})_0 (\xi_i^\dagger \overline{\sigma}^\mu \xi_j) + \text{h.c.}\ , \\ &(\widetilde{\psi}\widetilde{\psi})_0 (\nu_a^\dagger \xi_i^\dagger) + \text{h.c.}\ , \qquad (\widetilde{\psi}\widetilde{\psi})_0 (\xi_i^\dagger \xi_j^\dagger) + \text{h.c.}\ ,\end{aligned} \tag{2.66}$$

  where the charged fermion bilinears $(\widetilde{\psi}\widetilde{\psi})_0$ and $(\widetilde{\psi}^\dagger \widetilde{\psi})_0$ now have to be chosen according to Eqs. (2.42b) and (2.42c).

- The current-current type operators with no derivatives and with three charged fermions are

$$\begin{aligned}&(u_a^\dagger \overline{\sigma}^\mu d_b)(e_a^\dagger \overline{\sigma}_\mu \xi_i) + \text{h.c.}\ , \qquad (d_a^\dagger \overline{\sigma}^\mu u_b)(\overline{e}_a^\dagger \overline{\sigma}_\mu \xi_i) + \text{h.c.}\ , \\ &(\overline{u}_a d_b)(e_a^\dagger \xi_i^\dagger) + \text{h.c.}\ .\end{aligned} \tag{2.67}$$

  The operators $(\overline{d}_a^\dagger \overline{\sigma}^\mu \overline{u}_b)(e_a^\dagger \overline{\sigma}_\mu \xi_i)$, $(\overline{u}_a^\dagger \overline{\sigma}^\mu \overline{d}_b)(\overline{e}_a^\dagger \overline{\sigma}_\mu \xi_i)$, $(\overline{d}_a u_b)(\overline{e}_a^\dagger \xi_i^\dagger)$ and their hermitian conjugates do not contribute since they have to involve at least two light fermion mass insertions.

- The operators with one derivative and without charged fermions are

$$\begin{aligned}&(\nu_a^\dagger \overline{\sigma}^\mu \xi_i)(\xi_j D_\mu \xi_k) + \text{h.c.}\ , \quad (\nu_a^\dagger \overline{\sigma}_\mu \xi_i)(\xi_j \sigma^{\mu\nu} D_\nu \xi_k) + \text{h.c.}\ , \\ &(\xi_i^\dagger \overline{\sigma}^\mu \nu_a)(\xi_j D_\mu \xi_k) + \text{h.c.}\ , \quad (\xi_i^\dagger \overline{\sigma}_\mu \nu_a)(\xi_j \sigma^{\mu\nu} D_\nu \xi_k) + \text{h.c.}\ , \\ &(\nu_a^\dagger \overline{\sigma}^\mu \nu_b)(\xi_j D_\mu \xi_k) + \text{h.c.}\ , \quad (\nu_a^\dagger \overline{\sigma}_\mu \nu_b)(\xi_j \sigma^{\mu\nu} D_\nu \xi_k) + \text{h.c.}\ , \\ &(\nu_a^\dagger \overline{\sigma}^\mu \nu_b)(\nu_a D_\mu \xi_j) + \text{h.c.}\ , \quad (\nu_a^\dagger \overline{\sigma}_\mu \nu_b)(\nu_a \sigma^{\mu\nu} D_\nu \xi_j) + \text{h.c.}\ .\end{aligned} \tag{2.68}$$

- The operators with one derivative and with two charged fermions are

$$(\widetilde{\psi}^\dagger\overline{\sigma}_\mu\widetilde{\psi})_0(\nu_a D^\mu\xi_i) + \text{h.c.}\ , \quad (\widetilde{\psi}^\dagger\overline{\sigma}_\mu\widetilde{\psi})_0(\nu_a\overline{\sigma}^{\mu\nu}D_\nu\xi_i) + \text{h.c.}\ , \tag{2.69}$$
$$(\widetilde{\psi}^\dagger\overline{\sigma}_\mu\widetilde{\psi})_0(\xi_i D^\mu\xi_j) + \text{h.c.}\ , \quad (\widetilde{\psi}^\dagger\overline{\sigma}_\mu\widetilde{\psi})_0(\xi_i\overline{\sigma}^{\mu\nu}D_\nu\xi_j) + \text{h.c.}\ .$$

- The operators with one derivative and with three charged fermions are

$$(d_a^\dagger\overline{\sigma}^\mu u_b)(e_c D^\mu\widetilde{\xi}_i) + \text{h.c.}\ , \quad (d_a^\dagger\overline{\sigma}^\mu u_b)(e_c\overline{\sigma}^{\mu\nu}D_\nu\widetilde{\xi}_i) + \text{h.c.} \tag{2.70}$$

  As before, the operators $(\overline{u}_a^\dagger\overline{\sigma}^\mu\overline{d}_b)(e_c D^\mu\widetilde{\xi}_i)$, $(\overline{u}_a^\dagger\overline{\sigma}^\mu\overline{d}_b)(e_c\overline{\sigma}^{\mu\nu}D_\nu\widetilde{\xi}_i)$ and their hermitian conjugates do not contribute due to the necessary light fermion mass insertions.

2.4.4 Vector Portal Operators

At the electroweak scale, spin 1 mediators do not couple to the SM via operators of dimension $d = 5$. As a result, the corresponding GeV scale portal Lagrangian can only contain interactions that are suppressed at most by a factor of ϵ/M_W^2 rather than ϵ/M_W^3. At order ϵ/M_W^2, hidden (pseudo-)vector mediators couple to the SM via operators of dimension $d \leq 6$. One immediate consequence is that there are no portal operators with four SM fermions, since they would be at least of dimension $d = 7$. As before, the discussion can be restricted to portal Lagrangians involving only a single vector field V_μ.

As for the electroweak scale portal Lagrangian, there are no purely bosonic portal operators of dimension $d = 4$. The available operators of dimension $d = 6$ contain a total of six Lorentz indices, which can be contracted using either three metric tensors or using one Levi-Civita symbol and one metric tensor. The indices of the field strength tensors cannot be contracted with each other. For operators with a Levi-Civita tensor, this implies that at least one index from each of the field strength tensors has to be contracted by the Levi-Civita tensor.

It is convenient to define the shorthand notation

$$\partial V \equiv \partial^\alpha V_\alpha\ , \qquad V^2 \equiv V^\alpha V_\alpha\ , \tag{2.71}$$
$$V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu\ , \quad \hat{V}_{\mu\nu} \equiv \partial_\mu V_\nu + \partial_\nu V_\mu\ , \quad \widetilde{V}_{\mu\nu} \equiv 2\epsilon_{\mu\nu\alpha\beta}\partial^\alpha V^\beta\ .$$

1) Photon type operators:

- There are no operators with zero or three derivatives. Operators without any derivatives vanish due to the antisymmetry of the field strength tensor, since both of the Lorentz indices of the field strength tensor have to be contracted with a hidden field operator. Operators with three derivatives vanish since the derivatives can always be made to act on the SM field strength tensor. Therefore, these operators can always be eliminated using the SM equations of motion or the Bianchi identity.

- Operators with one derivative have to involve three hidden vector fields. The derivative can always be reshuffled such that it does not act on the field strength tensor, so all of the operators have to contain exactly one instance of the objects $V_{\mu\nu}$, $\hat{V}_{\mu\nu}$, and $\widetilde{V}_{\mu\nu}$. Accounting for the different possibilities of contracting Lorentz indices, the only available operators are

$$\begin{aligned} &V_\nu V^\mu \hat{V}_{\mu\alpha} F^{\nu\alpha} \ , && V_\nu V^\mu V_{\mu\alpha} F^{\nu\alpha} \ , && V_\nu V^\mu \widetilde{V}_{\mu\alpha} F^{\nu\alpha} \ , \\ &V^2 V_{\mu\nu} F^{\mu\nu} \ , && V^2 \widetilde{V}_{\mu\nu} F^{\mu\nu} \ , && V_\nu V^\mu V_{\mu\alpha} \widetilde{F}^{\nu\alpha} \ . \end{aligned} \tag{2.72}$$

- Operators with two derivatives have to involve two hidden field operators. A complete basis can be constructed from operators of the types

$$\begin{aligned} &\partial V V_{\mu\nu} F^{\mu\nu} \ , && V_\mu \partial^2 V_\nu F^{\mu\nu} \ , && V_{\mu\nu} V_\alpha \partial^\alpha F^{\mu\nu} \ , \\ &\partial V V_{\mu\nu} \widetilde{F}^{\mu\nu} \ , && V_\mu \partial^2 V_\nu \widetilde{F}^{\mu\nu} \ , && V_{\mu\nu} V_\alpha \partial^\alpha \widetilde{F}^{\mu\nu} \ , \\ &\widetilde{V}^{\mu\nu} V_{\mu\alpha} F^\alpha_{\ \nu} && \widetilde{V}^{\mu\nu} \hat{V}_{\mu\alpha} F^\alpha_{\ \nu} \end{aligned} \tag{2.73}$$

2) Gauge type operators: Portal operators with two field strength tensors can contain either two photon tensors or two gluon tensors, and either two hidden fields or one hidden field and one derivative.

- For operators with two hidden fields, a complete basis can be constructed from operators of the shapes

$$\begin{aligned} &V^2 X^{\mu\nu} X_{\mu\nu} \ , && V_\mu V^\nu X^{\mu\alpha} X_{\nu\alpha} \ , \\ &V^2 X^{\mu\nu} \widetilde{X}_{\mu\nu} \ , && V_\mu V^\nu X^{\mu\alpha} \widetilde{X}_{\alpha\nu} \ . \end{aligned} \tag{2.74}$$

- For operators with two field strength tensors and one derivative, and one hidden field, the derivative can act either on one of the field strength tensors or the hidden field operator. The derivative can always be reshuffled such that it acts on the hidden field. Accounting for the different possibilities of constructing the Lorentz indices, the only available operators are

$$\partial V X^{\mu\nu} X_{\mu\nu} \ , \qquad \partial V X^{\mu\nu} \widetilde{X}_{\mu\nu} \ , \qquad \partial_\mu V^\nu X^{\mu\alpha} \widetilde{X}_{\alpha\nu} \ , \tag{2.75}$$

3) Two fermion type operators: fermion current type operators can contain either a scalar-type fermion bilinear of the type $\psi\psi$, a vector-type fermion bilinear of the type $\psi^\dagger\overline{\sigma}^\mu\psi$, or a tensor-type fermion bilinear $\psi\overline{\sigma}^{\mu\nu}\psi$.

Portal operators with a scalar type fermion bilinear or a tensor type fermion bilinear cannot contain a SM field strength tensor, as such an operator would have to contain at least two more derivatives or hidden field operators, so that its dimension would be at least $d = 7$. Operators with a vector type fermion bilinear can contain either zero or one SM field strength tensor.

- Operators with a scalar-type fermion bilinear can contain either two hidden field operators or one hidden field operator and one derivative. If the derivative acts on either of the SM fermions, the operator can be eliminated using relation (2.45a). A complete basis can be constructed from the operators

$$V^2(\psi\psi)_0 + \text{h.c.} \ , \qquad \partial V(\psi\psi)_0 + \text{h.c.} \ . \tag{2.76}$$

- Operators with a vector type fermion bilinear and no field strength tensor can contain either zero, one, or two derivatives. The only available operators without derivatives are

$$V_\mu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \qquad V^2 V_\mu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ . \tag{2.77}$$

  Operators with one derivative have to involve two hidden field operators in order to be Lorentz invariant. The derivative can either act on one of the hidden fields or one of the SM fermions.

Accounting for different ways of contracting the Lorentz indices, the only available operators with one derivative are

$$\begin{aligned} &V_{\mu\nu}V^\nu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \quad \hat{V}_{\mu\nu}V^\nu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \qquad \widetilde{V}_{\mu\nu}V^\nu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \\ &\partial V V_\mu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \quad V_\mu V_\nu(\psi^\dagger\overline{\sigma}^\mu D^\nu\psi)_0 \ . \end{aligned} \tag{2.78}$$

Operators with two derivatives can only involve a single hidden field. If both derivatives act one the same SM fermion field, the corresponding operator can be eliminated using the SM equations. As a result, the derivatives can always be reshuffled such that at least one of them acts on the hidden field operator. However, operators involving either $V_{\mu\nu}$ or $\widetilde{V}_{\mu\nu}$ can be eliminated using relation (2.45f). The only available operators are

$$\partial^2 V_\mu(\psi^\dagger\overline{\sigma}^\mu\psi)_0 \ , \qquad \hat{V}_{\mu\nu}(\psi^\dagger\overline{\sigma}^\mu D^\nu\psi)_0 \ . \tag{2.79}$$

- Operators with a vector type fermion bilinear and one field strength tensor cannot contain any derivatives. Accounting for the two ways of contracting the Lorentz indices, the only available operators are

$$V^\mu(\psi^\dagger\overline{\sigma}^\nu X_{\mu\nu}\psi)_0 \ , \qquad V^\mu(\psi^\dagger\overline{\sigma}^\nu \widetilde{X}_{\mu\nu}\psi)_0 \ . \tag{2.80}$$

- Operators with a tensor-type fermion bilinear has to contain one hidden field operator and one derivative. Operators with two hidden vector field operators vanish due to the antisymmetry of $\overline{\sigma}^{\mu\nu}$. If the derivative acts on either of the SM fermions, the operator can be eliminated using relation (2.45c). Therefore, the only available operators are of the type

$$V_{\mu\nu}(\psi\overline{\sigma}^{\mu\nu}\psi)_0 + \text{h.c.} \ . \tag{2.81}$$

2.4.5 The Subset of $|\Delta F| = 1$ Portal Operators

In the remainder of this work, I am primarily interested in operators that mediate quark flavour changing transitions at low energies. In general, the relevant diagrams can be sorted into two types: Firstly, diagrams with one $|\Delta F| = 1$ SM Fermi interaction and one $|\Delta F| = 0$ portal

	d	Two Quark +h.c.	Quark Dipole +h.c.	Four fermion +h.c.
$S_{1,2}$	4	$S_i\,\overline{d}_a d_b$		
	5	$S_i S_j\,\overline{d}_a d_b$		
	6	$S_i S_j S_k\,\overline{d}_a d_b$ $\partial^2 S_i\,\overline{d}_a d_b$ $S_i \partial_\mu S_j\, d_a^\dagger \overline{\sigma}^\mu d_b$	$S_i\,\overline{d}_a \sigma_{\mu\nu} F^{\mu\nu} d_b$ $S_i\,\overline{d}_a \sigma_{\mu\nu} G^{\mu\nu} d_b$	
	7	$S_i S_j S_k S_l\,\overline{d}_a d_b$		$S_i\,(u_a^\dagger \overline{\sigma}^\mu d_b)(e_c^\dagger \overline{\sigma}_\mu \nu_d)$ $S_i\,(q_a^\dagger \overline{\sigma}^\mu q_b)(q_c^\dagger \overline{\sigma}_\mu q_d)$ $S_i\,(d_a^\dagger \overline{\sigma}^\mu d_b)(\nu_c^\dagger \overline{\sigma}_\mu \nu_d)$ $S_i\,(q_a^\dagger \overline{\sigma}^\mu q_b)(\overline{q}_c^\dagger \overline{\sigma}_\mu \overline{q}_d)$ $S_i\,(q_a^\dagger \overline{q}_b^\dagger)(\overline{q}_c q_d)$
$\xi_{1,2}$	6			$(\xi_i^\dagger\,\overline{\sigma}_\mu e_a)(d_b^\dagger \overline{\sigma}^\mu u_c)$ $(\xi_i^\dagger\,\overline{\sigma}_\mu \nu_a)(d_b^\dagger \overline{\sigma}^\mu d_c)$ $(\xi_i^\dagger\,\overline{\sigma}^\mu \xi_j)(d_b^\dagger \overline{\sigma}^\mu d_c)$
V_μ	4	$V_\mu\, d_a^\dagger \overline{\sigma}^\mu d_b$		

Table 2.2: Quark flavour changing GeV scale portal operators for mediators with Spin $S \leq 1$.

interaction, and secondly, diagrams with one $|\Delta F| = 1$ portal interaction but no higher dimensional SM interactions.

Higher dimensional portal interactions of dimension $d = 6$ or $d = 7$ only enter into diagrams of the second type. Consequently, I only include higher dimensional operators if they are quark flavour changing, and if they contribute in diagrams that are of the same order as the leading order diagrams of the first type. Quark flavour changing operators have to contain either two or four SM fermions, and at least one down-type quark, since there is only one up-type quark flavour in theory at low energies.

Previously, I only kept track of the suppression associated with higher dimensional operators. Now, I also account for the $(4\pi)^{-2}$ suppression factors associated with loop corrections. In terms of the NDA power counting, the SM Fermi interactions have to be counted as ${}^{(4\pi)^2}\!/\!{}_{M_W^2}$, where the $(4\pi)^2$ enhancement factor might be thought of as resulting from the operators being generated at tree level. For spin 0 and spin $1/2$ mediators, the $d \leq 5$ portal operators with $|\Delta F| = 0$ are suppressed by at most a factor of ϵ/M_W, and for spin 1 mediators, they are suppressed by at most a factor of ϵ. As a result, diagrams of the second type only contribute at the same order as a leading order diagram of the first type if the corresponding higher dimensional portal operators are suppressed at most by a factor of $(4\pi)^2\epsilon/M_W^3$ (for spin 0 and spin $1/2$ mediators), or a factor of $(4\pi)^2\epsilon/M_W^2$ (for spin 1 mediators).

There are two more things to consider when counting powers of (4π). Firstly, the portal coupling ϵ itself has to be associated with a factor of $1/4\pi$ in order for power counting to be self-consistent. This can be seen from considering e.g. the portal operator $\epsilon S_i\,\psi\psi$, which has to be for order ϵ rather than of order $4\pi\epsilon$, since it is of zeroth order in the $1/M_W$ expansion that underlies the NDA power counting in this chapter. Secondly, for integer spin mediators, the electroweak scale portal operators involve only at most a single hidden field. Therefore, the corresponding low energy portal operators with more than one hidden field operator have to involve at least one hidden sector interaction. Assuming that the hidden sector interactions are perturbative, this implies that such operators have to be suppressed by at least an additional factor of $1/4\pi$.

The relevant quark flavour violating operators are summarized in table 2.2. Ignoring the $(4\pi)^n$ power counting rules of NDA, one would have to include a number of additional portal operators:

- For spin 0 mediators, one would have to include operators of order $4\pi\epsilon/M_W^3$ and ϵ/M_W^3. At order $4\pi\epsilon/M_W^3$, one has

$$\begin{aligned} &S_iS_j\partial_\mu S_k\, d_a^\dagger\overline{\sigma}^\mu d_b\ , \qquad && S_iS_j\,\overline{d}_a\overline{\sigma}_{\mu\nu}X^{\mu\nu}d_b + \text{h.c.}\ , \\ &\partial_\nu S_i\, d_a^\dagger\overline{\sigma}_\mu X^{\mu\nu} d_b\ , && \partial_\nu S_i\, d_a^\dagger\overline{\sigma}_\mu \widetilde{X}^{\mu\nu} d_b\ . \end{aligned} \tag{2.82a}$$

  At order ϵ/M_W^3, one has

$$S_i\partial^2 S_j\,\overline{d}_a d_b + \text{h.c.}\ , \qquad \partial_\alpha S_i\partial^\alpha S_j\,\overline{d}_a d_b + \text{h.c.}\ . \tag{2.82b}$$

- For spin 0 mediators, one would have to include operators of order ${}^{4\pi\epsilon}/_{M_W^3}$. At order ${}^{4\pi\epsilon}/_{M_W^3}$, one has

$$
\begin{aligned}
&(\overline{d}_a d_b)(\nu_c \xi_i) + \text{h.c.}\ , && (\overline{d}_a \overline{\sigma}_{\mu\nu} d_b)(\nu_c \overline{\sigma}^{\mu\nu} \xi_i) + \text{h.c.}\ , \\
&(\overline{d}_a d_b)(\xi_i \xi_j) + \text{h.c.}\ , && (\overline{d}_a \overline{\sigma}_{\mu\nu} d_b)(\xi_i \overline{\sigma}^{\mu\nu} \xi_j) + \text{h.c.}\ , \\
&(\overline{d}_a d_b)(\nu_c^\dagger \xi_i^\dagger) + \text{h.c.}\ , && (\overline{d}_a u_b)(e_c \xi_i) + \text{h.c.}\ , \\
&(\overline{d}_a d_b)(\xi_i^\dagger \xi_j^\dagger) + \text{h.c.}\ , && (\overline{u}_a d_b)(e_c^\dagger \xi_i^\dagger) + \text{h.c.}\ , \\
& && (d_a^\dagger \overline{\sigma}^\mu u_b)(\overline{e}_c^\dagger \overline{\sigma}_\mu \xi_i) + \text{h.c.}\ ,
\end{aligned}
\tag{2.83a}
$$

which are $\propto {}^1/_{M_W^3}$ due to the necessary light fermion mass insertions, and

$$
\begin{aligned}
&(d_a^\dagger \overline{\sigma}_\mu d_b)(\nu_a D^\mu \xi_i) + \text{h.c.}\ , && (d_a^\dagger \overline{\sigma}_\mu d_b)(\nu_a \overline{\sigma}^{\mu\nu} D_\nu \xi_i) + \text{h.c.}\ , \\
&(d_a^\dagger \overline{\sigma}_\mu d_b)(\xi_i D^\mu \xi_j) + \text{h.c.}\ , && (d_a^\dagger \overline{\sigma}_\mu d_b)(\xi_i \overline{\sigma}^{\mu\nu} D_\nu \xi_j) + \text{h.c.}\ , \\
&(d_a^\dagger \overline{\sigma}^\mu u_b)(e_c D^\mu \xi_i) + \text{h.c.}\ , && (d_a^\dagger \overline{\sigma}^\mu u_b)(e_c \overline{\sigma}^{\mu\nu} D_\nu \xi_i) + \text{h.c.}\ .
\end{aligned}
\tag{2.83b}
$$

- For spin 1 mediators, one would have to include operators of order ${}^{4\pi\epsilon}/_{M_W^2}$ and ${}^{\epsilon}/_{M_W^2}$. At order ${}^{4\pi\epsilon}/_{M_W^2}$, one has

$$
V^2 V_\nu\, d_a^\dagger \overline{\sigma}^\nu d_b\ , \qquad V^\mu\, d_a^\dagger \overline{\sigma}^\nu X_{\mu\nu} d_b\ , \qquad V^\mu\, d_a^\dagger \overline{\sigma}^\nu \widetilde{X}_{\mu\nu} d_b\ . \tag{2.84a}
$$

At order ${}^{\epsilon}/_{M_W^2}$, one has

$$
\begin{aligned}
&V^2\, \overline{d}_a d_b + \text{h.c.}\ , && V_\mu\, \overline{d}_a^\dagger \overline{\sigma}^\mu \overline{d}_b\ , \\
&\partial V\, \overline{d}_a d_b + \text{h.c.}\ , && V_{\mu\nu}\, \overline{d}_a \overline{\sigma}^{\mu\nu} d_b + \text{h.c.}\ ,
\end{aligned}
\tag{2.84b}
$$

which are $\propto {}^1/_{M_W^2}$ due to the necessary light quark mass insertions, and

$$
\begin{aligned}
&V_{\mu\nu} V^\nu d_a^\dagger \overline{\sigma}^\mu d_b\ , && \partial V V_\mu d_a^\dagger \overline{\sigma}^\mu d_b\ , && V_\mu V_\nu d_a^\dagger \overline{\sigma}^\mu D^\nu d_b\ , \\
&\hat{V}_{\mu\nu} V^\nu d_a^\dagger \overline{\sigma}^\mu d_b\ , && \partial^2 V_\mu d_a^\dagger \overline{\sigma}^\mu d_b\ , && \hat{V}_{\mu\nu} d_a^\dagger \overline{\sigma}^\mu D^\nu d_b\ , \\
&\widetilde{V}_{\mu\nu} V^\nu d_a^\dagger \overline{\sigma}^\mu d_b\ .
\end{aligned}
\tag{2.84c}
$$

Chapter 3

PET Chiral Perturbation Theory

3.1 The External Current Picture

In this chapter, I derive a specific PET that couples the light pseudo scalar mesons of QCD to generic hidden sectors. At the level of the SM, the physics of these mesons is described by chiral perturbation theory ($= \chi$PT), and the corresponding χPT Lagrangian is well known [30, 10, 31, 32, 33, 11, 34, 35]. An elementary description of the light mesons on the quark-gluon level is unfeasible, since quantum chromo dynamics (=QCD) is non-perturbative at low energies, say, below the charm mass $m_c \sim 1.5\,\mathrm{GeV}$. Hence, the χPT portal Lagrangian, rather than the elementary QCD portal Lagrangian, is the suitable tool for studying the coupling of the SM mesons to generic hidden sectors.

Following the restrictions established in chapter 2, I consider hidden sector models that couple to the SM via a single light hidden mediator with spin ≤ 1 and a mass below $M_K \approx 500\,\mathrm{MeV}$. The corresponding set of portal operators has been constructed in section 2.4, which provides the starting point for the discussion in this chapter. Neglecting electroweak corrections, one has to include only the portal operators with dimension $d \leq 5$. However, I am particularly interested in hidden sector induced, quark flavour changing processes, such as charged Kaon decays $K^+ \to \pi^+ S$, which are mediated by higher dimensional operators associated with virtual $W^{\pm}$ boson exchanges. To capture these transitions, one also has to include the dimension $d = 6$ and $d = 7$ operators collected in table 2.2.

To deduce the corresponding PET χPT Lagrangian, I do not consider the hidden fields to be dynamical degrees of freedom, but rather external currents that function either as sources or drains of energy and momentum associated with SM excitations. Within the SM, this external current picture is the standard approach to the coupling of χPT to the SM photons at leading order, see e.g. [10, 31, 32, 36, 34, 35, 37]. The main downside of this approach is that it neglects χPT operators induced by virtual photon exchanges, which start to appear at order $\mathcal{O}(e^2)$. Within the SM, these operators have to be accounted for, and they are necessary to accurately capture e.g. the ${}^{\epsilon'}/_{\epsilon}$ ratio, or the mass splitting between neutral and charged pions. In particular, $\Delta I = {}^1/_2$ rule enhanced electromagnetic penguin contributions are important for the correct SM estimation of the ${}^{\epsilon'}/_{\epsilon}$ ratio [38, 39, 40]. For extensive listings of these operators see e.g. [41, 42, 43, 44, 45, 46].

In contrast, the PET framework is constructed on the assumption that it is sufficient to capture the coupling to hidden sectors at order $\mathcal{O}(\epsilon)$, so that the external current picture is suitable for capturing all of the necessary effective portal operators in χPT. In what follows, I will use the external current description to capture the coupling of QCD to both hidden sectors *and* the remainder of the SM, neglecting the χPT operators generated by virtual photon exchanges. This way, the coupling of QCD to external sectors can be summarized in terms of 10 generic external currents, that I denote as l_μ, r_μ, χ, a, ϑ, ξ, $t_{\mu\nu}$, A, B, and C.

Except for a, all of these currents are present in the SM. In particular, the impact of the currents l_μ, r_μ, χ, and ϑ is well understood, and their inclusion at and beyond leading order is discussed in many standard introductions to χPT [10, 31, 32, 34]. The leading order χPT operators with $t_{\mu\nu}$ have been listed in [37]. The remaining currents A, B, C, and ξ are constant in the standard model, and all of them couple to higher order operators O_i that are induced by virtual heavy gauge boson exchanges. Essentially, they are the Wilson coefficients associated with the effective operators O_i. Hence, the resulting contributions to χPT can be obtained by appealing directly to the transformation behaviour of the operators O_i under chiral rotations of the quark fields [47, 48, 49, 50, 51].

Compared to the SM case, the novelty of the work presented in this chapter consists of three main contributions:

1. I add the external current a. It does not appear in the SM, so that its inclusion is of no relevance for the construction of χPT in the SM. If the Higgs boson was light, an effective operator $HG_{\mu\nu}G^{\mu\nu}$ would contribute to a, and the impact of such a vertex in χPT has been studied in [52]. Here, I use the external current picture to generalize the approach taken there to account for generic hidden scalars.

2. In the SM, the currents A, B, C, and ξ are constant, and the resulting modifications in χPT can be found by appealing directly to the behaviour of the operators O_i under chiral rotations. However, it is difficult to generalize this approach to the case with spacetime dependent currents. Using the external current approach, I extend the description in χPT to include generic, spacetime dependent currents A, B, C, and ξ.

3. The final PET χPT Lagrangian contains a number of new low energy constants (=LECs) associated with operators containing the external currents a, A, B, C, and ξ. Here, I estimate the LECs associated to a, A, B, and C. For the A, B and C LECs, I combine the strategies used in [52, 53, 54], where large N_c factorization rules are combined with the well-known low-energy realizations of QCD quark bilinears [30, 31, 32]. For the a LECs, I use the strategy of [52], where the conformal anomaly of QCD is exploited to obtain a low-energy realization for the gluon kinetic term $G_{\mu\nu}G^{\mu\nu}$.

The remainder of this chapter is structured as follows: In section 3.2, I give a short review of QCD at low energies, with a focus on the conformal anomaly of QCD, the large N_c limit, and chiral symmetry. In section 3.3, I summarize the coupling of QCD to the other sectors of the SM and to generic hidden sectors, and construct the resulting external currents. In section 3.4, I derive the actual χPT portal Lagrangian, and in section 3.5 I estimate the new LECs.

3.2 Aspects of QCD at Low Energies

For the sake of completeness, I summarize few standard aspects of QCD at low energies that are relevant for the discussion in the remainder of this chapter. In particular, I discuss the conformal anomaly of QCD, the large N_c limit, aspects of chiral symmetry, and the Pseudo Nambu-Goldstone bosons (=PNGBs).

3.2.1 The QCD Lagrangian

QCD is a $\mathrm{SU}(N_c = 3)$ gauge theory with $N_f = 6$ quark colour triplets that transform as members of the fundamental representation of the gauge group. In pure QCD, the quarks remain massless, but within the SM, quark masses are generated by electroweak scale Yukawa interactions involving the Higgs boson.

The 6 quark fields can be grouped into three light quark flavours u, d, and s with masses well below 1 GeV, and three heavy quark flavours c, b, and t with masses well above 1 GeV. Usually, the heavy quark fields are integrated out at energies below the charm quark mass $m_c \sim 1.5\,\mathrm{GeV}$, so that QCD contains only the $N_f = 3$ light quark flavours and the $N_c^2 - 1 = 8$ gluon fields associated with the gauge symmetry. It is convenient to decompose the resulting QCD Lagrangian as

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{mass}} + \mathcal{L}_{\theta} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{ghost}} \ , \tag{3.1}$$

where

$$\mathcal{L}_{\mathrm{kin}} \equiv \mathrm{i}\, q^{\dagger}\overline{\sigma}^{\mu} D_{\mu} q + \mathrm{i}\, \overline{q}^{\dagger}\sigma^{\mu} D_{\mu}\overline{q}^{\dagger} - \frac{\omega_s}{16\pi^2} G_{\mu\nu}G^{\mu\nu} \ , \tag{3.2a}$$

$$\mathcal{L}_{\mathrm{mass}} \equiv -(\overline{q} M_q q + \mathrm{h.c.}) \ , \qquad \mathcal{L}_{\theta} \equiv -\frac{\theta}{16\pi^2}\widetilde{G}_{\mu\nu}G^{\mu\nu} \ . \tag{3.2b}$$

$\omega_s \equiv {}^{2\pi}/_{\alpha_s}$ is the inverse of the QCD fine structure constant, $M_q = \mathrm{diag}(m_u, m_d, m_s)$ is the quark mass matrix, and θ is the QCD theta angle. The chiral quark fields $q = (u, d, s)$, $\overline{q} = (\overline{u}, \overline{d}, \overline{d})$ are defined to be triplets in flavour space. As before, I follow the two-component notation of [26]. The covariant derivatives contain only the gluon fields A_{μ}, and colour traces in all terms are implied. The θ term can be written as a total derivative,

$$\widetilde{G}_{\mu\nu}G^{\mu\nu} \propto \partial_\mu \epsilon^{\mu\nu\alpha\beta}(A_\nu G_{\alpha\beta} - \frac{2}{3}A_\nu A_\alpha A_\beta) \ , \tag{3.3}$$

but it does not vanish, since the topologically nontrivial vacuum structure of non-abelian Yang-Mills theories allows for extended field configurations with both a finite euclidian action and nonzero values for the gluon fields at infinity. Experimentally, one finds $|\theta| \lesssim 10^{-10}$[55].

The final two contributions result from the gauge fixing procedure of non-abelian gauge theories. $\mathcal{L}_{\mathrm{GF}}$ denotes the gauge fixing Lagrangian, and $\mathcal{L}_{\mathrm{ghost}}$ is the corresponding ghost Lagrangian. In covariant gauges, they become

$$\mathcal{L}_{\mathrm{GF}} = \frac{1}{\xi}\partial_\mu A^\mu \partial_\nu A^\nu \ , \qquad\qquad \mathcal{L}_{\mathrm{ghost}} = 2\partial_\mu \overline{c} D^\mu c \ , \tag{3.4}$$

where ξ is the gauge fixing parameter, $c = \lambda_a c^a$ and $\overline{c} = \lambda_a \overline{c}^a$ are the ghost fields of QCD, and D^μ is the covariant derivative of the adjoint representation. As before, colour traces are implied.

3.2.2 Conformal Anomaly

The kinetic term (3.2a) is invariant under the scale transformation

$$g^{\mu\nu} \to e^{-2t} g^{\mu\nu} \ , \tag{3.5}$$

where $t \in \mathbb{R}$. Even without external currents, this scale invariance is broken by the quark mass term and by the running of the QCD fine structure constant $\alpha_s = {}^{2\pi}/_{\omega_s}$[56, 57, 58]. The size of this symmetry breaking can be quantified in terms of the trace of the improved energy-momentum tensor,

$$\begin{aligned}\Theta_{\mu\nu}(x) \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{QCD}}}{\delta g^{\mu\nu}(x)} = q^\dagger \overline{\sigma}_{(\mu}\, \mathrm{i}\, D_{\nu)} q + \overline{q}^\dagger \sigma_{(\mu}\, \mathrm{i}\, D_{\nu)} \overline{q} \\ - \frac{\omega_s}{32\pi^2} G_{\mu\alpha} G^\alpha_{\ \nu} - \eta_{\mu\nu}\mathcal{L}_{\mathrm{QCD}} \ ,\end{aligned} \tag{3.6a}$$

where the symmetrized indices are defined such that $T_{(\mu\nu)} \equiv 1/2(T_{\mu\nu} + T_{\nu\mu})$. At tree-level, $\Theta_{\mu\nu}$ can be simplified by using the classical equations of motion for the quark fields, which yields

$$\Theta_{\mu}^{\ \mu} \equiv \overline{q} M_q q + \text{h.c.} \ . \tag{3.6b}$$

The part of the symmetry breaking that is associated with the running of α_s is captured by loops corrections to the trace relation (3.6b). In dimensional regularization, the transformation $g^{\mu\nu} \to e^{-2t} g_{\mu\nu}$ modifies the renormalization scale μ, giving $\mu \to e^t \mu$. Hence, the renormalized coupling $\omega_s(\mu)$ has to be replaced with $\omega_s(e^t\mu)$, modifying the QCD Lagrangian (3.1). Working within the $\overline{\text{MS}}$ scheme, ω_s runs according to the renormalization group equation

$$\frac{\mathrm{d}\omega_s}{\mathrm{d}t} = -\left(\frac{4\pi}{\alpha_s}\right)^2 \beta_s(\alpha_s) = \beta_0 + \frac{\alpha_s}{4\pi}\beta_1 + \dots \ , \tag{3.7}$$

where the two leading coefficients of the beta function are [59, 55]

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f \ , \quad \beta_1 = \frac{34}{3}N_c^2 - 2\frac{N_c^2 - 1}{2N_c}N_f - \frac{10}{3}N_c N_f \ . \tag{3.8}$$

Accounting for the running at 1-loop, this yields the modified trace relation[60]

$$\Theta_{\mu}^{\mu} \equiv -\frac{\beta_0}{32\pi^2} G_{\mu\nu}G^{\mu\nu} + \overline{q} M_q q + \text{h.c.} \ . \tag{3.9}$$

In principle, the 1-loop trace relation receives further corrections from the anomalous dimensions of the quark masses[60], giving

$$M_q \to (\mathbb{1} + \gamma_M) M_q \ . \tag{3.10}$$

However, only the contribution from the running of ω_s is relevant to the discussion in the remainder of this chapter, since it can be used to deduce the low energy realization of the gluon kinetic term by re-expressing it as a linear combination of the Θ_{μ}^{μ}, the quark mass term, and other operators. For this purpose, the anomalous dimensions of the quark masses only yield a small correction compared to the contribution generated by the quark masses at tree level. Hence, it is sufficient to use relation (3.9) instead.

Aside from considerations of symmetry breaking, it is worth noting that the running of α_s has profound consequences for the behaviour of QCD at low energies. Most strikingly, the renormalization group equation (3.7) exhibits a Landau pole at $\Lambda_{\text{QCD}}^{\overline{\text{MS}}} = (332 \pm 17)\,\text{MeV}$[55], which is ultimately what causes the the theory to be nonperturbative at low energies.

3.2.3 Large N_c QCD

At low energies, QCD becomes nonperturbative, that is, the perturbative expansion in powers of α_s no longer yields well defined experimental predictions. Instead, a physically meaningful theory can be obtained by expanding QCD in powers of $\delta \equiv {}^1/N_c = {}^1/3$, with the strict large N_c limit corresponding to $N_c \to \infty$. In this section, I introduce the basic power counting of the large N_c expansion in QCD using material collected from [61, 62, 63].

At first glance, the large N_c approach seems questionable, since one might expect that numerical prefactors could easily compensate for the "small" prefactor of ${}^1/3$. However, the large N_c expansion qualitatively reproduces a number of features of QCD at low energies, such as the OZI rule [64, 65, 66], which states that QCD diagrams are expected to be suppressed if they can be separated into disconnected subdiagrams by dropping all of the gluon lines. Furthermore, the large N_c expansion produces excellent predictions for certain observables, such as the neutral Kaon mixing parameter B_K. In the strict large N_c limit, one obtains $B_K = 0.75$, compared to $B_K \approx (0.7625 \pm 0.97)$ from lattice QCD [67, 68, 69, 70]. That being said, improvements beyond the strict large N_c limit are technically challenging, and due to relatively large expansion parameter ${}^1/3$, the corresponding beyond leading order contributions may be sizable.

On a technical level, the $N_c \to \infty$ limit has to be taken such that the QCD scale Λ_{QCD} remains finite, since Λ_{QCD} defines the typical energy scale of the nonperturbative physics in QCD. For instance, if the limit $N_c \to \infty$ is taken while keeping g_s fixed, one finds that $\Lambda_{\text{QCD}} \to 0$, and QCD remains perturbative at arbitrarily low energies. To retain a finite QCD scale Λ_{QCD}, the strong coupling constant g_s has to be rescaled such that $\alpha_s' = \alpha_s N_c$ remains constant. Hence, each factor of g_s^2 scales as $1/N_c$.

In terms of power counting, this implies that one has to keep track of factors of g_s as well as of explicit factors of N_c. Powers of g_s are counted just as in perturbative QCD:

- Working with canonically normalized fields, vacuum-to-vacuum diagrams in QCD can contain three-point vertices that scale as g_s, and four-point vertices that scale as g_s^2.
- Each power of the QCD theta angle θ is associated with an extra factor of g_s^2.
- Correlation functions with gauge singlet quark bilinear can be evaluated by introducing terms $\propto J\psi\psi$, which contribute as two-point vertices that scale as g_s^0.
- Correlations functions with glueballs can be evaluated by introducing terms $\propto JGG$, which contribute as a combination of two-point, three-point, and four-point vertices that scale as g_s^0, g_s^1, and g_s^2, respectively.
- If V_n denotes the total number of n-point vertices, the number of internal lines is given as $2P = 2V_2 + 3V_3 + 4V_4$.

In summary, each diagram scales as

$$g_s^{V_3+2V_4+2V_\theta} = g_s^{2P-2V+2V_\theta} \ , \tag{3.11}$$

where $V = V_2 + V_3 + V_4$ is the total number of vertices, and V_θ denotes the number of vertices with a θ angle.

Since the QCD Lagrangain contains no explicit factors of N_c, each factor of N_c is associated with a closed colour loop. To count these colour loops, it is convenient to use an alternative notation for Feynman diagrams that was invented by 't Hooft [71]. In this notation, gluon propagators are represented by a double line, since they carry two colour indices, while quark propagators are represented by a single line, since they carry only one colour index. The full Feynman rules for the double line notation can be found in [71, 61, 62, 63]. Using the double line notation, each QCD diagram defines a corresponding polyhedron:

- The three- and four point vertices corresponds to the vertices of the polyhedron, and each internal line, build from an alternating series of propagators and two-point vertices, corresponds to the edge of the polyhedron.

- In diagrams with no internal quark lines, each face of this polyhedron now corresponds to a closed colour loop.

- Internal quark lines have to form closed loops that are not associated with a factor of N_c, since the quark lines only carry a single colour index. Declaring the inside of each of these loops to be another face of the polyhedron, one finds that the number of colour loops is $L = F - Q$, where F denotes the number of faces and Q denotes the number of quark loops inside the diagram.

This correspondence to polyhedrons is useful, since the numbers of vertices, edges, and faces of any polyhedron are related to each other by Eulers theorem, which states that $F - P + V = 2 - 2H$, where H is the genus of the polyhedron. Roughly, H can be thought of as counting the number of holes enclosed by the polyhedron. Putting everything together, one finds that each connected diagram scales as

$$\begin{aligned} g_s^{2P-2V+2V_\theta} N_c^{F-Q} &= (g_s^2 N_c)^{P-V+V_\theta} N_c^{2-2H-Q-V_\theta} \\ &\propto N_c^{2-2H-Q-V_\theta} \ . \end{aligned} \tag{3.12}$$

This formula shows that connected diagrams scale at most as N_c^2, and each quark loop and each factor of θ is suppressed by a factor of $\delta = {}^1\!/_{N_c}$. If the number of quark loops and θ insertions is kept fix, such as for pure gluon dynamics, subleading diagrams are suppressed by powers of δ^2. Disconnected diagrams scale as the product of the scaling behaviour of each of their constituent diagrams, so they may scale as arbitrary powers of N_c.

Importantly, the above discussion does not just apply to vacuum diagrams, but also to connected diagrams contributing to correlation functions build from products of gauge invariant quark bilinears $\mathcal{B}_n \propto \psi\psi$ and gluon bilinears $\mathcal{G}_m \propto GG$, with indices n and m iterating over some collection of such gauge singlets. Using the bare fields, one finds that connected correlation functions scale as

$$\langle 0|\mathcal{T}\Big\{\prod_n \mathcal{G}_m\Big\}|0\rangle_{\text{conn.}} \propto N_c^2\ , \quad \langle 0|\mathcal{T}\Big\{\prod_n \mathcal{B}_n\Big\}|0\rangle_{\text{conn.}} \propto N_c\ , \tag{3.13}$$

which implies that they diverge as $N_c \to \infty$. However, the norms of asymptotic states have to remain finite as $N_c \to \infty$. To achieve this, the superfluous factors of N_c are absorbed into the wave function renormalization of $\mathcal{B}_n$ and $\mathcal{G}_n$. Rescaling $\mathcal{B}_n \to N_c^{-1/2} B_n$ and $\mathcal{G}_n \to N_c^{-1} G_n$, one finds that renormalized correlation functions scale as

$$\langle 0|\mathcal{T}\Big\{\prod_m G_m\Big\}|0\rangle_{\text{conn.}} \propto N_c^{2-m}, \quad \langle 0|\mathcal{T}\Big\{\prod_n B_n\Big\}|0\rangle_{\text{conn.}} \propto N_c^{(2-n)/2}, \tag{3.14}$$

so that the connected parts of higher order correlation functions vanish in the strict large N_c limit $\delta \to 0$. In particular, one has

$$\langle 0|\mathcal{T}\{G_a G_b\}|0\rangle = \langle 0|G_a|0\rangle\langle 0|G_b|0\rangle \times \big(1 + O(\delta^2)\big)\ , \tag{3.15}$$

$$\langle 0|\mathcal{T}\{B_a B_b\}|0\rangle = \langle 0|B_a|0\rangle\langle 0|B_b|0\rangle \times \big(1 + O(\delta)\big)\ . \tag{3.16}$$

since the connected parts scale as N_c^0, while the disconnected parts with two constituent diagrams scale as N_c (for the mesons B_n) and N_c^2 (for the glueballs G_m), respectively.

3.2.4 Chiral Symmetry

Now, I review a few aspects of chiral symmetry, following the general introductions given in [10, 33, 11, 34, 35].

In addition to being scale invariant, the kinetic term (3.2a) is also invariant under chiral flavour rotations of the shape

$$q \to Vq\ , \qquad \overline{q} \to \overline{q}\,\overline{V}\ , \tag{3.17}$$

where $V, \overline{V} \in \mathrm{U}(3)$ are unitary 3×3 matrices. The set of all possible chiral rotations is a representation of the symmetry group $G \equiv \mathrm{U}(3)_L \times \mathrm{U}(3)_R = \mathrm{U}(1)_L \times \mathrm{U}(1)_R \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, and the 18 Noether currents associated with G are

$$L_\mu^a = \frac{1}{2} q^\dagger \lambda^a \overline{\sigma}_\mu q\ , \qquad R_\mu^a = \frac{1}{2}\overline{q}\lambda^a \sigma_\mu \overline{q}^\dagger\ , \tag{3.18}$$

where $\lambda_0 \equiv \sqrt{2/3}\mathbb{1}$, and the λ_i with $i = 1, \ldots, 8$ denote the Gell-Mann matrices. The normalization for λ_0 is chosen such that $\langle \lambda_a \lambda_b \rangle = 2\delta_{ab}$. One also obtains the 18 conserved charges

$$Q_L^a \equiv \int \mathrm{d}^3 x L_0^a \; , \qquad\qquad Q_R^a \equiv \int \mathrm{d}^3 x R_0^a \; . \tag{3.19}$$

Finally, G contains an invariant subgroup $H \equiv \mathrm{U}(3)_V = \mathrm{U}(1)_V \times \mathrm{SU}(3)_V$ that is associated with the set of chiral rotations with $\overline{V} = V^\dagger$. Notice that the conserved charge Q_V^0 associated with the $\mathrm{U}(1)_V$ symmetry is the baryon number of QCD.

The chiral symmetry of the kinetic Lagrangian (3.2a) is broken by the quark masses and the axial anomaly of QCD. It is useful to study this explicit symmetry breaking in more detail. The explicit symmetry breaking via the quark masses is straightforward, since the mass term

$$\overline{q} M_q q + \mathrm{h.c.} \tag{3.20}$$

is evidently not invariant under the transformation (3.17). As the quark masses are unequal, $m_u < m_d < m_s$, the mass term (3.20) breaks not only G, but also the $\mathrm{SU}(3)_V$ symmetry of H, so that only $\mathrm{U}(1)_V$ symmetry associated with baryon number conservation is left unbroken.

Next, consider the axial anomaly. When rotating the quark fields according to (3.17), the accompanying transformation of the path integral measure generates an additional term in the action [72, 73, 74],

$$\mathrm{i}\langle \ln V + \ln \overline{V} \rangle \; \omega(x) \; , \qquad \text{where} \qquad \omega(x) = \frac{1}{16\pi^2} \widetilde{G}_{\mu\nu} G^{\mu\nu} \; , \tag{3.21}$$

so that the generating functional is only left unaltered if $\ln\det \overline{V} = -\ln\det V$. Hence, the $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ symmetry contained in G is explicitly broken down to the $\mathrm{U}(1)_V$ symmetry contained in H, even in the absence of quark masses. For the divergence of the axial Noether current, one finds the Adler-Bell-Jackiew anomaly [75, 76, 77],

$$\partial_\mu (R_\mu^0 - L_\mu^0) = \omega(x) \; . \tag{3.22}$$

In the presence of a fixed gluon background, the size of the symmetry breaking can be quantified in terms of the winding number,

$$n \equiv \int \mathrm{d}^4 x \; \omega(x) \; . \tag{3.23}$$

In general, the QCD generating functional has to sum over gauge field configurations with arbitrary winding numbers. In this case, the size of the symmetry breaking can be quantified in terms of the "topological susceptibility" of QCD, which is defined to be the square average of the vacuum winding number density per spacetime volume,

$$\tau \equiv \frac{1}{V}\langle 0|n^2|0\rangle = -\,\mathrm{i}\int \mathrm{d}^4x\ \langle 0|\mathcal{T}\left\{\omega(x)\omega(0)\right\}|0\rangle\ . \tag{3.24}$$

Splitting spacetime into an infinite grid of hypercubes with finite volume, the path integral for each hypercube can be decomposed into a contribution from field configurations with integer valued winding number, and a remainder with fractional winding number between 0 and 1. The contribution from integer winding numbers can be thought of as the superposition of multi-instanton states that constitutes the true vacuum of QCD. It is possible to integrate out this instanton background, which was first done by 't Hooft [78, 79]. This way, one finds an additional contribution to the QCD action that can be seen to explicitly break the anomalous axial symmetry,

$$\kappa\, e^{\mathrm{i}\,\theta}\det(\overline{q}\otimes q) + \text{h.c.}\ , \tag{3.25}$$

where the constant κ is connected to the topological susceptibility τ. The tensor product indicates that the flavour indices are not contracted with each other, so that $\overline{q}\otimes q$ denotes a matrix in flavour space. $\det(\overline{q}\otimes q)$ is an interpolating field for the pseudoscalar meson associated with the axial charge operator Q^0_A.

Notice that the axial anomaly (3.22) vanishes in the strict large N_c limit. Since diagrams contributing to the axial anomaly have to involve at least one quark loop, one has

$$\begin{aligned}&\langle 0|\mathcal{T}\Big\{[\partial^\mu(R-L)^0_\mu]\prod_n B_n\prod_m G_m\Big\}|0\rangle_{\text{conn.}}\\ &= \langle 0|\mathcal{T}\Big\{\omega\prod_n B_n\prod_m G_m\Big\}|0\rangle_{\text{conn.}} \propto N_c^{-(n+2m)/2}\ ,\end{aligned} \tag{3.26}$$

where $\omega \propto {}^1\!/\!{}_{N_c}$, as in canonical normalization $\omega \propto g_s^2\widetilde{G}G$. Since $\omega|0\rangle$ has negative parity, one has $\langle 0|\omega|0\rangle = 0$. Hence, all of the correlation functions (3.26) vanish in the large N_c limit. Assuming that the Fock space at low energies is spanned by the operators B_n and G_m, this implies that the chiral anomaly does indeed vanish for $N_c \to \infty$. For large but finite N_c, the effect of the axial anomaly can be reintroduced by evaluating the 't Hooft operator (3.25) at leading order in the large N_c expansion. Working at this level of accuracy, one obtains the logarithmic determinant

$$\kappa\, e^{\mathrm{i}\,\theta} \ln \det(\overline{q} \otimes q) + \text{h.c.}\ . \tag{3.27}$$

3.2.5 Pseudo Nambu-Goldstone Bosons

Finally, I discuss the PNGBs of QCD. As before, I follow the general introductions given in [10, 33, 11, 34, 35].

Even in the absence of any explicit symmetry breaking, the QCD vacuum is not invariant under the action of G. More precisely, the finite value of the chiral quark condensate

$$B_0 \equiv \langle 0|\overline{q}_a q_b + \text{h.c.}|0\rangle \neq 0 \tag{3.28}$$

implies that the nine axial charge operators $Q_A^a \equiv Q_R^a - Q_L^a$ do not annihilate the vacuum,

$$Q_A^a|0\rangle \neq 0\ . \tag{3.29}$$

This relation is what is meant when the group generators associated with the axial charges Q_A^a are said to be "spontaneously broken". In contrast, the nine vector charge operators $Q_V^a = Q_R^a + Q_L^a$ do annihilate the vacuum, $Q_V^a|0\rangle = 0$, so that the associated finite chiral rotations with $\overline{V} = V^\dagger$ leave the vacuum unchanged.

If QCD contained no source of explicit symmetry breaking, the Goldstone theorem would imply the existence of 9 massless, pseudoscalar Nambu-Goldstone bosons (=NGB) $\widetilde{\phi}_a$, one for each generator of G that does not annihilate the vacuum. In this limit, the typical energy scale of the spontaneous symmetry breaking can be quantified in terms of the matrix element

$$\langle 0|(R-L)_a^\mu(x)|\widetilde{\phi}_b(p)\rangle = \mathrm{i}\,\delta_{ab}\frac{p^\mu}{\sqrt{(2\pi)^3 2p_0}} \cdot F e^{ixp}\ , \tag{3.30}$$

where the shape of the matrix element (3.30) is fixed by Lorentz invariance, with the decay constant F as the only free parameter.

In reality, chiral symmetry is broken explicitly by the light quark masses and the axial anomaly of QCD, which can be captured in terms of the chiral symmetry breaking Lagrangian

$$\mathcal{L}_{\text{broken}} \equiv -\overline{q} M_q q - \kappa e^{\mathrm{i}\,\theta} \det(\overline{q} \otimes q) \ . \tag{3.31}$$

Due to this explicit symmetry breaking, one obtains nine massive PNGBs ϕ_a, rather than the massless NGBs $\widetilde{\phi}_a$. The size of the mass matrix M_{ab} is determined by the size of the underlying explicit symmetry breaking,

$$M_{ab}^2 \propto \mathcal{L}_{\text{broken}} \ . \tag{3.32}$$

The symmetry of the matrix element (3.30) is also broken explicitly, with each PNGB obtaining a distinct decay constant, $F \to F_a$. It is conventional to quantify the size of the spontaneous symmetry breaking in terms of the Pion decay constant F_π, since corrections from explicit symmetry breaking are expected to be small for the Pions in accordance with the small masses of the u and d quarks. Matching the matrix element (3.30) to the Pion decay width, one finds $F_\pi \approx 93\,\text{MeV}$ [55].

The explicit symmetry breaking due to (3.31) vanishes in the chiral limit, which is obtained by taking $M_q \to 0$ and $N_c \to \infty$. If it is possible to treat $\mathcal{L}_{\text{broken}}$ as a perturbation about the chiral limit, the PNGBs modes can be constructed in the unperturbed theory. Assuming that the other resonances of QCD can not be excited, the resulting pseudoscalar fields can then be used as the fundamental building block of χPT. In this approach, the explicit symmetry breaking then has to be reintroduced as a perturbation on the level of χPT.

In the chiral limit, the QCD action is invariant under the general chiral rotations (3.17) with $(V, \overline{V}) \in G = \mathrm{U}(3)_L \times \mathrm{U}(3)_R$. Rotations that are part of the invariant subgroup $H \cong \mathrm{U}(3)_V$ leave the vacuum unchanged, while general chiral rotations can map physically different vacua onto each other, so that the symmetry of the theory under G is spontaneously broken. The set of distinct vacua is given the coset ${}^{G}\!/\!_{H} \cong \mathrm{U}(3)_A$, which is isomorphic to the set of unitary 3×3 matrices. According to the Goldstone theorem, each spontaneously broken generator of G

corresponds to a massless NGB. Intuitively, the NGB modes are those excitations for which the field value at each point in space remains within the orbit of the vacuum state. More rigorously, the spin 0 mesons of QCD are created by the bosonic quark bilinears ${B_a}^{\dot{b}} \equiv q_a \overline{q}^{\dot{b}}$. The NGB modes can be isolated explicitly by means of a singular value decomposition,

$$B = UD\overline{U} = (U\overline{U})(\overline{U}^\dagger D\overline{U}) = g\hat{B} \ , \tag{3.33}$$

where D is some diagonal matrix and the unitary matrix $g = U\overline{U} \in \mathrm{U}(3)_A \cong {}^{G}\!/\!{}_{H}$ collects the degrees of freedom that create and annihilate the NGB modes. The matrix g is the fundamental building block of χPT. It can be parametrized as

$$g = \exp\left(\mathrm{i}\,\frac{\Phi}{f}\right) \ , \qquad\qquad \Phi = \phi_a \lambda_a \ , \tag{3.34}$$

where the constant f is a free parameter that has to be determined by matching it to the pion decay constant (**??**), and the ϕ_a are the individual NGB fields. Since g is a unitary matrix, its determinant has to be a complex phase,

$$\det g = e^{\mathrm{i}\,\psi} \ , \qquad\qquad \psi = -\,\mathrm{i} \ln \det g = \frac{\sqrt{6}\,\phi_0}{f} \ . \tag{3.35}$$

It is conventional to write g in terms of states with definite charge and strangeness,

$$\begin{aligned} \pi_0 &\equiv \phi_3 \ , & \pi^\pm &\equiv \frac{1}{\sqrt{2}}(\phi_1 \mp \mathrm{i}\,\phi_2) \ , \\ \eta_0 &\equiv \phi_0 \ , & K^\pm &\equiv \frac{1}{\sqrt{2}}(\phi_4 \mp \mathrm{i}\,\phi_5) \ , \\ \eta_8 &\equiv \phi_8 \ , & K^0 = (\overline{K}^0)^\dagger &\equiv \frac{1}{\sqrt{2}}(\phi_6 - \mathrm{i}\,\phi_7) \ . \end{aligned} \tag{3.36}$$

Collectively, the π^0, $\pi^\pm$, K^0, $\overline{K}^0$, $K^\pm$, and η^8 states transform as an octet under H, while the η^0 transforms as a singlet under H.

Experimentally, there are nine light pseudoscalar mesons with masses below $1\,\mathrm{GeV}$, the heaviest of which is the η' meson with $M_{\eta'} = 958\,\mathrm{MeV}$.

The remaining eight mesons are much lighter, with masses ranging from $M_\pi = 135 - 139\,\text{MeV}$ for the Pions to $M_K = 493 - 497\,\text{GeV}$ for the Kaons and $M_\eta = 548\,\text{MeV}$ for the η meson.

The central insight exploited by χPT is that these nine pseudoscalar mesons can be identified with the PNGBs (3.36). For the Pions and Kaons, this identification is straightforward, but the situation is more complicated for the η mesons. In fact, the η and η' mesons are the result of mass mixing between the octet PNGB η^8 and the singlet PNGB η^0.

So far, I have talked of χPT as a single effective theory of the light pseudoscalar mesons. In fact, the internal mass hierarchy of the light pseudoscalar mesons leads to an ambiguity in terms of which PNGBs need to be used to construct χPT. Focusing on Kaon decays, there are two relevant candidate versions:

- The U(3) version of χPT, which is constructed by using the full nonet (3.36) of PNGBs associated with the explicitly broken chiral symmetry group $G = \text{U}(3)_L \times \text{U}(3)_R$ of QCD.
- The SU(3) version of χPT which is constructed by using only the octet of PNGBs associated with the explicitly broken subgroup $G' \equiv \text{SU}(3)_L \times \text{SU}(3)_R \subset G$.

The U(3) version of χPT captures all nine pseudoscalaer mesons. To construct it, one has to assume that the explicit symmetry breaking due to the light quark masses and the axial anomaly can be treated as a perturbation about the chiral limit, in which G is not broken explicitely. To obtain the chiral limit for G, one has to neglect the quark masses and take $N_c \to \infty$, where the large N_c limit is necessary to have the axial anomaly vanish. Hence, the U(3) version of χPT relies on the validity of the large N_c expansion.

On the other hand, the SU(3) version of χPT only captures the dynamics of the Kaons and Pions, but it can be constructed without appealing to the large N_c limit. In this theory, the explicit symmetry breaking due to the axial anomaly is not considered to be perturbatively small, so that one retains only the eight PNGBs associated with the explicit symmetry breaking of the subgroup G'. Besides the three Pions and the four Kaons, this version of χPT contains a final η^8 meson, which completes the octet. Neglecting the mass mixing with the η^0 meson, the η^8 can be approximately identified with the η meson.

To capture the coupling of χPT to external pseudoscalar fields via operators such as $S\, G_{\mu\nu}\widetilde{G}^{\mu\nu}$, it is necessary to keep track of the singlet η^0 meson. Since my goal is to construct a PET χPT Lagrangian that accounts for the coupling to *general* external sectors, I will use the U(3) version of χPT in the remainder of this work. See also section 3.4.1 for a more detailed discussion.

3.3 QCD in the Presence of External Currents

In this section, I construct the external currents that summarize the coupling of QCD to hidden sectors captured by the PET framework of chapter 2. For completeness sake, I also list the SM contributions to the external currents, which are well known.

Finally, I present two short computations that prepare the subsequent discussions in sections 3.4 and 3.5: In subsection 3.3.2, I compute the modifications that the trace relation (3.6b) receives in the presence of generic external currents. In subsection 3.3.3, I decompose the objects A, B, and C into contributions that transform as members of irreducible representations of G, an rewrite the four quark Lagrangian in terms of these constituent currents.

3.3.1 The Interaction Lagrangian

In the presence of sectors that are external to QCD, the QCD Lagrangian is augmented by an "interaction" sector that collects all of the perturbative interactions involving the external fields,

$$\mathcal{L}_{\text{QCD}} \to \mathcal{L}_{\text{QCD}} + \delta\mathcal{L}_{\text{int}} \ . \tag{3.37}$$

In this notation, the interaction Lagrangian contains both the electromagnetic and weak SM interactions as well as the coupling to hidden sectors.

In general, QCD couples to the remainder of the SM via the electromagnetic interactions mediated by the photons, and via the higher dimensional operators associated with virtual exchanges of the heavy SM bosons. In what follows, I neglect the higher dimensional SM operators, unless they mediate $|\Delta F| = 1$ transitions. To first order in $1/M_W^2$, the only available operators of this type are either dipole type operators or four quark operators.

Within the PET framework, the coupling to hidden sectors is mediated by the operators collected in section 2.4. Neglecting virtual heavy boson exchanges, the portal Lagrangian consists of operators with dimension $d \leq 5$, which contain either two quark fields with no derivatives or two gluon fields. As in the SM case, higher dimensional portal operators are negligible, unless they mediate $|\Delta F| = 1$ transitions. Furthermore, I also neglect portal operators for which the power counting rules of NDA a relative suppression of at least $^1/(4\pi)$ compared to the leading order portal operators. The relevant subset of higher dimensional operators has been collected in table 2.2. These operators either contain two quarks and no derivatives, four quarks, or a dipole type interaction.

In summary, the full interaction Lagrangian with SM and portal contributions can be cast as

$$\begin{aligned}\delta\mathcal{L}_{\text{int}} \equiv &-q^\dagger\overline{\sigma}^\mu l_\mu q - \overline{q}\sigma^\mu r_\mu \overline{q}^\dagger - a\, G_{\mu\nu}G^{\mu\nu} - \widetilde{a}\, G_{\mu\nu}\widetilde{G}^{\mu\nu} \\ &- \left[\overline{q}\,(s + \mathrm{i}\,p)q + \text{h.c.}\right] + \mathcal{L}_{\text{dipole}} + \mathcal{L}_{\overline{q}q\overline{q}q}\ ,\end{aligned} \tag{3.38}$$

where $\mathcal{L}_{\text{dipole}}$ collects the dipole contributions,

$$\mathcal{L}_{\text{dipole}} = -\overline{q}\,\overline{\sigma}^{\mu\nu}t_{\mu\nu}q - \overline{q}\,\xi\overline{\sigma}^{\mu\nu}G_{\mu\nu}q + \text{h.c.}\ , \tag{3.39}$$

and $\mathcal{L}_{\overline{q}q\overline{q}q}$ collects the four quark interactions,

$$\begin{aligned}\mathcal{L}_{\overline{q}q\overline{q}q} = &-A^{bd}_{ac}(q^{\dagger a}\overline{\sigma}^\mu q_b)(q^{\dagger c}\overline{\sigma}_\mu q_d) \\ &- B^{b\dot{d}}_{a\dot{c}}(q^{\dagger a}\overline{\sigma}^\mu q_b)(\overline{q}^\dagger_{\dot{c}}\overline{\sigma}_\mu \overline{q}^{\dot{d}}) - C^{\dot{b}d}_{a\dot{c}}(q^{\dagger a}\overline{q}^\dagger_{\dot{b}})(\overline{q}^{\dot{c}}q_d)\ .\end{aligned} \tag{3.40}$$

The external currents $k \in \{l_\mu, r_\mu, s, p, \xi, t_{\mu\nu}\}$ are 3×3 matrices in flavour space, while the external currents $M \in \{A, B, C\}$ transform as tensor products of matrices in flavour space. The two remaining currents a and $\widetilde{a}$ are flavour singlets. Further, the dotted and undotted *latin* indices in the four quark Lagrangian denote quantities that transform as members of the fundamental representation of the chiral flavour symmetry groups $\mathrm{U}_R(3)$ and $\mathrm{U}_L(3)$, respectively[1].

I decompose all of the currents into contributions from the SM and hidden sectors,

$$k = k^{\text{SM}} + k^{\text{BSM}}\ , \qquad M = M^{\text{SM}} + M^{\text{BSM}}\ . \tag{3.41}$$

The currents s, p, a, and $\widetilde{a}$ do not receive SM contributions. However, it is useful to combine s, p, and $\widetilde{a}$ with the corresponding chiral symmetry

[1] This notation is inspired by, and not to be confused with, the familiar dot notation used in supersymmetry, where the dotted and undotted *greek* indices denote quantities that transform as members of the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group.

breaking contributions in QCD proper,

$$\chi = M_q + s + \mathrm{i}\, p \ , \qquad\qquad \vartheta \equiv \theta + \widetilde{a} \ . \tag{3.42}$$

For the matrix valued currents, I define the orthonormal basis vectors $(Q_i^j)_{kl} \equiv \delta_{ik}\delta_{jl}$. I also separate these currents into a quark flavour conserving and a quark flavour violating part,

$$k = k_{\Delta F=0} + k_{\Delta F=1} \ , \tag{3.43}$$

where

$$k_{\Delta F=0} = \begin{pmatrix} k_{\overline{u}u} & 0 & 0 \\ 0 & k_{\overline{d}d} & 0 \\ 0 & 0 & k_{\overline{s}s} \end{pmatrix} \qquad k_{\Delta F=1} = \begin{pmatrix} 0 & k_{\overline{u}d} & k_{\overline{u}s} \\ k_{\overline{d}u} & 0 & k_{\overline{d}s} \\ k_{\overline{s}u} & k_{\overline{s}d} & 0 \end{pmatrix} \ . \tag{3.44}$$

The currents l_μ, r_μ, s, and p are hermitian, giving

$$k_{\overline{u}d} = \left(k_{\overline{d}u}\right)^\dagger \ , \qquad k_{\overline{u}s} = \left(k_{\overline{s}u}\right)^\dagger \ , \qquad k_{\overline{d}s} = \left(k_{\overline{s}d}\right)^\dagger \ . \tag{3.45}$$

SM Contributions

For completeness sake, I begin by summarizing the SM contributions to the external currents. In general, these contributions are well known, but the SM contributions to the four quark Lagrangian (3.40) are usually not written in terms of four objects like A, B, and C, see e.g. the treatments in[80, 81, 82, 83, 84, 85, 86, 50, 87]. Hence, this section is also intended to provide the explicit mapping between the standard notation for the four quark Lagrangian and the external current notation that I use in the remainder of this chapter.

The SM contributions l_μ^{SM} and r_μ^{SM} encode the electromagnetic coupling of QCD to the SM photons, and the weak coupling to leptonic charged and neutral currents. The electromagnetic coupling is mediated by the interaction Lagrangian

$$\mathcal{L}_{\mathrm{em}} \equiv -q^\dagger Q A_\mu \overline{\sigma}^\mu q - \overline{q} Q A_\mu \sigma^\mu \overline{q}^\dagger \ , \tag{3.46}$$

where $Q \equiv \mathrm{diag}(+1/3, -2/3, -2/3)$ is the quark charge matrix, which is not to be confused with the orthogonal basis matrices Q_i^j. At leading order in ${}^{1}\!/\!{}_{M_W^2}$, the coupling to leptonic charged currents is mediated by the interactions Lagrangian

$$\mathcal{L}_{\overline{\nu} l \overline{q} q} \equiv -2\sqrt{2} G_F \left[q^\dagger (V_{\overline{u}d} Q_1^2 + V_{\overline{u}s} Q_1^3) \overline{\sigma}^\mu q \right] J_{+\,\mu} + \text{h.c.} \ , \tag{3.47}$$

$$J_{+\,\mu} = \left(J_{-\,\mu}\right)^\dagger = \sum_{a \in e, \mu} \left(l_a^\dagger \overline{\sigma}^\mu \nu_a \right) \ , \tag{3.48}$$

and $G_F \equiv {}^{\sqrt{2} g^2}\!/\!{}_{8 M_W^2} \approx 1.17 \cdot 10^{-5}\,\text{GeV}^{-2}$ is the Fermi constant. Leptonic neutral current couplings are quark flavour conserving and hence negligible. In summary,

$$(r_\mu)^{\text{SM}}_{\Delta F = 0} = Q A_\mu \ , \qquad (l_\mu)^{\text{SM}}_{\Delta F = 0} = Q A_\mu \ , \tag{3.49}$$

and

$$(l_\mu)^{\text{SM}}_{\Delta F = 1} \equiv 2\sqrt{2}\, G_F \Big(\left[V_{\overline{u}d} Q_1^2 + V_{\overline{u}s} Q_1^3 \right] J_{+\,\mu} + \text{h.c.} \Big) \ . \tag{3.50}$$

The currents $t_{\mu\nu}^{\text{SM}}$ and ξ^{SM} encode electro- and chromomagnetic quark dipole interactions. At 1-loop, one has [88]

$$(t_{\mu\nu})^{\text{SM}}_{\overline{a}b} = \frac{1}{3} \frac{m_a}{(4\pi)^2} G_F \sum_{f = u,c,t} d_f V^\dagger_{\overline{a}f} V_{\overline{f}b} F_{\mu\nu} \ , \tag{3.51a}$$

$$\xi^{\text{SM}}_{\overline{a}b} = \frac{m_a}{(4\pi)^2} G_F \sum_{f = u,c,t} \widetilde{d}_f V^\dagger_{\overline{a}f} V_{\overline{f}b} \ , \tag{3.51b}$$

where the Wilson coefficients $d_f(\mu)$ and $\widetilde{d}_f(\mu)$ have to be determined by matching the low energy effective theory to the full SM. As they appear only in higher dimensional operators, I retain only the quark flavour changing $|\Delta F| = 1$ contributions. Hence,

$$t^{\text{SM}}_{\mu\nu} = Q M_q \left(Q_2^3 \frac{1}{m_d} (t_{\mu\nu})^{\text{SM}}_{\overline{d}s} + \text{h.c.} \right) \ , \tag{3.52a}$$

$$\xi^{\text{SM}} = M_q \left(Q_2^3 \frac{1}{m_d} \xi^{\text{SM}}_{\overline{d}s} + \text{h.c.} \right) \ . \tag{3.52b}$$

The SM contributions to the currents A^{SM}, B^{SM}, and C^{SM} are constant and function as coupling strengths for the SM four quark interactions. Usually, the four quark part of the SM Lagrangian is cast as [86, 50, 87]

$$\Delta\mathcal{L}_{\overline{q}q\overline{q}q} \equiv -2\sqrt{2}G_F\,|V|^2_{ds}\sum_i c_i(\mu)O_i + \text{h.c.}\ , \tag{3.53}$$

where $|V|^2_{ds} \equiv V_{\overline{u}d}V^*_{\overline{u}s}$, and the O_i denote four quark operators mediating purely hadronic $\Delta F = 1$ transitions. Neglecting electromagnetic penguins, there are six six relevant four quark operators [80, 81, 82, 83, 89, 85],

$$O_1 = (s^\dagger\overline{\sigma}^\mu u)(u^\dagger\overline{\sigma}_\mu d)\ , \qquad O_2 = (s^\dagger\overline{\sigma}^\mu d)(u^\dagger\overline{\sigma}_\mu u)\ , \tag{3.54a}$$

$$O_3 = (s^\dagger\overline{\sigma}^\mu d)(q^{\dagger a}\overline{\sigma}_\mu q_a)\ , \qquad O_4 = (s^\dagger\overline{\sigma}^\mu q_a)(q^{\dagger a}\overline{\sigma}_\mu d)\ , \tag{3.54b}$$

$$O_5 = (s^\dagger\overline{\sigma}^\mu d)(\overline{q}^\dagger_{\dot{a}}\overline{\sigma}_\mu\overline{q}^{\dot{a}})\ , \qquad O_6 = (s^\dagger\overline{q}^\dagger_{\dot{a}})(\overline{q}^{\dot{a}}d)\ . \tag{3.54c}$$

Rewriting the four quark Lagrangian in terms of A, B, and C, this gives the contributions

$$\begin{aligned}
(A^{\rm SM})^{bd}_{ac} &= 2\sqrt{2}G_F\,|V|^2_{ds}\Big[c_1(Q^1_3)^b_a(Q^2_1)^d_c + c_2(Q^2_3)^b_a(Q^1_1)^d_c \\
&\qquad\qquad + c_3(Q^2_3)^b_a\mathbb{1}^d_c + c_4\sum_i(Q^i_3)^b_a(Q^2_i)^d_c\Big] + \text{h.c.}\ , \\
(B^{\rm SM})^{b\dot{d}}_{a\dot{c}} &= 2\sqrt{2}G_F\,|V|^2_{ds}\,c_5(Q^2_3)^b_a(\mathbb{1}_{3\times3})^{\dot{d}}_{\dot{c}} + \text{h.c.}\ , \\
(C^{\rm SM})^{\dot{b}d}_{a\dot{c}} &= 2\sqrt{2}G_F\,|V|^2_{ds}\,c_6\sum_i(Q^i_3)^{\dot{b}}_a(Q^2_i)^d_{\dot{c}} + \text{h.c.}\ .
\end{aligned} \tag{3.55}$$

Hidden Sector Contributions

Now, I summarize the external current contributions generated by the GeV scale portal operators collected in section 2.4.

I begin by considering the contributions generated by portal operators of dimension $d \leq 5$. Using the the list of operators in section 2.4, one has

$$s = \epsilon\, C_{s_i}\, S_i\ , \qquad a = \frac{\epsilon}{M_W}C_{a_i}\, S_i\ , \qquad r^{\rm BSM}_\mu = \epsilon\, C_{r,v}\, V_\mu\ , \tag{3.56}$$

$$p = \epsilon\, C_{p_i}\, S_i\ , \qquad \widetilde{a} = \frac{\epsilon}{M_W}\widetilde{C}_{a_i}\, S_i\ , \qquad l^{\rm BSM}_\mu = \epsilon\, C_{l,v}\, V_\mu\ ,$$

where the Wilson coefficients C_{a_i} and $\widetilde{C}_{a_i}$ are flavour singlets, while the remaining Wilson coefficients C_X are 3×3 matrices in flavour space that contain both quark flavour conserving and quark flavour changing contributions. Since the portal operators have to be uncharged, one has the restriction

$$(C_X)_{\overline{d}u} = (C_X)_{\overline{u}d} = (C_X)_{\overline{s}u} = (C_X)_{\overline{s}u} = 0. \tag{3.57}$$

Higher dimensional operators contribute to the currents l_μ, s, p, ξ, $t_{\mu\nu}$, A, B, and C. As before, I include only $|\Delta F| = 1$ contributions. For the currents l_μ, s, p these contributions yield the modifications

$$\begin{aligned} s \to s + \epsilon\, G_F \Big[& m_{\mathrm{LE}}\, C_{s_i s_j}\, S_i\, S_j + C_{s_i s_j s_k}\, S_i\, S_j\, S_k \\ & + C_{s_i, \partial^2}\, \partial^2 S_i + \frac{1}{M_W} C_{s_i s_j s_k s_l}\, S_i\, S_j\, S_k\, S_l \Big] , \\ p \to p + \epsilon\, G_F \Big[& m_{\mathrm{LE}}\, C_{p_i p_j}\, S_i\, S_j + C_{p_i p_j p_k}\, S_i\, S_j\, S_k \\ & + C_{p_i, \partial^2}\, \partial^2 S_i + \frac{1}{M_W} C_{p_i p_j p_k p_l}\, S_i\, S_j\, S_k\, S_l \Big] , \end{aligned} \tag{3.58}$$

and

$$\begin{aligned} l_\mu^{\mathrm{BSM}} \to l_\mu^{\mathrm{BSM}} + \epsilon\, G_F \big[& C_{l, S_i \partial S_j}\, (S_i \overleftrightarrow{\partial}_\mu S_j) + \frac{C_{l, S_i \nu_a \nu_b}}{M_W}\, S_i (\nu_a^\dagger \overline{\sigma}_\mu \nu_b) \\ & + \frac{C_{l, S_i e_a \nu_b}}{M_W}\, S_i (e_a^\dagger \overline{\sigma}_\mu \nu_b) + C_{l, e_a \xi} (\overline{e}_a^\dagger \overline{\sigma}_\mu \xi_i) \\ & + C_{l, \nu_a \xi_i} (\overline{\nu}_a^\dagger \overline{\sigma}_\mu \xi_i) + C_{l, \xi_i \xi_j} (\xi_i^\dagger \overline{\sigma}_\mu \xi_j) + \text{h.c.} \Big] , \end{aligned}$$

where m_{LE} is some low energy mass scale that has to be inserted to ensure that the Lagrangian is of dimension $d = 4$. The normalization is chosen such that the higher dimensional operators are suppressed by an explicit factor of G_F. As I only include higher dimensional operators that are quark flavour violating and uncharged, most of the entries in the Wilson coefficient matrices C_X vanish. Up to hermitian conjugates, the only nonvanishing entries are

$$(C_{l, S_i e \nu})_{\overline{d}u} \ , \quad (C_{l, S_i e \nu})_{\overline{s}u} \ , \quad (C_{l, e \xi_i})_{\overline{d}u} \ , \quad (C_{l, e \xi_i})_{\overline{s}u} \ , \quad (C_X)_{\overline{d}s} \tag{3.59}$$

for the remaining Wilson coefficient matrices. The dipole currents ξ and $t_{\mu\nu}$ receive the contributions

$$\xi^{\mathrm{BSM}} = \epsilon\, G_F\, C_{\xi S_i}\, S_i \ , \qquad t^{\mathrm{BSM}}_{\mu\nu} = \epsilon\, G_F\, C_{tS_i}\, S_i F_{\mu\nu} \ , \tag{3.60}$$

where the only nonvanishing entries in the Wilson coefficient matrices are

$$(C_{\xi S_i})_{\overline{d}s} \ , \qquad (C_{\xi S_i})_{\overline{s}d} \ , \qquad (C_{tS_i})_{\overline{d}s} \ , \qquad (C_{tS_i})_{\overline{s}d} \ . \tag{3.61}$$

The contributions to A^{BSM}, B^{BSM}, and C^{BSM} are generated by a set of four quark operators that mirror the SM four quark operators (3.54a). Neglecting electromagnetic penguin operators, one has

$$\begin{aligned}
\widetilde{O}_1 &= (s^\dagger\overline{\sigma}^\mu u)(u^\dagger\overline{\sigma}_\mu d)\, S_i \ , & \widetilde{O}_2 &= (s^\dagger\overline{\sigma}^\mu d)(u^\dagger\overline{\sigma}_\mu u)\, S_i \ , \\
\widetilde{O}_3 &= (s^\dagger\overline{\sigma}^\mu d)(q^{\dagger a}\overline{\sigma}_\mu q_a)\, S_i \ , & \widetilde{O}_4 &= (s^\dagger\overline{\sigma}^\mu q_a)(q^{\dagger a}\overline{\sigma}_\mu d)\, S_i \ , \\
\widetilde{O}_5 &= (s^\dagger\overline{\sigma}^\mu d)(\overline{q}^\dagger_{\dot{a}}\overline{\sigma}_\mu\overline{q}^{\dot{a}})\, S_i \ , & \widetilde{O}_6 &= (s^\dagger\overline{q}^\dagger_{\dot{a}})(\overline{q}^{\dot{a}} d)\, S_i \ .
\end{aligned} \tag{3.62}$$

These operators yield the contributions

$$\begin{aligned}
(A^{\mathrm{BSM}})^{bd}_{ac} &= 2\sqrt{2}\, G_F\, |V|^2_{ds}\Big[\widetilde{c}_{i,1}(Q^1_3)^b_a(Q^2_1)^d_c + \widetilde{c}_{i,2}(Q^2_3)^b_a(Q^1_1)^d_c \\
&\qquad + \widetilde{c}_{i,3}(Q^2_3)^b_a\mathbb{1}^d_c + \widetilde{c}_{i,4}\sum_j (Q^j_3)^b_a(Q^2_j)^d_c\Big]\epsilon\frac{S_i}{M_W} + \text{h.c.} \ , \\
(B^{\mathrm{BSM}})^{b\dot{d}}_{a\dot{c}} &= 2\sqrt{2}\, G_F\, |V|^2_{ds}\, \widetilde{c}_{i,5}(Q^2_3)^b_a(\mathbb{1}_{3\times 3})^{\dot{d}}_{\dot{c}}\epsilon\frac{S_i}{M_W} + \text{h.c.} \ , \\
(C^{\mathrm{BSM}})^{\dot{b}d}_{a\dot{c}} &= 2\sqrt{2}\, G_F\, |V|^2_{ds}\, \widetilde{c}_{i,6}\sum_j (Q^j_3)^{\dot{b}}_a(Q^2_j)^d_{\dot{c}}\epsilon\frac{S_i}{M_W} + \text{h.c.} \ .
\end{aligned} \tag{3.63a}$$

For the four quark operators, I define

$$x_{i,j} \equiv \frac{\widetilde{c}_{i,j}}{c_i} \ , \qquad d_i \equiv c_i\left(1 + \sum_j x_{j,i}\frac{\epsilon}{M_W}S_j\right) \ . \tag{3.64}$$

In general, the BSM Wilson coefficient matrices C_x and $\widetilde{c}_{i,j}$ encode the dependence on UV and electroweak scale physics. At order ϵ/M^3_W, the anomalous dimensions of the $\widetilde{c}_{i,j}$ do not receive any contributions from hidden sector interactions. Such a contribution would have to be generated by a 1PI diagram with at least one hidden vertex, which would imply the presence of at least two portal vertices. However, each portal vertex carries a factor of ϵ, giving a total parametric suppression of at least ϵ^2. On the other hand, SM diagrams will contribute equally to the anomalous dimensions of c_i and $\widetilde{c}_{j,i}$ if the hidden scalar field is evaluated at zero momentum. Therefore, I expect that the ratio $x_{i,j} = \widetilde{c}_{i,j}/c_i$ is approximately renormalization scale independent.

3.3.2 Explicit Breaking of Scale Invariance

The various external currents explicitly break scale invariance and chiral invariance, which implies that they contribute to the divergence of the conserved currents (3.18) and to the trace of the improved energy momentum tensor $\Theta_{\mu\nu}$,

Tp prepare the estimation of the new LECs in section 3.5, I compute the tree-level modifications of Θ^μ_μ in the presence of generic external currents. Using the general definition of the energy momentum tensor given in (3.6a), one finds

$$\begin{aligned}\Theta_{\mu\nu} = {} & q^\dagger \overline{\sigma}_{(\mu}(\mathrm{i}\, D - l)_{\nu)} q + \overline{q}\sigma_{(\mu}(\mathrm{i}\, D - r)_{\nu)}\overline{q}^\dagger \\ & - 2(\overline{q}\xi\overline{\sigma}^\alpha{}_{(\mu} G_{\alpha\nu)} q + \overline{q}\overline{\sigma}^\alpha{}_{(\mu} t_{\alpha\nu)} q + \text{h.c.}) \\ & - \frac{1}{16\pi^2}(\omega_s + a) G_{\mu\alpha} G^\alpha{}_\nu - \eta_{\mu\nu}(\mathcal{L}_{\text{QCD}} + \delta\mathcal{L}_{\text{int}}) \ . \end{aligned} \tag{3.65}$$

As in the case of pure QCD, one can use classical equations of motion to simplify this expression. In the presence of the external currents, the classical equations of motion for the quark fields become

$$\begin{aligned}\overline{\sigma}^\mu \,\mathrm{i}\, D_\mu \overline{q}^{\dot a} = {} & (\chi^* q^\dagger + \sigma^{\mu\nu} t^*_{\mu\nu} q^\dagger + \xi^* \sigma^{\mu\nu} G_{\mu\nu} q^\dagger + \overline{\sigma}^\mu r^T_\mu \overline{q})^{\dot a} \\ & - \left[B^{b\dot a}_{c\dot d}\, \sigma^\mu \overline{q}^{\dot d}(q^{\dagger c}\overline{\sigma}_\mu q_b) - C^{\dot a d}_{b\dot c}\, q^\dagger_b (\overline{q}^{\dot c} q_d)\right] \ ,\end{aligned} \tag{3.66a}$$

$$\begin{aligned}\overline{\sigma}^\mu \,\mathrm{i}\, D_\mu q_a = {} & (\chi^\dagger \overline{q}^\dagger - \sigma^{\mu\nu} t^\dagger_{\mu\nu} \overline{q}^\dagger - \xi^\dagger \sigma^{\mu\nu} G_{\mu\nu} \overline{q}^\dagger + \overline{\sigma}^\mu l_\mu q)_a \\ & + \left[2A^{bd}_{ac}\, \overline{\sigma}^\mu q_b (q^{\dagger c}\overline{\sigma}_\mu q_d) \right. \\ & \left. \quad + B^{bd}_{a\dot c}\,\overline{\sigma}_\mu q_b (\overline{q}^{\dot c}\sigma^\mu \overline{q}^\dagger_{\dot d}) + C^{\dot b d}_{a\dot c}\, \overline{q}^\dagger_{\dot b}(\overline{q}^{\dot c} q_d)\right] \ ,\end{aligned} \tag{3.66b}$$

$$\sigma^\mu \,\mathrm{i}\, D_\mu \overline{q}^\dagger_{\dot a} = (\chi q + \overline{\sigma}^{\mu\nu} t_{\mu\nu} q + \xi \sigma^{\mu\nu} G_{\mu\nu} q - \sigma^\mu r_\mu \overline{q}^\dagger)_{\dot a} \tag{3.66c}$$

$$
\begin{aligned}
&+ \left[B^{bd\dot{}}_{c\dot{a}}\, \sigma^\mu \overline{q}^\dagger_{\dot{d}} (q^{\dagger c} \overline{\sigma}_\mu q_b) + C^{\dot{b}d}_{c\dot{a}}\, q_d (q^{\dagger c} \overline{q}^\dagger_{\dot{b}})\right] , \\
\sigma^\mu \,\mathrm{i}\, D_\mu q^{\dagger\, a} = {} & (\chi^T \overline{q} - \overline{\sigma}^{\mu\nu} t^T_{\mu\nu} \overline{q} - \xi^T \sigma^{\mu\nu} G_{\mu\nu} \overline{q} - \sigma^\mu l^T_\mu q^\dagger)^a \qquad (3.66\mathrm{d}) \\
&- \left[2A^{ad}_{bc}\, \sigma^\mu q^{\dagger b} (q^{\dagger c} \overline{\sigma}_\mu q_d) \right.\\
&\quad \left. + B^{bd\dot{}}_{a\dot{c}}\, \sigma_\mu q^\dagger_b (\overline{q}^{\dot{c}} \sigma^\mu \overline{q}^\dagger_{\dot{d}}) - C^{\dot{c}a}_{d\dot{b}}\, \overline{q}^{\dot{b}} (\overline{q}^\dagger_{\dot{c}} q^{\dagger d})\right] ,
\end{aligned}
$$

Using the quark field equations of motion to simplify this expression at tree level, one finds the tree-level trace relation

$$
\Theta^\mu_\mu \equiv (\overline{q}\chi q + \text{h.c.}) + \mathcal{L}_{\text{dipole}} - \mathcal{L}_{\bar{l}l\bar{q}q} + 2\mathcal{L}_{\overline{q}q\overline{q}q} \ . \qquad (3.67)
$$

Keeping track of the ω_s running, the modified trace relation becomes

$$
\begin{aligned}
\Theta^\mu_\mu \equiv {} & -\frac{\beta_0}{32\pi^2} G^a_{\mu\nu} G^{\mu\nu}_a \qquad (3.68) \\
& + (\overline{q}\chi q + \text{h.c.}) + \mathcal{L}_{\text{dipole}} - \mathcal{L}_{\bar{l}l\bar{q}q} + 2\mathcal{L}_{\overline{q}q\overline{q}q} \ .
\end{aligned}
$$

3.3.3 Generalized Chiral Symmetry

The external currents explicitly break the chiral symmetry of QCD in the sense that the full QCD action with external currents is not invariant under the transformation (3.17). However, the generating functional

$$
\mathcal{Z}_{\text{QCD}} = \mathcal{Z}_{\text{QCD}}\left[\chi, l_\mu, r_\mu, \xi, t_{\mu\nu}, A, B, C\right] \qquad (3.69)
$$

is invariant under the local $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ transformation

$$
\begin{aligned}
l_\mu &\to V\, l_\mu\, V^\dagger - \mathrm{i}\, V \partial_\mu V^\dagger \ , & \chi &\to \overline{V}^\dagger\, \chi\, V^\dagger \ , \qquad (3.70\mathrm{a}) \\
r_\mu &\to \overline{V}^\dagger r_\mu\, \overline{V} - \mathrm{i}\, \overline{V}^\dagger \partial_\mu \overline{V} \ , & \xi &\to \overline{V}^\dagger\, \xi\, V^\dagger \ , \\
\vartheta &\to \vartheta - \mathrm{i}\,\mathrm{tr}\ln V\overline{V} \ , & t_{\mu\nu} &\to \overline{V}^\dagger\, t_{\mu\nu}\, V^\dagger \ ,
\end{aligned}
$$

$$
\begin{aligned}
A_a{}^b{}_c{}^d &\to (V)_a{}^u (V^\dagger)_v{}^b (V)_c{}^x (V^\dagger)_y{}^d A_u{}^v{}_x{}^y \ , \qquad (3.70\mathrm{b}) \\
B_a{}^b{}_{\dot{c}}{}^{\dot{d}} &\to (V)_a{}^u (V^\dagger)_v{}^b (\overline{V}^\dagger)_{\dot{c}}{}^{\dot{x}} (\overline{V})_{\dot{y}}{}^{\dot{d}} B_u{}^v{}_{\dot{x}}{}^{\dot{y}} \ , \\
C_a{}^{\dot{b}}{}_{\dot{c}}{}^d &\to (V)_a{}^u (\overline{V})_{\dot{y}}{}^{\dot{b}} (\overline{V}^\dagger)_{\dot{c}}{}^{\dot{x}} (V)_v{}^d C_u{}^{\dot{y}}{}_{\dot{x}}{}^v \ ,
\end{aligned}
$$

since the resulting modifications can be eliminated by means of the variable transformation

$$
q_a \to (Vq)_a \ , \qquad \overline{q}^{\dot{a}} \to (\overline{q}\overline{V})^{\dot{a}} \ . \qquad (3.71)
$$

As before, the dotted and undotted latin indices to denote quantities transforming as members of the fundamental representations of $\mathrm{U}(3)_R$ and $\mathrm{U}(3)_L$, respectively.

In standard treatments of χPT [10, 11, 34, 35], an analogous symmetry that transforms only the currents l_μ, r_μ, χ, and ϑ is used to construct the χPT action in the presence of these external currents, and in [37], this procedure has been extended to account for the tensor currents $t_{\mu\nu}$.

Here, I consider the generalized symmetry defined by both (3.70a) and (3.70b), which also transforms the external currents A, B, C, and ξ, in order to construct the full action of χPT in the presence of all 10 external currents. For this purpose, it is useful to decompose the currents A, B, and C into constituents transforming as members of irreducible representations of $\mathrm{U}(3)_R \times \mathrm{U}(3)_L$. First, consider the current A. It only carries lefthanded indices, and transforms as a member of the representation

$$\begin{aligned}(3 \otimes \overline{3}) \otimes (3 \otimes \overline{3}) &= (8 \oplus 1) \otimes (8 \oplus 1) \\ &= 27_S \oplus 8_S \oplus 1_S \oplus 8_A \oplus 1_A + \dots \ ,\end{aligned} \tag{3.72}$$

where subscripts on the righthand side denote irreducible representations that are either fully symmetric and fully antisymmetric. The dots ... signify that I have neglected contributions from mixed symmetric representations. A receives no contributions from these representations, since $(q^{\dagger a}\overline{\sigma}^\mu q_b)(q^{\dagger c}\overline{\sigma}_\mu q_d)$ has to be symmetric under $(a,b) \leftrightarrow (c,d)$. Hence, A can be decomposed as

$$\begin{aligned}A_a{}^b{}_c{}^d = \frac{1}{3}\delta_{[a}{}^{[b}\delta_{c]}{}^{d]}\mathcal{A}_1 + 4\delta_{[a}{}^{[b}\mathcal{A}_{8c]}{}^{d]} \\ + \frac{1}{6}\delta_{(a}{}^{(b}\delta_{c)}{}^{d)}\mathcal{S}_1 + \frac{4}{5}\delta_{(a}{}^{(b}\mathcal{S}_{8c)}{}^{d)} + \mathcal{S}_{27a}{}^b{}_c{}^d \ ,\end{aligned} \tag{3.73a}$$

where[2]

$$\begin{aligned}\mathcal{A}_1 &\equiv A_{[x}{}^{[x}{}_{y]}{}^{y]} \ , & \mathcal{S}_1 &\equiv A_{(x}{}^{(x}{}_{y)}{}^{y)} \ , \\ \mathcal{A}_{8a}{}^b &\equiv A_{[a}{}^{[b}{}_{x]}{}^{x]} - \frac{1}{3}\delta_a{}^b\mathcal{A}_1 \ , & \mathcal{S}_{8a}{}^b &\equiv A_{(a}{}^{(b}{}_{x)}{}^{x)} - \frac{1}{3}\delta_a{}^b\mathcal{S}_1 \ ,\end{aligned} \tag{3.73b}$$

[2] As before, I (anti-)symmetrize indices such that $T^{(\mu\nu)} = 1/2(T^{\mu\nu} + T^{\nu\mu})$ and $T^{[\mu\nu]} = 1/2(T^{\mu\nu} - T^{\nu\mu})$.

$$\mathcal{S}_{27}{}_{a\ c}^{\ b\ d} \equiv A_{(a\ c)}^{\ (b\ d)} - \frac{4}{5}(\delta_{(a}{}^{(b}\mathcal{S}_{8}{}_{c)}{}^{d)} - \frac{1}{6}\delta_{(a}{}^{(b}\delta_{c)}{}^{d)}\mathcal{S}_1 \ , \tag{3.73c}$$

Next, consider B and C. At order in ϵ/M_W^3, only the lefthanded quarks couple to the $W^\pm$ bosons. In principle, the SM mass terms and the hidden sector Yukawa interactions of the type $S\overline{q}q$ can induce chirality flips, but the corresponding four quark operators only contribute at order ϵ/M_W^4. Hence, both B and C have to transform as singlets under $\mathrm{U}(3)_R$, giving

$$B_{a\ \dot{c}}^{\ b\ \dot{d}} \equiv B_{8a}{}^{b}\delta_{\dot{c}}{}^{\dot{d}} + \frac{1}{9}\delta_a{}^b\delta_{\dot{c}}{}^{\dot{d}}B_1 \ , \quad C_{a\ \dot{c}}^{\ \dot{b}\ d} \equiv C_{8a}{}^{b}\delta_{\dot{c}}{}^{\dot{d}} + \frac{1}{9}\delta_a{}^b\delta_{\dot{c}}{}^{\dot{d}}C_1 \ , \tag{3.74a}$$

where

$$B_{8a}{}^{b} \equiv \frac{1}{3}\Big(B_{a\ \dot{x}}^{\ b\ \dot{x}} - \frac{1}{3}\delta_a{}^b B_1\Big) \ , \qquad B_1 \equiv B_{x\ \dot{y}}^{\ x\ \dot{y}} \ , \tag{3.74b}$$

$$C_{8a}{}^{b} \equiv \frac{1}{3}\Big(C_{a\ \dot{x}}^{\ b\ \dot{x}} - \frac{1}{3}\delta_a{}^b C_1\Big) \ , \qquad C_1 \equiv C_{x\ \dot{y}}^{\ x\ \dot{y}} \ . \tag{3.74c}$$

The operators associated with the singlet contributions $\mathcal{A}_1$, $\mathcal{S}_1$, B_1, C_1 are quark flavour conserving and therefore negligible. In terms of the octet and 27-plet constituent currents, the four quark Lagrangian is

$$\begin{aligned}\mathcal{L}_{\overline{q}q\overline{q}q} = &-\mathcal{S}_{27}{}_{a\ c}^{\ b\ d} O_{\mathcal{S}_{27}}{}_{b\ d}^{\ a\ c} \\ &- \langle \mathcal{S}_8 O_{\mathcal{S}_8} + \mathcal{A}_8 O_{\mathcal{A}_8} + B_8 O_{B_8} + C_8 O_{C_8}\rangle \ .\end{aligned} \tag{3.75a}$$

where $\langle\cdot\rangle$ denotes a quark flavour trace, and the individual contributions are

$$\begin{aligned}
O_{\mathcal{S}_8}{}_a{}^b \equiv\ & \frac{2}{5}\big[(q^{\dagger a}\overline{\sigma}^\mu q_b)(q^{\dagger x}\overline{\sigma}_\mu q_x) + (q^{\dagger a}\overline{\sigma}^\mu q_x)(q^{\dagger x}\overline{\sigma}_\mu q_b)\big] \\
& - \frac{2}{15}\delta_a{}^b\big[(q^{\dagger y}\overline{\sigma}^\mu q_y)(q^{\dagger x}\overline{\sigma}_\mu q_x) + (q^{\dagger y}\overline{\sigma}^\mu q_x)(q^{\dagger x}\overline{\sigma}_\mu q_y)\big] \ , \\
O_{\mathcal{A}_8}{}_a{}^b \equiv\ & 2\big[(q^{\dagger a}\overline{\sigma}^\mu q_b)(q^{\dagger x}\overline{\sigma}_\mu q_x) - (q^{\dagger a}\overline{\sigma}^\mu q_x)(q^{\dagger x}\overline{\sigma}_\mu q_b)\big] \\
& - \frac{2}{3}\delta_a{}^b\big[(q^{\dagger y}\overline{\sigma}^\mu q_y)(q^{\dagger x}\overline{\sigma}_\mu q_x) - (q^{\dagger y}\overline{\sigma}^\mu q_x)(q^{\dagger x}\overline{\sigma}_\mu q_y)\big] \ , \\
O_{B_8}{}_a{}^b \equiv\ & (q^{\dagger b}\overline{\sigma}^\mu q_a)(\overline{q}^\dagger_{\dot{x}}\overline{\sigma}_\mu\overline{q}^{\dot{x}}) - \frac{1}{3}\delta_a{}^b(q^{\dagger y}\overline{\sigma}^\mu q_y)(\overline{q}^\dagger_{\dot{x}}\overline{\sigma}_\mu\overline{q}^{\dot{x}}) \ , \\
O_{C_8}{}_a{}^b \equiv\ & (q^{\dagger b}\overline{q}^\dagger_{\dot{x}})(\overline{q}^{\dot{x}} q_a) - \frac{1}{3}\delta_a{}^b(q^{\dagger y}\overline{q}^\dagger_{\dot{x}})(\overline{q}^{\dot{x}} q_y) \ ,
\end{aligned}$$

$$O_{\mathcal{S}_{27}}{}_{b\,d}^{\,a\,c} \equiv \frac{1}{2}\left[(q^{\dagger a}\overline{\sigma}^{\mu}q_b)(q^{\dagger c}\overline{\sigma}_{\mu}q_d) + (q^{\dagger a}\overline{\sigma}^{\mu}q_d)(q^{\dagger c}\overline{\sigma}_{\mu}q_b)\right] - \delta_{(a}^{\ (b}O_{\mathcal{S}_8}{}_{c)}^{\ d)}$$
$$- \frac{1}{12}\delta_{(a}^{\ (b}\delta_{c)}^{\ d)}\left[(q^{\dagger x}\overline{\sigma}^{\mu}q_x)(q^{\dagger y}\overline{\sigma}_{\mu}q_y) + (q^{\dagger x}\overline{\sigma}^{\mu}q_y)(q^{\dagger y}\overline{\sigma}_{\mu}q_x)\right] \, .$$

The octet contributions $M_8 \in \{\mathcal{A}_8, \mathcal{S}_8, B_8, C_8\}$ are traceless, hermitian 3×3 matrices, and can be written as a linear combination of the Gell-Mann matrices. The uncharged $|\Delta F| = 1$ contributions are proportional to Q_2^3 and Q_3^2, which project unto $s \leftrightarrow d$ transitions. Since I neglect quark flavour conserving contributions, I replace

$$M_8 \to Q_3^2 \langle Q_2^3 M_8 \rangle + \text{h.c.} \,, \tag{3.76a}$$

where

$$\langle Q_3^2 B_8 \rangle = |V|_{ds}^2 \; d_5 \,, \;\; \langle Q_3^2 \mathcal{S}_8 \rangle = |V|_{ds}^2 \frac{1}{4}\left[(d_1 + d_2) + 5(d_3 + d_4)\right] , \tag{3.76b}$$
$$\langle Q_3^2 C_8 \rangle = |V|_{ds}^2 \; d_6 \,, \; \langle Q_3^2 \mathcal{A}_8 \rangle = |V|_{ds}^2 \frac{1}{4}(d_2 - d_1 + d_4 - d_3) \,,$$

and $d_i \equiv c_i(1 + \sum_j x_{j,i} \cdot \epsilon^{S_j/M_W})$ as before.

The 27-plet contribution $(\mathcal{S}_{27})_{b\,d}^{\,a\,c}$ is fully tracelesss and symmetric under any contraction and permutation of indices. As with the octet operators, I keep only the uncharged $|\Delta F| = 1$ contributions, which are proportional to $(\mathcal{S}_{27})_{s\,q}^{\,d\,q}$ with $q = u, d, s$ and permutations thereof. Using the flavour structure of the O_i and $\widetilde{O}_i$ four quark operators, one finds

$$\mathcal{S}_{27}{}_{d\,s}^{\,s\,s} = \mathcal{S}_{27}{}_{d\,d}^{\,s\,d} = -\frac{1}{2}\mathcal{S}_{27}{}_{d\,u}^{\,s\,u} \,, \tag{3.77}$$

so that one can replace

$$\begin{aligned}
\mathcal{S}_{27}{}_{a\,c}^{\,b\,d} O_{\mathcal{S}_{27}}{}_{b\,d}^{\,a\,c} \to\ & 4\,\mathcal{S}_{27}{}_{d\,u}^{\,s\,u} O_{\mathcal{S}_{27}}{}_{s\,u}^{\,d\,u} \\
& + 2\,\mathcal{S}_{27}{}_{d\,d}^{\,s\,d} O_{\mathcal{S}_{27}}{}_{s\,d}^{\,d\,d} + 2\,\mathcal{S}_{27}{}_{d\,s}^{\,s\,s} O_{\mathcal{S}_{27}}{}_{s\,s}^{\,d\,s} + \text{h.c.} \\
=\ & 6\mathcal{S}_{27}{}_{d\,u}^{\,s\,u} O_{\mathcal{S}_{27}}{}_{s\,u}^{\,d\,u} + 4\mathcal{S}_{27}{}_{d\,d}^{\,s\,d} O_{\mathcal{S}_{27}}{}_{s\,d}^{\,d\,d} \\
& + 2\mathcal{S}_{27}{}_{d\,u}^{\,s\,u} O_{\mathcal{S}_{27}}{}_{s\,d}^{\,d\,d} + 2\mathcal{S}_{27}{}_{d\,d}^{\,s\,d} O_{\mathcal{S}_{27}}{}_{s\,u}^{\,d\,u} + \text{h.c.} \\
=\ & 5\,\mathcal{S}_{27}{}_{d\,u}^{\,s\,u} O_{\mathcal{S}_{27}}{}_{s\,u}^{\,d\,u} + \text{h.c.} \,.
\end{aligned} \tag{3.78}$$

Hence, the four quark Lagrangian can be cast as

$$\begin{aligned}
\mathcal{L}_{\overline{q}q\overline{q}q} =\ & -5\,\mathcal{S}_{27}{}_{d\,u}^{\,s\,u}(O_{\mathcal{S}_{27}})_{s\,u}^{\,d\,u} - \langle Q_2^3 \mathcal{S}_8 \rangle \langle Q_3^2 O_{\mathcal{S}_8} \rangle \\
& - \langle Q_2^3 \mathcal{A}_8 \rangle \langle Q_3^2 O_{\mathcal{A}_8} \rangle - \langle Q_2^3 B_8 \rangle \langle Q_3^2 O_{B_8} \rangle - \langle Q_2^3 C_8 \rangle \langle Q_3^2 O_{C_8} \rangle \,.
\end{aligned} \tag{3.79}$$

3.4 Construction of the PET χPT Lagrangian

In this section, I construct a generalized action for χPT in the presence of the external currents collected in the interaction Lagrangian (3.38), which captures the portal interactions between the light pseudoscalar mesons and hidden sectors at order $\mathcal{O}(\epsilon)$.

3.4.1 General Considerations

To construct the generalized χPT action, I follow the same general strategy used for deriving the action of χPT in the SM, see e.g. the general introductions in [30, 10, 31, 32, 33, 11, 34, 35].

In this approach, the coupling to external sectors is included using the external current picture, and the associated explicit symmetry breaking of the QCD chiral symmetry is considered to be a perturbation about the chiral limit, in which chiral symmetry is only broken spontaneously. χPT itself is then constructed in two steps: First, the PNGB are constructed for the unperturbed, that is, spontaneously broken, theory. This way, one can deduce the transformation behaviour of the PNGBs under general chiral rotations. Using both the PNGB of the chiral limit and the external currents as building blocks, the actual χPT Lagrangian is then constructed using a spurion analysis.

U(3) vs. SU(3) Chiral Perturbation Theory

As the first step, one has to decide which version of χPT to use. For many applications, it is possible to avoid having to rely on the large N_c limit by working exclusively within the SU(3) version of χPT. However, to account for the coupling of QCD to the flavour blind $\widetilde{a}$ current, one has to retain the singlet PNGB η^0 associated with the explicitly broken $\mathrm{U}(1)_A$ symmetry, since the interaction term $\widetilde{a}\widetilde{G}_{\mu\nu}G^{\mu\nu}$ hadronizes such that $\widetilde{a}$ couples only to the singlet PNGB[3]. One might think to avoid this problem by first eliminating the $\widetilde{a}\widetilde{G}_{\mu\nu}G^{\mu\nu}$ coupling by means of a local, axial rotation, in order to then work with the SU(3) of χPT. However, the current $\widetilde{a}$ would reappear as a flavour blind contribution

[3]Intuitively, this may be expected due to $\widetilde{G}_{\mu\nu}G^{\mu\nu}$ itself transforming as a pseudoscalar flavour singlet.

to the other pseudoscalar current p, and it turns out that such a flavour blind contribution also couples only to the η^0. This is hardly surprising, since a field redefinition should not alter the underlying physics. Hence, the singlet PNGB has to be included for a full accounting of the coupling of χPT to hidden sectors, which implies that one has to work within the full U(3) version of χPT, just as is the case for studies of the effect of isospin-breaking on the ${}^{\epsilon'}\!/_{\epsilon}$ ratio [90].

Symmetries

Besides the PNGB fields, the χPT Lagrangian can contain the external currents or their derivatives. The χPT generating functional has to remain invariant under the local chiral transformation of the external currents (3.70a). Combining the field redefinition (3.71) with relation (3.33), one finds that the χPT action should be invariant under (3.70a), if the PNGB fields are simultaneously shifted as

$$g \to V g \overline{V} \ , \qquad \psi = \!\!\to \psi - \mathrm{i}\langle \ln V + \ln \overline{V} \rangle \ . \tag{3.80}$$

As with gauge transformations, objects with derivatives ∂_μ are not necessarily covariant under the local field redefinition (3.80). Following the standard approach, I use the external currents l_μ and r_μ to construct covariant derivatives

$$\begin{aligned}
\mathrm{i}\, D_\mu T^{b_1 \dots b_m,\ \dot d_1 \dots \dot d_q}_{a_1 \dots a_n,\ \dot c_1 \dots \dot c_p} \equiv{}& \mathrm{i}\, \partial_\mu T^{\ b_1 \dots b_m,\ \dot d_1 \dots \dot d_q}_{a_1 \dots a_n,\ \dot c_1 \dots \dot c_q} \\
&+ \sum_{i=1}^{n} (l_\mu)^x_{a_i} T^{\ b_1 \dots b_m,\ \dot d_1 \dots \dot d_q}_{a_1 \dots a_{i-1}\, x\, a_{i+1} \dots a_n,\ \dot c_1 \dots \dot c_p} \\
&+ \sum_{k=1}^{p} (r_\mu)^{\dot x}_{\dot c_k} T^{\ b_1 \dots b_m,\ \dot d_1 \dots \dot d_q}_{a_1 \dots a_n,\ \dot c_1 \dots \dot c_{k-1}\, \dot x\, \dot c_{k+1} \dots \dot c_p} \\
&- \sum_{j=1}^{m} (l_\mu)^{b_j}_x T^{\ b_1 \dots b_{i-1}\, x\, b_{j+1} \dots b_m,\ \dot d_1 \dots \dot d_q}_{a_1 \dots a_n,\ \dot c_1 \dots \dot c_p} \\
&- \sum_{l=1}^{q} (r_\mu)^{\dot d_l}_{\dot x} T^{\ b_1 \dots b_m,\ \dot d_1 \dots \dot d_{l-1}\, \dot x\, \dot d_{l+1} \dots \dot d_q}_{a_1 \dots a_n,\ \dot c_1 \dots \dot c_p} \ .
\end{aligned} \tag{3.81}$$

For g and ψ, this gives

$$D_\mu g \equiv \partial_\mu g + \mathrm{i}\, l_\mu g - \mathrm{i}\, g\, r_\mu \ , \tag{3.82a}$$

$$D_\mu \psi \equiv \mathrm{i}\langle g D_\mu g^\dagger \rangle = \partial_\mu \psi - \langle r_\mu - l_\mu \rangle \ , \tag{3.82b}$$

where as before $r_\mu = r_\mu^{\mathrm{SM}} + r_\mu^{\mathrm{BSM}}$ and $l_\mu = l_\mu^{\mathrm{SM}} + l_\mu^{\mathrm{BSM}}$ contain both electroweak and hidden current contributions. In full analogy with actual gauge fields, I also define the "field strength tensors"

$$L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu - \mathrm{i}[l_\mu, l_\nu] \ , \tag{3.83a}$$

$$R_{\mu\nu} = g\Big(\partial_\mu r_\nu - \partial_\nu r_\mu - \mathrm{i}[r_\mu, r_\nu]\Big) g^\dagger \ . \tag{3.83b}$$

In principle, one also has to keep track of the covariant derivatives for the remaining external currents. At the level of accuracy in this work, only derivatives of ϑ contribute to the PET χPT Lagrangian. Hence, it is sufficient to define

$$D_\mu \vartheta = \partial_\mu \vartheta - \langle r_\mu - l_\mu \rangle \ . \tag{3.84}$$

Power Counting

In terms of power counting, the U(3) version of the χPT Lagrangian is defined as a simultaneous expansion in powers of p^2/Λ_χ^2 and $1/N_c$, where $\Lambda_\chi \equiv 4\pi F_\pi = 1.169\,\mathrm{GeV}$ is defined in terms of the Pion decay constant [36, 11]. Accounting for weak and hidden sector contributions, one also has to count powers of $1/M_W^2$ and ϵ.

First, consider the power counting rules for the p^2/Λ_χ^2 expansion. Since F_π defines the typical energy scale of χPT, this expansion has to be justified by means of the $1/4\pi$ counting scheme of NDA [21]. According to the rules of NDA, each derivative counts as $\partial^2 \sim p^2/\Lambda_\chi^2$, while powers of the PNGB fields ϕ_a have to be counted as $\mathcal{O}(1)$. Further, the l_μ and r_μ currents have to be counted as derivatives, $l_\mu, r_\mu \sim p/\Lambda_\chi$. The power counting of the remaining external currents is determined by the induced PNGB masses, since physical matrix elements have to be expanded in M_π^2/Λ_χ^2 as well as p^2/Λ_χ^2. Using relation (3.32), one finds $M_\pi^2 \propto \chi, \xi, t_{\mu\nu}$, so that one has to count these currents as p^2/Λ_χ^2 as well. Alternatively, the power counting of χ, ξ, and $t_{\mu\nu}$ can also be deduced from the scaling behaviour of on-shell matrix elements, see [91, 34, 37].

Second, consider the $^1/_{N_c}$ expansion. For the most part, I follow the N_c counting rules established in [11]. Using canonically normalized fields, the external current $\widetilde{a}$ appears in conjunction with a factor of $g_s^2 = {}^{(g_s')^2}/_{N_c}$, where g_s' is the rescaled QCD coupling that is held fix when taking the large N_c limit. At the level of χPT, this implies that each factor of a and ϑ is associated with a power of $^1/_{N_c}$, while the remaining external currents have to be counted as $\mathcal{O}(1)$. Furthermore, each flavour trace in the χPT Lagrangian has to be associated with a closed quark loop, which implies that it counts as $^1/_{N_c}$. In particular, one has $\psi \sim {}^1/_{N_c}$ for each power of ψ. Accounting for the currents a, A, B, and C, I alter these rules in two ways: Firstly, the currents a also appears conjunction with a factor of $g_s^2 = {}^{(g_s')^2}/_{N_c}$, so that also has to be associated with a power of $^1/_{N_c}$. Second, flavour traces $\langle\cdot\rangle$ have to be counted as $\mathcal{O}(1)$ when they appear in operators involving the four quark currents A, B, C, since vertices such as $q^\dagger\overline{\sigma}^\mu q q^\dagger\overline{\sigma}_\mu q$ are not accounted for in the large N_c power counting rule (3.12).

Finally, one has to specify how $^{p^2}/_{\Lambda_\chi^2}$ and $^1/_{N_c}$ and should be counted in relation to each other. In the remainder of this chapter, I follow the approach of [11] and adopt the counting $^1/_{N_c} \sim {}^{p^2}/_{\Lambda_\chi^2} \sim \delta$, with δ being some generic small parameter. When considering Kaon decays, this counting is sensible, since $^{M_K^2}/_{\Lambda_\chi^2} \approx {}^1/_4$ turns out to be numerically comparable to $^1/_3$. At lower energies, i.e. at $^s/_{\Lambda_\chi^2} \approx {}^{m_\pi^2}/_{\Lambda_\chi^2} \approx 0.01$, the small momentum is a much better expansion parameter than $^1/_{N_c}$, so that the standard $^{p^2}/_{\Lambda_\chi^2} \sim \delta$, $^1/_{N_c} \sim 1$ power counting used in the SU(3) version of χPT is more appropriate.

Using the $^1/_{N_c} \sim {}^{p^2}/_{\Lambda_\chi^2} \sim \delta$ counting, one has

$$g, A, B, C \sim 1 \; ; \quad \partial_\mu, r_\mu, l_\mu \sim \sqrt{\delta} \; ; \quad \psi, \langle\cdot\rangle, a, \vartheta, \chi, \xi, t_{\mu\nu} \sim \delta \; , \tag{3.85}$$

with the one exception that flavour traces have to be counted as $\langle\cdot\rangle \sim 1$ in operators with A, B, or C.

3.4.2 The generalized χPT Action

Neglecting the explicit symmetry breaking, the χPT action can contain only g and its derivatives. Once the various sources of electroweak symmetry breaking are included, the χPT action also contains the external currents a, ϑ, l_μ, r_μ, χ, ξ, $t_{\mu\nu}$, A, B, and C.

Even without electroweak corrections, the SM χPT action already contains contributions involving the external currents ϑ, l_μ, r_μ, χ. Typically, this part of the action is derived without assuming a specific shape for the generic external currents ϑ, l_μ, r_μ, and χ, so that it is fully general, see e.g. [36, 11]. The remaining currents a, ξ, $t_{\mu\nu}$, A, B, and C are either suppressed by a factor of ϵ or $1/M_W^2$, so they can only appear linearly. Hence, I decompose the generalized χPT action as

$$\mathcal{S}_\chi \equiv \mathcal{S}_{\chi,s} + \delta\mathcal{S}_{\chi,\text{dipole}} + \delta\mathcal{S}_{\chi,\bar{q}q\bar{q}q} + \delta\mathcal{S}_{\chi,a} \ , \tag{3.86}$$

where

$$S_{\chi,s}[\vartheta, l_\mu, r_\mu, \chi] = S_{\chi,\text{SM}}\Big|_{G_F \to 0} \tag{3.87}$$

denotes the part of the action that mirrors the action for χPT in the SM without electroweak contributions. The dipole action $\delta\mathcal{S}_{\chi,\text{dipole}}$ collects contributions that involve ξ or $t_{\mu\nu}$, and the four quark action $\delta\mathcal{S}_{\chi,\bar{q}q\bar{q}q}$ collects contributions that involve A, B, or C. Both $\delta\mathcal{S}_{\chi,\text{dipole}}$ and $\delta\mathcal{S}_{\chi,\bar{q}q\bar{q}q}$ contain SM contributions that are well-known, and can be found in e.g. [48, 49, 50, 37]. They can also involve the currents ϑ, χ, l_μ, and r_μ, but not the external current a. All terms with a, and in particular the electroweak corrections with both a and the electroweak currents ξ, $t_{\mu\nu}$, A, B, and C, are collected in $\delta\mathcal{S}_{\chi,a}$. Some of the contributions in $\delta\mathcal{S}_{\chi,a}$ have been derived already in [52], which studied the coupling of χPT to a light Higgs boson. Here, I study the coupling of χPT to the more general case of a generic scalar current a, which includes the case of a light Higgs boson as a special case.

It is interesting to note that the leading order contributions to the SM χPT action $\mathcal{S}_{\chi,s}$ are suppressed by factor of δ^2 compared to the leading order diagrams in the large N_c limit in QCD, which are expected to scale as $\mathcal{O}(N_c^2 p^0)$. For the SM χPT action $\mathcal{S}_{\chi,s}$, I consider both the leading order contributions at $\mathcal{O}(\delta^2)$, and the next-to-leading order contributions at order $\mathcal{O}(\delta^3)$. There are two reasons for this: First, certain important processes, such as neutral Pion decays $\pi \to \gamma\gamma_{\text{Dark}}$ only appear at order $\mathcal{O}(\delta^3)$. Second, the $\mathcal{O}(\delta^3)$ contributions turn out to relevant for the leading order estimation of the new low energy constants (=LECs) that appear in the weak sector in the presence of hidden sectors.

In contrast, I explicitly construct only the leading order contributions for $\delta\mathcal{S}_{\chi,\text{dipole}}$, $\delta\mathcal{S}_{\chi,\bar{q}q\bar{q}q}$, and $\delta\mathcal{S}_{\chi,a}$. These contributions enter at order $\mathcal{O}(\delta^2)$ for $\delta\mathcal{S}_{\chi,\bar{q}q\bar{q}q}$, and at order $\mathcal{O}(\delta^3)$ for $\delta\mathcal{S}_{\chi,\text{dipole}}$ and $\delta\mathcal{S}_{\chi,a}$. While next-to-leading order corrections are expected to be important for quantitative determination of decay widths and scattering amplitudes, a complete construction of the next-to-leading order contributions is beyond the scope of this work. The leading order contributions are expected to be sufficient to constrain the order of magnitude of the portal sector Wilson coefficients defined in section 3.5.

The $\Delta F = 0$ SM χPT Action with External Currents

I begin by summarizing the SM χPT action $S_{\chi,s}[l_\mu, r_\mu, \vartheta, \chi]$, which contains the modifications due to the presence of generic external currents l_μ, r_μ, χ, and ϑ. At leading order, one has [36, 11]

$$\mathcal{L}^{(2)}_{\chi,\text{s}} = \frac{f^2}{4}\langle D^\mu g^\dagger D_\mu g\rangle + \frac{f^2}{4} b\langle g\chi + g^\dagger\chi^\dagger\rangle - \frac{1}{12} f^2 m_\psi^2 \left(\psi - \vartheta\right)^2 \ . \tag{3.88}$$

The operators $\propto (\psi - \vartheta)$ and $\mathrm{i}\langle g\chi - \text{h.c.}\rangle$ are forbidden by parity conservation, while $D_\mu\psi D^\mu\psi$ is of order $\mathcal{O}(\delta^3)$ due to the additional flavour trace. In terms of power counting, the first two terms in Eq. (3.88) contribute as $\mathcal{O}(p^2 N_c)$, while the final singlet mass term $\propto m_\psi^2$ contributes as $\mathcal{O}(p^0 N_c^0)$.

Neglecting higher order corrections, the parameters f, b, and m_ψ are in simple correspondence with the Pion decay constant F_π, the chiral quark condensate B_0, and the vacuum susceptibility τ,

$$F_\pi = f\left(1 + \mathcal{O}(\delta)\right) \ , \qquad B_0 = b\left(1 + \mathcal{O}(\delta)\right) \ , \tag{3.89a}$$

$$\tau = \frac{1}{6} f^2 m_\psi^2 \left(1 + \mathcal{O}(\delta)\right) \ . \tag{3.89b}$$

The leading order classical equation of motion can be obtained by varying the Lagrangian (3.86) with respect to g [30, 10, 34]. Since $g + \delta g$ has to be unitary, δg can be cast as

$$g + \delta g \equiv (\mathbb{1} + \mathrm{i}\,\lambda_a\Delta_a)g + \mathcal{O}(\Delta^2) \ , \tag{3.90}$$

where the Δ_a with $a = 0, \ldots, 8$ are arbitrary real-valued functions. For $a \neq 0$, it is sufficient to vary the first two terms in (3.86), giving

$$0 \stackrel{!}{=} \langle \lambda_a \, \mathrm{i}(D^2 g g^\dagger - g D^2 g^\dagger + g\chi - \chi^\dagger g^\dagger) \rangle \ . \tag{3.91}$$

For $a = 0$, one also has to vary the final term, since $\delta\psi = \sqrt{6}\,\Delta_0/f$. This gives

$$0 \stackrel{!}{=} \langle \lambda_0 \, \mathrm{i}(D^2 g g^\dagger - g D^2 g^\dagger + g\chi - \chi^\dagger g^\dagger) \rangle - \frac{1}{\sqrt{6}} f m_\psi^2 \, (\psi - \vartheta) \ . \tag{3.92}$$

Since any hermitian 3×3 matrix can be written as $M = \frac{1}{2}\lambda_a \langle \lambda_a M \rangle$, this implies the matrix-valued equation of motion

$$0 \stackrel{!}{=} \mathrm{i}(D^2 g g^\dagger - g D^2 g^\dagger + g\chi - \chi^\dagger g^\dagger) - \frac{1}{6} f m_\psi^2 (\psi - \vartheta) \ \mathbb{1} \ , \tag{3.93}$$

so that in higher order contributions, the object $(g D^2 g^\dagger - \text{h.c.})$ can always be eliminated in favour of contributions containing χ or ψ. At the same time, one has the general identity

$$0 = D^2 (g g^\dagger) = D^2 g g^\dagger + g D^2 g^\dagger + 2 D_\mu g D^\mu g^\dagger \ , \tag{3.94}$$

so that the object $(g D^2 g^\dagger + \text{h.c.})$ can always be eliminated in favour of contributions containing the object $g D_\mu g^\dagger$. For the construction of the remaining contributions to the χPT action, it is useful to define the fully lefthanded objects,

$$L_\mu = \mathrm{i}\, g D_\mu g^\dagger \ , \qquad\qquad X = b\, g\chi \ . \tag{3.95}$$

It can be shown that L_μ is the χPT low energy realization of the lefthanded Noether current (3.18).

At order $\mathcal{O}(\delta^3)$, the action picks up two types of contributions: First, there are contribution obtained by using the standard methods of constructing effective Lagrangians. Second, there is the Wess-Zumino-Witten (WZW) term $\mathcal{S}_{\text{WZW}}$, which is an anomalous contribution that cannot be written straightforwardly in terms of a Lagrangian density [92]. The non-anomalous contribution is [11]

$$\begin{aligned}\mathcal{L}^{(3)}_{\chi,\mathrm{s}} = {} & L_2\langle L_\mu L_\nu L^\mu L^\nu\rangle + (2L_2 + L_3)\langle L_\mu L^\mu L_\nu L^\nu\rangle \\ & + L_5\langle L_\mu L^\mu (X + X^\dagger)\rangle + L_8\langle XX + X^\dagger X^\dagger\rangle \\ & - \mathrm{i}\, L_9\langle L^\mu L^\nu (L_{\mu\nu} + R_{\mu\nu})\rangle + L_{10}\langle R_{\mu\nu} L^{\mu\nu}\rangle \\ & + f_\pi^2 \Lambda_1 D_\mu\psi D^\mu\psi - \mathrm{i}\, f_\pi^2\Lambda_2(\psi - \vartheta)\langle X - X^\dagger\rangle \\ & + H_0 D_\mu\vartheta D^\mu\vartheta + H_1\langle R_{\mu\nu}R^{\mu\nu} + L_{\mu\nu}L^{\mu\nu}\rangle + H_2\langle X^\dagger X\rangle\ .\end{aligned} \tag{3.96}$$

As before, the operator $(\psi - \vartheta)^3$ is forbidden by parity conservation. The numbering of the L_i is chosen such that it coincides with the standard numbering of the $\mathcal{O}(p^4)$ operators in the SU(3) χPT Lagrangian [10, 34, 35], where the theory is not expanded in $1/N_c$. The LEC's L_i and Λ_i can be fixed by experimental observations, but the H_i operators do not depend on g. As a result, they do not contribute to perturbative computations of S-matrix elements [30, 10, 34]. Nonetheless, the corresponding terms are necessary to renormalize loop graphs with multiple external currents attached to the diagram.

Next, consider WZW term. It can be written as an integral over a five dimensional submanifold in the nine dimensional space spanned by the possible field values of g. The connection to the conventional way of writing an action is established by identifying the four dimensional boundary of this submanifold with Minkowsky spacetime. An elegant derivation of the WZW action in the presence of external currents has been performed in [92]. Accounting for the external currents, one has

$$\begin{aligned}\mathcal{S}_{\mathrm{WZW}} = {} & \frac{\mathrm{i}\, N_c}{240\pi^2}\int \mathrm{d}\omega_{ijklm}\Sigma^{ijklm} \\ & - \frac{\mathrm{i}\, N_c}{48\pi^2}\int \mathrm{d}^4x\ \epsilon^{\mu\nu\alpha\beta}\Big(W_{\mu\nu\alpha\beta}(l, r, g) - W_{\mu\nu\alpha\beta}(l, r, \mathbb{1})\Big)\,,\end{aligned} \tag{3.97a}$$

where the function $W_{\mu\nu\alpha\beta}(r, l, g)$ depends on g and on the external currents l^μ and r^μ. A complete expression for $W_{\mu\nu\alpha\beta}(r, l, g)$ can be found in section 5.3 of [10]. Here, I only keep terms that are at most quadratic in the external currents. In this approximation, one has [10]

$$
\begin{aligned}
W_{\mu\nu\alpha\beta}(r,l,g) = \langle & -\mathrm{i}\, L^0_\mu g^\dagger (\partial_\nu r_\alpha) g l_\beta + L^0_\mu L^0_\nu g^\dagger r_\alpha g l_\beta \\
& - \mathrm{i}\, L^0_\mu \{ l_\nu, \partial_\alpha l_\beta \} - \frac{1}{2} L^0_\mu l_\nu L^0_\alpha l_\beta + L^0_\mu L^0_\nu L^0_\alpha l_\beta \rangle \\
& - (g \leftrightarrow g^\dagger, l^\mu \leftrightarrow r^\mu) \ ,
\end{aligned}
\tag{3.98a}
$$

where I have defined the lefthanded pure QCD Noether current as $L^0_\mu \equiv \mathrm{i}\, g^\dagger \partial_\mu g$.

The WZW has to be included, since the χPT action defined by (3.88) and (3.125a) is independently invariant under $x^\mu \to -x^\mu$, and $g \to g^\dagger$ [10, 92]. However, the full QCD Lagrangian is only invariant under parity, which corresponds to the simultaneous transformation $x^\mu \to -x^\mu$, $g \to g^\dagger$[4]. Due to this superfluous symmetry, certain experimentally observed processes, such as $K^+K^- \to \pi^0\pi^+\pi^-$ or the aforementioned $\pi \to \gamma\gamma$ cannot be mediated by the non-anomalous contributions to the χPT action. Instead, both of these processes are mediated by the WZW action, which breaks the superfluous symmetry.

Dipole Contributions

The dipole sector collects operators that contain ξ and $t_{\mu\nu}$. At order $\mathcal{O}(\delta^2)$, there are no operators with $t_{\mu\nu}$, since such operators would need to contain at least two additional derivatives in order to be Lorentz invariant, so that they can only contribute at order $\mathcal{O}(\delta^3)$. For the same reason, operators with ξ can only contain g, and no derivatives. Hence, the only available dipole operator is the mass-like term

$$
\langle \Xi + \mathrm{h.c.} \rangle \ , \tag{3.99}
$$

giving

$$
\delta \mathcal{L}^{(2)}_{\chi,\mathrm{dipole}} = \kappa_1 \frac{f^2}{4} \langle \Xi \rangle + \mathrm{h.c.} \ , \tag{3.100}
$$

where $\Xi = b\, g \xi$.

[4]Recall that the PNGBs are *pseudo*-scalars

At order $\mathcal{O}(\delta^3)$, the dipole action contains contributions with both $t_{\mu\nu}$ and ξ. Operators with derivatives acting on the external currents can be eliminated using partial integration. For $t_{\mu\nu}$, one has the operators

$$\langle T_{\mu\nu} L^\mu L^\nu \rangle + \text{h.c.} \,, \qquad \langle T_{\mu\nu} (L^{\mu\nu} + R^{\mu\nu}) \rangle + \text{h.c.} \,, \tag{3.101}$$

where $T_{\mu\nu} = f\, g t_{\mu\nu}$ is the fully lefthanded version of $t_{\mu\nu}$. Notice that this list of operators is consistent with the prior result obtained in [37]. There, one finds two additional operators that are quadratic in $t_{\mu\nu}$. In the present work, I count $t_{\mu\nu} \sim {}^1\!/\!M_W^2$, so that these quadratic terms would only contribute at order ${}^1\!/\!M_W^4$.

For ξ, one has the operators

$$\mathrm{i}(\psi - \vartheta)\langle \Xi \rangle + \text{h.c.} \,, \qquad \langle \Xi L_\mu L^\mu \rangle + \text{h.c.} \,, \tag{3.102a}$$

$$\langle \Xi X^\dagger \rangle + \text{h.c.} \,, \qquad \langle \Xi X \rangle + \text{h.c.} \,. \tag{3.102b}$$

In terms of the action, one obtains

$$\begin{aligned} \delta\mathcal{L}^{(3)}_{\chi,\text{dipole}} = \kappa_2 \langle \Xi L_\mu L^\mu \rangle + \kappa_3 \langle \Xi X \rangle + \kappa_4 \langle \Xi X^\dagger \rangle + \mathrm{i}\, \kappa_5 (\psi - \vartheta) \langle \Xi \rangle \\ + \lambda_1 \langle T_{\mu\nu} L^\mu L^\nu \rangle + \lambda_2 \langle T_{\mu\nu} (L^{\mu\nu} + R_{\mu\nu}) \rangle + \text{h.c.} \,. \end{aligned} \tag{3.103}$$

Again, $\langle \Xi X^\dagger \rangle$ does not depend on g. In total, the constants κ_i and λ_i are 7 free parameters.

Four Quark Contributions

The four quark sector of the χPT Lagrangian collects operators that contain the currents A, B, and C. As before, operators with derivatives acting on the external currents can be eliminated using partial integration. Using the decomposition (3.72), the four quark contributions can be split into an octet contribution involving factors of $\mathcal{M}_8 \in \{\mathcal{A}_8, \mathcal{S}_8, B_8, C_8\}$, and a 27-plet contribution involving factors of $\mathcal{S}_{27}$,

$$\mathcal{L}_{\chi,\bar{q}q\bar{q}q} = \mathcal{L}_{\chi,8} + \mathcal{L}_{\chi,27} \,. \tag{3.104}$$

In the octet Lagrangian, the matrices $(\mathcal{M}_8)_a^b$ carry two lefthanded indices that need to be contracted, but no Lorentz indices. To order δ, the only available fully lefthanded objects are

$${L_\mu}_a^{\ b} \,, \qquad (g D_\mu D_\nu g^\dagger)_a^{\ b} \,, \qquad X_a^{\ b} \,, \qquad {L_{\mu\nu}}_a^{\ b} \,, \qquad {R_{\mu\nu}}_a^{\ b} \,. \tag{3.105}$$

Since $(g^\dagger D_\mu D_\nu g)$ and the field strength tensors are already of order δ, their Lorentz indices would have to be contracted within themselves. However, the field strength tensors are traceless, and the structure $g^\dagger D^2 g$ can be eliminated using identity (3.94) and the equations of motion (3.93). Hence, the only available octet operators are

$$\langle M_8 \hat{L}_\mu \hat{L}^\mu \rangle \ , \qquad \langle M_8 \hat{L}_\mu \rangle \langle L^\mu \rangle \ , \qquad \langle M_8 X + \text{h.c.} \rangle \ . \tag{3.106}$$

where $\hat{L}_\mu \equiv L_\mu - {}^1\!/\!{}_3 \langle L_\mu \rangle$ is the traceless octet contribution to the lefthanded Noether current. Using the replacement (3.76a), one obtains the octet Lagrangian

$$\begin{aligned} \mathcal{L}^{(2)}_{\chi,8} = & -\frac{1}{4} f^4 \Big[\langle Q^2_3 (s_8 \mathcal{S}_8 + a_8 \mathcal{A}_8 + b_8 B_8 + c_8 C_8) \rangle \langle Q^3_2 \hat{L}_\mu \hat{L}^\mu \rangle \\ & \qquad + \langle Q^2_3 (s^s_8 \mathcal{S}_8 + a^s_8 \mathcal{A}_8 + b^s_8 B_8 + c^s_8 C_8) \rangle \langle Q^3_2 \hat{L}^\mu \rangle \langle L_\mu \rangle \\ & \qquad + \langle Q^2_3 (\overline{s}_8 \mathcal{S}_8 + \overline{a}_8 \mathcal{A}_8 + \overline{b}_8 B_8 + \overline{c}_8 C_8) \rangle \langle Q^3_2 (X + \text{h.c.}) \rangle \Big] \\ & + \text{h.c.} \ , \end{aligned} \tag{3.107}$$

where the Wilson coefficients a_X, s_X, b_X, and c_X are free parameters. To establish contact with the conventional notation for the octet Lagrangian in the SM, I define

$$\begin{aligned} \frac{G_F}{\sqrt{2}} |V|^2_{ds} \, g_X &\equiv \langle Q^2_3 (a_X \mathcal{A}^{\text{SM}}_8 + s_X \mathcal{S}^{\text{SM}}_8 \\ & \qquad + b_X B^{\text{SM}}_8 + c_X C^{\text{SM}}_8) \rangle \ , \\ \frac{G_F}{\sqrt{2}} |V|^2_{ds} h_{X,i} \, {}^{\epsilon S_i}\!/\!{}_{M_W} &\equiv \langle Q^2_3 (a_X \mathcal{A}^{\text{BSM}}_8 + s_X \mathcal{S}^{\text{BSM}}_8 \\ & \qquad + b_X B^{\text{BSM}}_8 + c_X C^{\text{BSM}}_8) \rangle \ , \end{aligned} \tag{3.108a}$$

and further

$$G_X \equiv \frac{G_F}{\sqrt{2}} |V|^2_{ds} f^2 \, g_X \ , \qquad H_{X,i} \equiv \frac{G_F}{\sqrt{2}} |V|^2_{ds} f^2 \, h_{X,i} \ . \tag{3.108b}$$

In the 27-plet Lagrangian, the current $(\mathcal{S}_{27})_b{}^a{}_d{}^c$ has four lefthanded flavour indices that need to be contracted. Considering the building blocks in (3.105), the only available 27-plet operator is

$$\mathcal{S}_{27}{}_b{}^a{}_d{}^c \, L_\mu{}_a{}^b \, L^\mu{}_c{}^d \ , \tag{3.109}$$

By construction, $(\mathcal{S}_{27})_b{}^a{}_d{}^c$ is fully tracelesss and symmetric under $(b,a) \leftrightarrow (d,c)$. As with the octet operators, I am only interested in the uncharged and strangeness violating terms, which are proportional to either $(\mathcal{S}_{27})_s{}^d{}_q{}^q$ with $q = u, d, s$ or permutations thereof. Thus,

$$\begin{aligned} 2\mathcal{S}_{27b}{}^a{}_d{}^c \hat{L}_{\mu a}{}^b \, \hat{L}^\mu{}_c{}^d &= \mathcal{S}_{27b}{}^a{}_d{}^c \big[\hat{L}_{\mu a}{}^b \, \hat{L}^\mu{}_c{}^d + \hat{L}_{\mu a}{}^d \, \hat{L}^\mu{}_c{}^b\big] \\ &\to 4\,\mathcal{S}_{27s}{}^d{}_u{}^u \big[\hat{L}_{\mu d}{}^u \, \hat{L}^\mu{}_u{}^s + 2\hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_u{}^u + \hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_d{}^d\big] \\ &\quad + 4\,\mathcal{S}_{27s}{}^d{}_d{}^d \big[2\hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_d{}^d + \hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_u{}^u\big] + \text{h.c.}\,. \end{aligned} \tag{3.110}$$

Using relation (3.77), one finds

$$\mathcal{S}_{27b}{}^a{}_d{}^c \hat{L}_{\mu a}{}^b \, \hat{L}^\mu{}_c{}^d \to \mathcal{S}_{27s}{}^d{}_u{}^u \big[2\hat{L}_{\mu d}{}^u \, \hat{L}^\mu{}_u{}^s + 3\hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_u{}^u\big] + \text{h.c.}\,. \tag{3.111}$$

Accordingly, the 27-plet Lagrangian becomes

$$\mathcal{L}^{(2)}_{\chi,27} = -\frac{1}{4} f^4 \, s_{27} \mathcal{S}_{27s}{}^d{}_u{}^u \Big[\hat{L}_{\mu d}{}^s \, \hat{L}^\mu{}_u{}^u + \frac{2}{3} \hat{L}_{\mu d}{}^u \, \hat{L}^\mu{}_u{}^s\Big] + \text{h.c.}\,. \tag{3.112}$$

As with the octet Lagrangian, I define

$$\frac{G_F}{\sqrt{2}} |V|^2_{ds} \, g_{27} = s_{27} \, \mathcal{S}^{\text{SM}}_{27}{}_s{}^d{}_u{}^u \,, \tag{3.113a}$$

$$\frac{G_F}{\sqrt{2}} |V|^2_{ds} \, h_{27,i} \, {}^{\epsilon S_i}\!/_{M_W} \equiv s_{27} \mathcal{S}^{\text{BSM}}_{27}{}_s{}^d{}_u{}^u \,, \tag{3.113b}$$

and

$$G_{27} \equiv \frac{G_F}{\sqrt{2}} |V|^2_{ds} f^2 \, g_{27} \,, \qquad H_{27,i} \equiv \frac{G_F}{\sqrt{2}} |V|^2_{ds} f^2 \, h_{27,i} \,. \tag{3.113c}$$

For both the octet and 27-plet Lagrangians in (3.107) and (3.112), the terms involving $\lambda_7 = {}^{(Q^3_2 - Q^2_3)}\!/_{2\,\mathrm{i}}$ contribute only for $\operatorname{Im} V_{us} V^\dagger_{du} \neq 0$, corresponding to the SM CP violation generated by the CKM matrix.

As it should, the SM contribution to $\mathcal{L}_{\chi,\bar{q}q\bar{q}q}$ is identical to the standard result obtained in [47, 48] The constants a_X, s_X, b_X, and c_X are 13 new free parameters that are not present in the SM. In addition, the 6 SM Wilson coefficients encoded within A, B, and C also have to be treated as free parameters, as their running for energies well below $m_c \sim 1.5\,\text{GeV}$ is unknown due to the unperturbative nature of QCD.

Scalar to Gluon Coupling

The scalar current a is chirally invariant and counts as $\delta \sim {}^{1}\!/\!_{N_c}$. Hence, the order $\mathcal{O}(\delta^3)$ operators with a are just the same as the order $\mathcal{O}(\delta^2)$ operators discussed in the previous sections, except with an additional factor of a. Once again, any operators with derivatives acting on a can be eliminated using partial integration, and any contributions with $D^2 g g^\dagger$ or $g D^2 g^\dagger$ can be eliminated using identity (3.94) or the equations of motion (3.93). Explicitly, one obtains

$$\begin{aligned}\mathcal{L}^{(3)}_{\chi,a} = a\frac{f^2}{4}\Big[&\frac{1}{2}\alpha_1\langle L_\mu L^\mu\rangle + \alpha_2\langle X\rangle - \alpha_3\frac{1}{3}m_\psi^2(\psi-\vartheta)^2 \\ &- \beta_1 G_8\langle Q_2^3 \hat{L}^\mu \hat{L}_\mu\rangle - \beta_2 G_8^s\langle Q_2^3 \hat{L}_\mu\rangle\langle L^\mu\rangle \\ &- \beta_3 \overline{G}_8\langle Q_2^3(X+\text{h.c.})\rangle + \gamma_1\kappa_1\langle \Xi + \text{h.c.}\rangle \\ &- \gamma_2 G_{27}\Big(\hat{L}_{\mu}{}_d{}^s\,\hat{L}^\mu{}_u{}^u + \frac{2}{3}\hat{L}_{\mu}{}_d{}^u\,\hat{L}^\mu{}_u{}^s\Big) + \text{h.c.}\Big]\end{aligned} \tag{3.114}$$

where the constants α_i, β_i, and γ_i are 8 new LECs that are not present in the SM χPT action.

3.4.3 Transition to the Physical Vacuum

The SM sector of the χPT Lagrangian contains a well-known tadpole term,

$$\begin{aligned}\mathcal{L}_\chi \supset& -\mathrm{i}\,\frac{f}{4}b\Big[\overline{G}_8\langle Q_2^3[M_q,\Phi]\rangle - \kappa_1\langle \xi^{\text{SM}}\Phi\rangle\Big] + \text{h.c.} \\ =& -\mathrm{i}\,\frac{f}{4}\widetilde{G}_8 b\langle [Q_2^3, M_q]\Phi\rangle + \text{h.c.}\ ,\end{aligned} \tag{3.115}$$

where

$$\widetilde{G}_8 \equiv \overline{G}_8 + \kappa_1\frac{1}{m_d}\xi^{\text{SM}}_{ds}\ . \tag{3.116}$$

Following the SM procedure [48], I eliminate this tadpole term by expanding g around its vacuum expectation value. This amounts to the chiral field redefinition

$$g \equiv W\,\widetilde{g}\,\overline{W}\ , \qquad \widetilde{\psi} \equiv -\,\mathrm{i}\ln\left[\det\widetilde{g}\right] = \psi - \mathrm{i}\langle \ln W + \ln\overline{W}\rangle\ , \tag{3.117}$$

where

$$
\begin{aligned}
W &\equiv e^{-\,\mathrm{i}(\alpha_L\lambda_7+\beta_L\lambda_6)} = \mathbb{1} + \mathcal{O}(G_F)\,, \\
\overline{W} &\equiv e^{+\,\mathrm{i}(\alpha_R\lambda_7+\beta_R\lambda_6)} = \mathbb{1} + \mathcal{O}(G_F)\,.
\end{aligned}
\tag{3.118}
$$

The parameters α_X and β_X are given as

$$
\begin{aligned}
\frac{\beta_L}{\alpha_L} = \frac{\beta_R}{\alpha_R} &= \tan\left(\arg(\widetilde{G}_8)\right)\,, \\
(\alpha_L \pm \alpha_R) &= \pm\arctan\left(|\widetilde{G}_8|\frac{m_s \pm m_d}{m_s \mp m_d}\right) \approx \pm|\widetilde{G}_8|\,,
\end{aligned}
\tag{3.119}
$$

and the corresponding the vacuum expectation value is

$$
g_0 \equiv e^{\mathrm{i}(\alpha_R-\alpha_L)\lambda_7+\mathrm{i}(\beta_R-\beta_L)\lambda_6}\,. \tag{3.120}
$$

After the field redefinition (3.117), one obtains the diagonalized quark mass matrix

$$
M_q' \equiv \overline{W} M_q(\mathbb{1} - \widetilde{G}_8 Q_2^3 - \widetilde{G}_8^\dagger Q_3^2) W\,. \tag{3.121}
$$

Its entries correspond to the experimentally determined quark masses. I also define

$$
\begin{aligned}
X' = b\,\widetilde{g}\chi'\,, \qquad \chi' &= s' + \mathrm{i}\,p' \\
&\equiv \overline{W}[\chi(\mathbb{1} - \widetilde{G}_8 Q_2^3 - \widetilde{G}_8^\dagger Q_3^2) + f^2\kappa_1\xi^{\mathrm{BSM}}]W\,,
\end{aligned}
\tag{3.122}
$$

where s' and p' denote the hermitian and antihermitian part of χ', being distinct linear combinations of the naive external currents s, p, and ξ.

Due to the chiral symmetry (3.70a), the interaction terms in the Lagrangian (3.86) are formally unaffected by the chiral rotation (3.117), provided that the external currents are redefined such that they absorb the unitary matrices W and $\overline{W}$,

$$
\widetilde{l}_\mu \equiv W^\dagger\, l_\mu\, W\,, \quad \widetilde{r}_\mu \equiv \overline{W}^\dagger\, r_\mu\, \overline{W}\,, \quad \widetilde{\vartheta} \equiv \vartheta - \mathrm{i}\langle \ln W + \ln \overline{W}\rangle\,. \tag{3.123a}
$$

The external current a does not pick up any corrections, since it is a scalar with respect to chiral rotations, and the currents ξ, $t_{\mu\nu}$, A, B, and C do not pick up any corrections at order ${}^1\!/_{M_W^2}$, since they are already of order ${}^1\!/\!M_W^2$. For the sake of notational simplicity, I hereafter suppress the explicit $\sim$ appearing in $\widetilde{g}$, $\widetilde{\psi}$, $\widetilde{l}_\mu$, $\widetilde{r}_\mu$, and $\widetilde{\vartheta}$.

To conclude this section, I summarize the individual contributions to the generalized χPT action after the field redefinition (3.117). At $\mathcal{O}(\delta^2)$, one has

$$\mathcal{L}^{(2)}_{\chi,\mathrm{s}} = \frac{f^2}{4}\langle L_\mu L^\mu\rangle + \frac{f^2}{4}\langle X' + \text{h.c.}\rangle - \frac{1}{12} f^2 m_\psi^2 (\psi - \vartheta)^2 \ , \tag{3.124a}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\chi,8} = &-\frac{f^2}{4}\Big[G_8\langle Q_2^3 \hat{L}_\mu \hat{L}^\mu\rangle + G_8^s \langle Q_2^3 \hat{L}^\mu\rangle\langle L_\mu\rangle\Big] + \text{h.c.} \ , \\ &-\frac{f^2}{4}\Big[H_{8,i}\langle Q_2^3 \hat{L}_\mu \hat{L}^\mu\rangle + H_{8,i}^s \langle Q_2^3 \hat{L}^\mu\rangle\langle L_\mu\rangle \\ &\qquad + \overline{H}_{8,i}\langle Q_2^3 (X' + \text{h.c.})\rangle\Big]\epsilon \frac{S_i}{M_W} + \text{h.c.} \ ,\end{aligned} \tag{3.124b}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\chi,27} = -\frac{f^2}{4}(G_{27} + H_{27,i}\,\epsilon\frac{S_i}{M_W})\Big[&(\hat{L}_\mu)_d^{\ s}(\hat{L}^\mu)_u^{\ u} \\ &+ \frac{2}{3}(\hat{L}_\mu)_d^{\ u}(\hat{L}^\mu)_u^{\ s}\Big] + \text{h.c.} \ ,\end{aligned} \tag{3.124c}$$

At $\mathcal{O}(\delta^3)$, one has

$$\begin{aligned}\mathcal{L}^{(3)}_{\chi,\mathrm{s}} = &\, L_2 \langle L_\mu L_\nu L^\mu L^\nu\rangle + (2L_2 + L_3)\langle L_\mu L^\mu L_\nu L^\nu\rangle \\ &+ L_5 \langle L_\mu L^\mu (X' + \text{h.c.})\rangle + L_8\langle X' X' + \text{h.c.}\rangle \\ &- \mathrm{i}\, L_9 \langle L^\mu L^\nu (L_{\mu\nu} + R_{\mu\nu})\rangle + L_{10}\langle R_{\mu\nu} L^{\mu\nu}\rangle \\ &+ f_\pi^2 \Lambda_1 D_\mu \psi D^\mu \psi - \mathrm{i}\, f_\pi^2 \Lambda_2 (\psi - \vartheta)\langle X' - \text{h.c.}\rangle \\ &+ H_0 D_\mu \vartheta D^\mu \vartheta + H_1 \langle R_{\mu\nu} R^{\mu\nu} + L_{\mu\nu} L^{\mu\nu}\rangle + H_2 \langle X'^\dagger X'\rangle \ ,\end{aligned} \tag{3.125a}$$

$$\begin{aligned}\mathcal{S}_{\mathrm{WZW}} = &\,\frac{\mathrm{i}\, N_c}{240\pi^2}\int \mathrm{d}\omega_{ijklm}\Sigma^{ijklm} \\ &- \frac{\mathrm{i}\, N_c}{48\pi^2}\int \mathrm{d}^4 x\ \epsilon^{\mu\nu\alpha\beta}\Big(W_{\mu\nu\alpha\beta}(l, r, g) - W_{\mu\nu\alpha\beta}(l, r, \mathbb{1})\Big) \ ,\end{aligned} \tag{3.125b}$$

$$\begin{aligned}\delta\mathcal{L}^{(3)}_{\chi,\mathrm{dipole}} = &\,\kappa_2 \langle \Xi L_\mu L^\mu\rangle + \kappa_3\langle \Xi X\rangle + \kappa_4 \langle \Xi X^\dagger\rangle + \mathrm{i}\,\kappa_5(\psi - \vartheta)\langle \Xi\rangle \\ &+ \lambda_1 \langle T_{\mu\nu} L^\mu L^\nu\rangle + \lambda_2 \langle T_{\mu\nu}(L^{\mu\nu} + R_{\mu\nu})\rangle + \text{h.c.} \ .\end{aligned} \tag{3.125c}$$

$$\begin{aligned}\mathcal{L}^{(3)}_{\chi,a} = a\frac{f^2}{4}\Big[&\frac{1}{2}\alpha_1 \langle L_\mu L^\mu\rangle + \alpha_2 \langle X'\rangle - \alpha_3 \frac{1}{3} m_\psi^2 (\psi - \vartheta)^2 \\ &- \beta_1 G_8 \langle Q_2^3 \hat{L}^\mu \hat{L}_\mu\rangle - \beta_2 G_8^s \langle Q_2^3 \hat{L}_\mu\rangle\langle L^\mu\rangle\end{aligned}$$

$$
\begin{aligned}
&- (\beta_3 - \alpha_2)\overline{G}_8 \langle Q_2^3 (X' + \text{h.c.})\rangle + \gamma_1 \kappa_1 \langle \Xi + \text{h.c.}\rangle \\
&- \gamma_2 G_{27} \Big(\hat{L}_{\mu d}{}^{s}\, \hat{L}^{\mu}{}_{u}{}^{u} + \frac{2}{3} \hat{L}_{\mu d}{}^{u}\, \hat{L}^{\mu}{}_{u}{}^{s} \Big) + \text{h.c.} \Big] \, . \qquad (3.125\text{d})
\end{aligned}
$$

Notice that $\mathcal{L}^{(2)}_{\chi,\mathrm{s}}$ has now absorbed all of the mass-like terms, which is a well-known result, see e.g. chapter in [93] or section 5 in [94] that also implies that $\overline{G}_8$ does not contribute to SM decays. This reshuffling has also occurred in $\mathcal{L}^{(3)}_a$, yielding the replacement

$$
\beta_3 \to \beta_3 - \alpha_2 \; . \tag{3.126}
$$

Previously, this has been shown in [52], which studied the coupling of χPT to a light Higgs boson.

3.5 Low Energy Constants

In this section, I estimate the 12 new LEC's α_i, β_i, γ_i, $\overline{g}_8$, and $h_{X,i}$ that cannot be constrained by SM observations. For the LECs $\overline{g}_8$ and h_X, which appear in the four quark Lagrangian, I follow the strategies used in [52, 53, 54]. There, the large N_c factorization rules (3.15) are combined with the well-known low-energy realizations of gauge singlet quark bilinears in order to reexpress the g_X terms of the LECs that appear in $S_{\chi,\mathrm{SM}}[\vartheta, l_\mu, r_\mu, \chi]$. For the LECs α_i, β_i, and γ_i, which appear in the scalar to gluon interaction Lagrangian, I follow the strategy used in [52], where the conformal anomaly was exploited in order to reexpress the gluon kinetic term $G_{\mu\nu}G^{\mu\nu}$ in terms of the trace of the energy momentum tensor.

3.5.1 Low energy Realizations of QCD Gauge Singlets

To estimate the new LEC's, it is necessary to find nonperturbative approximations for QCD correlation functions in the limit of small $\delta \sim {}^{p^2}\!/\!_{\Lambda_\chi^2} \sim {}^{1}\!/\!_{N_c}$. In general, the physical state of QCD can be encoded in terms of the density matrix $\hat{\rho}$. For any collection $\mathcal{O}_a$ of local, gauge invariant operators, the corresponding quantum expectation values are

$$\mathrm{tr}\{\hat{\rho}\,\mathcal{O}_a(x)\} = \frac{\delta}{\delta J_a(x)} \mathcal{Z}_{\mathrm{QCD}}[J_a] \ , \tag{3.127}$$

where

$$\mathcal{Z}_{\mathrm{QCD}}[J_a] \equiv \int \mathcal{D}\phi_{\mathrm{QCD}}\,\hat{\rho}\,O_a(x) e^{\mathrm{i}\,S[\phi_{\mathrm{QCD}}]+\mathrm{i}\int \mathrm{d}^4x J_a(x)\mathcal{O}_a(x)} \tag{3.128}$$

is the QCD generating functional, and ϕ_{QCD} symbolically denotes the QCD quark and gluon fields. J_a denotes a generic collection of external currents, such as, but not limited to, the ones collected in the interaction Lagrangian (3.38). In the presence of physical external currents, the coupling $\mathrm{i}\int \mathrm{d}^4x J_a(x)\mathcal{O}_a(x)$ already appears as part of the action and does not have to be added by hand.

By construction, the χPT generating functional is the small δ approximation of the full QCD generating functional,

$$\mathcal{Z}_{\mathrm{QCD}}[J_a] = \mathcal{Z}_\chi^{(n)}[J_a] + \dots \ , \tag{3.129}$$

where the superscript denotes that I include terms in the action of order $\mathcal{O}(\delta^n)$ or less. Hence,

$$
\begin{aligned}
\operatorname{tr}\{\hat\rho\, \mathcal{O}_a(x)\} &= \frac{\delta}{\delta J_a(x)} \mathcal{Z}_\chi^{(n)}[J_a] + \mathcal{O}(\delta^{(n+1-k)}) \qquad (3.130)\\
&= \operatorname{tr}\left\{\hat\rho\, \frac{\delta S_\chi^{(n)}}{\delta J_a}\right\} + \mathcal{O}(\delta^{n+1-k})\ ,
\end{aligned}
$$

where the external current is taken to scale as $J_a(x) \sim \delta^k$. Since this identity has to hold for any physical choice of $\hat\rho$, one can simply identify

$$
\mathcal{O}_a(x) = \frac{\delta S_\chi^{(n)}}{\delta J_a} + \mathcal{O}(\delta^{n+1-k}) \qquad (3.131)
$$

This way, it is possible to obtain well defined approximations for the various QCD gauge singlet operators appearing in the interaction Lagrangian (3.38), which I use for estimating the LEC's for the χPT Lagrangian.

The low energy realizations for the gauge singlets associated with the currents l_μ, r_μ, ϑ, and χ are well known. Using the SM action $S_{\chi,\mathrm{SM}}[\vartheta, l_\mu, r_\mu, \chi]$, one obtains

$$
\begin{aligned}
q^{\dagger a}\overline{\sigma}^\mu q_b &= \frac{1}{2} f^2 L^{\mu\, a}_{\ b} + \mathcal{O}(\delta^{5/2})\,, \qquad (3.132a)\\
\overline{q}^\dagger_{\dot b}\overline{\sigma}^\mu \overline{q}^{\dot a} &= \frac{1}{2} f^2 (g^\dagger L^\mu\, g)^{\ \dot a}_{\dot b} + \mathcal{O}(\delta^{5/2})\,,\\
\frac{1}{16\pi^2} G_{\mu\nu}\widetilde{G}^{\mu\nu} &= \frac{1}{6} f^2 m_\psi^2 \psi + \mathcal{O}(\delta^2)\,,
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{q}^{\dot a} q_b = (q^{\dagger b}\overline{q}^\dagger_{\dot a})^\dagger = -b\big[&\frac{1}{4} f^2 - \mathrm{i}\, f^2 \Lambda_2(\psi - \vartheta) \qquad (3.132b)\\
&+ L_5 L_\mu L^\mu + 2L_8 X + H_2 X^\dagger\big]_b^{\ x} g_x^{\ \dot a}\\
&+ \mathcal{O}(\delta^3)\ .
\end{aligned}
$$

For the gauge singlets associated with the currents a, ξ, and $t_{\mu\nu}$, I obtain

$$
\begin{aligned}
\overline{q}^{\dot a} \sigma^{\mu\nu} q_b &= 0 + \mathcal{O}(\delta^2)\,,\\
\overline{q}^{\dot a} \overline{\sigma}^{\mu\nu} \lambda_x G^x_{\mu\nu} q_b &= -\kappa_1 \frac{1}{4} f^2 b\, g_x^{\ \dot a} + \mathcal{O}(\delta^2)\,,
\end{aligned} \qquad (3.133a)
$$

and

$$\frac{1}{16\pi^2}G_{\mu\nu}G^{\mu\nu} = \frac{f^2}{4}\Big[\frac{1}{2}\alpha_1\langle L_\mu L^\mu\rangle - \beta_1 G_8\langle Q_2^3\hat{L}^\mu\hat{L}_\mu\rangle - \beta_2 G_8^s\langle Q_2^3\hat{L}_\mu\rangle\langle L^\mu\rangle + \gamma_1\kappa_1\langle\Xi + \text{h.c.}\rangle - \gamma_2 G_{27}\Big(\hat{L}_{\mu d}{}^{s}\,\hat{L}^{\mu}{}_{u}{}^{u} + \frac{2}{3}\hat{L}_{\mu d}{}^{u}\,\hat{L}^{\mu}{}_{u}{}^{s}\Big) + \alpha_2\langle X\rangle - \beta_3\overline{G}_8\langle Q_2^3(X + \text{h.c.})\rangle - \alpha_3\frac{1}{3}m_\psi^2(\psi-\vartheta)^2 + \text{h.c.} + \mathcal{O}(\delta^3)\Big] \ . \tag{3.133b}$$

For the χPT operators with A, B, and C, I only keep track of the octet and 27-plet contributions, so I obtain low-energy realizations for the corresponding octet and 27-plet four quark operators,

$$5\,O_{\mathcal{S},27}{}_{d}{}^{s}{}_{u}{}^{u} = \frac{1}{4}f^4\,s_{27}\Big[\hat{L}_{\mu d}{}^{s}\,\hat{L}^{\mu}{}_{u}{}^{u} + \frac{2}{3}\hat{L}_{\mu d}{}^{u}\,\hat{L}^{\mu}{}_{u}{}^{s}\Big] + \mathcal{O}(\delta^3) \ , \tag{3.134a}$$

$$O_{X}{}_{d}{}^{s} = \frac{1}{4}f^4\Big[k_8(\hat{L}_\mu\hat{L}^\mu)_d{}^s + k_8^s\langle L_\mu\rangle\hat{L}^{\mu}{}_{d}{}^{s} + \overline{k}_8(X + \text{h.c.})_d{}^s\Big] + \mathcal{O}(\delta^3) \ , \tag{3.134b}$$

where now $O_X \in \{O_{\mathcal{S}_8}, O_{\mathcal{A}_8}, O_{B_8}, O_{C_8}\}$, and $k_X \in \{s_X, a_X, b_X, c_X\}$.

Using the low-energy realizations (3.132a)-(3.134b), one also finds the seemingly trivial approximations

$$-\bar{q}\chi q + \text{h.c.} = \mathcal{L}^{(2)}_{\chi,\text{mass}} + \mathcal{O}(\delta^3) \ , \quad \mathcal{L}_{\text{dipole}} = \mathcal{L}^{(2)}_{\chi,\text{dipole}} + \mathcal{O}(\delta^3) \ , \quad \mathcal{L}_{\bar{q}q\bar{q}q} = \mathcal{L}^{(2)}_{\chi,\bar{q}q\bar{q}q} + \mathcal{O}(\delta^3) \ , \quad \mathcal{L}_{\bar{l}l\bar{q}q} = \mathcal{L}^{(2)}_{\chi,\bar{l}l\bar{q}q} + \mathcal{O}(\delta^3) \ , \tag{3.135}$$

where now

$$\mathcal{L}^{(2)}_{\chi,\text{mass}} \equiv \frac{f^2}{4}\langle X + \text{h.c.}\rangle \ , \tag{3.136a}$$

$$\mathcal{L}^{(2)}_{\chi,\text{dipole}} \equiv \kappa_1\frac{f^4}{4}\langle\Xi + \text{h.c.}\rangle \ , \tag{3.136b}$$

$$\mathcal{L}^{(2)}_{\chi,\bar{l}l\bar{q}q} \equiv +\frac{f^2}{2}\frac{G_F}{\sqrt{2}}4J^\mu_{\text{cc}+}\big\langle\big[V_{ud}Q_2^1 + V_{us}Q_3^1\big]L^{\text{em}}_\mu\big\rangle + \text{h.c.} \ , \tag{3.136c}$$

so that the mass term includes only the quark mass contribution to the PNGB masses.

Finally, I also require the low-energy realization of the trace of the improved energy momentum tensor in order to implement the matching strategy of [52]. In contrast to the other low-energy realizations, it can be computed directly in χPT. One has

$$\Theta^\mu_\mu = 2\mathcal{L}^{(2)}_{\chi,\mathrm{kin}} - 4\mathcal{L}^{(2)}_\chi + \mathcal{O}(\delta^3) \ , \tag{3.137a}$$

where the "kinetic" Lagrangian is

$$\begin{aligned}
\mathcal{L}^{(2)}_{\chi,\mathrm{kin}} &\equiv g^{\mu\nu}\frac{\delta\mathcal{L}^{(2)}_\chi}{\delta g^{\mu\nu}} \qquad\qquad (3.137\mathrm{b})\\
&= \frac{1}{4}f^2\Big[\frac{1}{2}\langle L^\mu L_\mu\rangle - \frac{G_F}{\sqrt{2}}4J^\mu_{\mathrm{cc}+}\big\langle\big[V_{ud}Q^1_2 + V_{us}Q^1_3\big]L_\mu\big\rangle \\
&\qquad - G_8\langle Q^3_2\hat{L}_\mu\hat{L}\mu\rangle - G^s_8\langle Q^3_2\hat{L}^\mu\rangle\langle\hat{L}_\mu\rangle \\
&\qquad - G_{27}\Big((\hat{L}_\mu)_d{}^s(\hat{L}^\mu)_u{}^u + \frac{2}{3}(\hat{L}_\mu)_d{}^u(\hat{L}^\mu)_u{}^s\Big)\Big] \\
&\qquad + \mathrm{h.c.} + \mathcal{O}(\delta^3) \ .
\end{aligned}$$

The term involving the leptonic charged current $J^\mu_{\mathrm{cc}+} = J^a_{\mathrm{cc}+}e^\mu_a$ has to be varied with respect to the vierbein e^μ_a rather than the metric tensor, which gives rise to a relative prefactor of $1/2$. After reabsorbing this interaction into the kinetic term $\langle L_\mu L^\mu\rangle$, one has a leftover term with a prefactor of $-1/2$.

Using the kinetic term (3.137b), the trace of the energy momentum tensor is

$$\begin{aligned}
\Theta^\mu_\mu = -\frac{1}{4}f^2\Big[&\langle L^\mu L_\mu\rangle + 2\frac{G_F}{\sqrt{2}}4J^\mu_{\mathrm{cc}+}\big\langle\big[V_{ud}Q^1_2 + V_{us}Q^1_3\big]L_\mu\big\rangle \qquad (3.138)\\
&- 2G_8\langle Q^3_2\hat{L}_\mu\hat{L}^\mu\rangle + 4\langle X\rangle - \frac{2}{3}f^2m^2_\psi(\psi-\vartheta)^2 \\
&- 2G^s_8\langle Q^3_2\hat{L}^\mu\rangle\langle\hat{L}_\mu\rangle - 4\overline{G}_8\langle Q^3_2(X+\mathrm{h.c.})\rangle \\
&+ 4\kappa_1\langle\Xi\rangle - 2G_{27}\Big((\hat{L}_\mu)_d{}^s(\hat{L}^\mu)_u{}^u + \frac{2}{3}(\hat{L}_\mu)_d{}^u(\hat{L}^\mu)_u{}^s\Big)\Big] \\
+ \mathrm{h.c.} \ .&
\end{aligned}$$

3.5.2 LEC's for Scalar to Gluon Coupling

To estimate the LECs α_i, β_i, and γ_i, I combine the low energy realizations (3.135) with the modified trace relation (3.9). Solving for the scalar to gluon coupling, the trace relation becomes

$$-\frac{\beta_0}{16\pi}G_{\mu\nu}G^{\mu\nu} = \Theta_\mu^{\ \mu} + (-\overline{q}\chi q + \text{h.c.}) \\ - \mathcal{L}_{\text{dipole}} + \mathcal{L}_{\bar{l}l\bar{q}q} - 2\mathcal{L}_{\bar{q}q\bar{q}q} \ . \tag{3.139}$$

The low energy realization (3.133b) of the gluon kinetic term on the lefthand side of this equation explicitly contains the α_i, β_i, and γ_i coefficients. Using the low energy realizations (3.135) for the operators on the righthand side, one obtains

$$\begin{aligned}\frac{\beta_0}{16\pi}G_{\mu\nu}G^{\mu\nu} \equiv \frac{f^2}{4}\Big[&\langle L^\mu L_\mu\rangle + 3\langle X\rangle - \frac{2}{3}m_\psi^2(\psi-\vartheta)^2 \\ &- 4G_8\langle Q_2^3\hat{L}_\mu\hat{L}^\mu\rangle - 4G_8^s\langle Q_2^3\hat{L}^\mu\rangle\langle\hat{L}_\mu\rangle \\ &+ 5\kappa_1\langle\Xi\rangle - 5\overline{G}_8\langle Q_2^3(X+\text{h.c.})\rangle \\ &- 4G_{27}\big((\hat{L}_\mu)_d^{\ s}(\hat{L}^\mu)_u^{\ u} + \frac{2}{3}(\hat{L}_\mu)_d^{\ u}(\hat{L}^\mu)_u^{\ s}\big)\Big] \\ + \text{h.c.} \ .&\end{aligned} \tag{3.140}$$

Comparing this expression with the realization (3.133b), one finds

$$\begin{aligned}&\beta_0\,\alpha_1 = 2 + \mathcal{O}(\delta)\,, \quad \beta_0\,\alpha_2 = 3 + \mathcal{O}(\delta)\,, \quad \beta_0\,\alpha_3 = 4 + \mathcal{O}(\delta)\,, \\ &\beta_0\,\beta_1 = 4 + \mathcal{O}(\delta)\,, \quad \beta_0\,\beta_2 = 4 + \mathcal{O}(\delta)\,, \quad \beta_0\,\beta_3 = 5 + \mathcal{O}(\delta)\,, \\ &\beta_0\,\gamma_1 = 5 + \mathcal{O}(\delta)\,, \quad \beta_0\,\gamma_2 = 4 + \mathcal{O}(\delta)\,.\end{aligned} \tag{3.141}$$

3.5.3 LEC's for the octet and 27-plet Terms

To estimate $\overline{g}_8$ and the $h_{X,i}$, I proceed in four steps:

- Following the strategy used in [52, 53, 54], I fix the constants a_X, s_X, b_X, and c_X by combining the low energy realizations (3.132a) – (3.133b) of the quark bilinears with the large N_c factorization rules (3.15), in order to obtain alternative low energy realizations for the four quark operators. This yields expressions for g_X and h_X as linear combinations of the $c_i(\mu = M_K)$ and $\widetilde{c}_i(\mu = M_K)$ Wilson coefficients of the four quark operators O_i and $\widetilde{O}_i$.
- Second, I neglect the penguin operators O_3 – O_5 and $\widetilde{O}_3$ – $\widetilde{O}_5$. In [54], this has been done to estimate the SM LEC g_8^s, and it is

expected to be reasonable for an order of magnitude estimation of the coupling constants, since these penguin operators are generated at loop level, and therefore suppressed by factors of $1/4\pi$. In principle, the final penguin operators O_6 and $\widetilde{O}_6$ are also suppressed in this way, but they are enhanced by chiral effects and therefore contribute at leading order. This way, one obtains estimates for the Wilson coefficients $c_1(\mu = M_K)$, $c_2(\mu = M_K)$, $c_6(\mu = M_K)$ in terms of the SM LECs g_8, g_8^s, and g_{27}, f, b, L_5, and H_2.

- Third, I estimate H_2 by following the strategy used in [52], where the combination $(2L_5 + H_2)$ has been related to the possibility of flavour symmetry violating chiral quark condensates. Since H_2 cannot be measured directly, the discussion in [52] had to rely on an old computation using QCD sum rules [95], but lattice computations [96] now produce more accurate estimates for the values of the chiral quark condensates.
- Finally, I use the fact that the ratios $x_{i,j}(\mu = M_K) \approx x_{i,j}(\mu = m_c)$ are expected to be approximately renormalization scale invariant in order to estimate the $h_{X,i}$ in terms of the Wilson coefficients $c_i(\mu = M_K)$.

Moving on to the actual computation, I start by combining the large N_c factorization rules (3.15) with the low energy realizations (3.132a) – (3.133b). This gives

$$(O_{\mathcal{S}_{27}})_d{}^s{}_u{}^u = \frac{1}{4} f^4 \Big[\frac{3}{5}(\hat{L}^\mu)_d{}^s(\hat{L}_\mu)_u{}^u + \frac{2}{5}(\hat{L}^\mu)_d{}^u(\hat{L}_\mu)_u{}^s\Big] + \mathcal{O}(\delta^3) \ , \tag{3.142a}$$

$$(O_{\mathcal{S}_8})_d{}^s = \frac{1}{4} f^4 \, \frac{2}{5}\big[(L^\mu L_\mu)_d{}^s + (L^\mu)_d{}^s\langle L_\mu\rangle\big] + \mathcal{O}(\delta^3) \ , \tag{3.142b}$$

$$(O_{\mathcal{A}_8})_d{}^s = \frac{1}{4} f^4 \, 2\big[(L^\mu L_\mu)_d{}^s - (L^\mu)_d{}^s\langle L_\mu\rangle\big] + \mathcal{O}(\delta^3) \ , \tag{3.142c}$$

$$(O_{B_8})_d{}^s = \frac{1}{4} f^4 (L^\mu)_d{}^s\langle L_\mu\rangle + \mathcal{O}(\delta^3) \ , \tag{3.142d}$$

$$\begin{aligned}(O_{C_8})_d{}^s = \frac{1}{4} f^2 b^2 \big[&2L_5(L_\mu L^\mu)_d{}^s \\ &+ (2L_8 + H_2)(X' + \text{h.c.})_d{}^s\big] + \mathcal{O}(\delta^3) \ .\end{aligned} \tag{3.142e}$$

In the last replacement, it is necessary to include order δ^2 contributions for the low energy realizations of the quark bilinears, since the order

δ contributions vanish. In principle, one also has to consider finite N_c corrections to the factorization rule (3.15). However, if they are supposed to contribute at order δ^2, these corrections cannot involve any of the external currents or derivatives, which would be associated with further factors δ. Hence, they do not contribute to portal interactions at leading order.

Comparing the above expressions (3.142a) – (3.142e) with the direct low energy realizations (3.132a) – (3.133b) obtained from the weak chiral Lagrangian, one finds

$$b_8 = \overline{s}_8 = \overline{b}_8 = \overline{a}_8 = 0 + O(\delta) \ , \tag{3.143a}$$

$$\begin{aligned} a_8 &= 2 + O(\delta) \ , \quad & a_8^s &= -\frac{2}{3} + O(\delta) \ , \quad & b_8^s &= 1 + O(\delta) \ , \\ s_8 &= \frac{2}{5} + O(\delta) \ , \quad & s_8^s &= \frac{2}{3} + O(\delta) \ , \quad & s_{27} &= 3 + O(\delta) \ , \end{aligned} \tag{3.143b}$$

$$\begin{aligned} c_8 &= 2\frac{b^2}{f^2} L_5 + O(\delta) \ , \qquad & \overline{c}_8 &= \frac{b^2}{f^2}(2L_8 + H_2) + O(\delta) \ , \\ c_8^s &= \frac{4}{3}\frac{b^2}{f^2} L_5 + O(\delta) \ . \end{aligned} \tag{3.143c}$$

The appearance of the $b^2/f^2 \sim 10^3$ prefactor in front of the c_x coefficients is usually referred to as "chiral enhancement" because the density-density structure $(\overline{q}^\dagger q^\dagger)(\overline{q}q)$ of the corresponding four quark operator distinguishes it from the current-current structures $(q^\dagger\overline{\sigma}^\mu q)(q^\dagger\overline{\sigma}_\mu q)$ and $(q^\dagger\overline{\sigma}^\mu q)(\overline{q}^\dagger\overline{\sigma}_\mu\overline{q})$ of the other four-quark operators. This chiral enhancement is what compensates for the loop suppression factor of the penguin diagrams generating O_6 and $\widetilde{O}_6$, causing them to be important at leading order. The other penguin operators $O_3 - O_5$ and $\widetilde{O}_3 - \widetilde{O}_5$ are not enhanced. Neglecting them, one obtains

$$\begin{aligned} g_8 &\approx \frac{3}{5}c_1 - \frac{2}{5}c_2 + c_8 c_6 \ , \qquad & g_8^s &\approx -\frac{2}{5}c_1 + \frac{3}{5}c_2 \ , \\ \overline{g}_8 &= \overline{c}_8 c_6 \ , & g_{27} &\approx \frac{3}{5}(c_1 + c_2) \ , \\ h_8 &\approx \frac{3}{5}x_1 c_1 - \frac{2}{5}x_2 c_2 + c_8 x_6 c_6 \ , & h_8^s &\approx -\frac{2}{5}x_1 c_1 + \frac{3}{5}x_2 c_2 \ , \\ \overline{h}_8 &= \overline{c}_8 x_6 c_6 \ , & h_{27} &\approx \frac{3}{5}(x_1 c_1 + x_2 c_2) \ . \end{aligned} \tag{3.144}$$

Inverting this system of equations, one obtains

$$c_1(\mu = m_K) \approx g_{27} - g_8^s \approx -0.25 g_8 \ , \tag{3.145a}$$

$$c_2(\mu = m_K) \approx \frac{2}{3} g_{27} + g_8^s \approx +0.34 g_8 \ , \tag{3.145b}$$

and

$$\begin{aligned} \overline{g}_8 = \overline{c}_8 c_6(\mu = m_K) &\approx \frac{2L_8 + H_2}{2L_5}(g_8^s + g_8 - \frac{1}{3} g_{27}) \\ &\approx \frac{2L_8 + H_2}{2L_5} \cdot 1.25 g_8 \ . \end{aligned} \tag{3.145c}$$

The value of prefactor $2L_8 + H_2$ cannot be fitted to direct experimental observations, since H_2 does not contribute to perturbative computations. However, it is connected to flavour symmetry violating light quark condensates. Using the low-energy realization (3.133b), one obtains

$$\begin{aligned} \langle 0 | \overline{q}^{\dagger \dot{a}} q_b | 0 \rangle + \text{h.c.} = &-b\Big(\frac{f^2}{4} \delta_b{}^{\dot{a}} + (2L_8 + H_2)(b\chi)_b{}^{\dot{a}}\Big) \\ &+ \text{h.c.} + \mathcal{O}(\delta^2) \ , \end{aligned} \tag{3.146}$$

so that in the SM

$$\begin{aligned} x \equiv \frac{\langle 0 | \overline{s}s + \text{h.c.} | 0 \rangle}{\langle 0 | \overline{u}u + \text{h.c.} | 0 \rangle} - 1 &= \frac{8}{F_\pi^2}(M_{K^\pm}^2 - M_{\pi^\pm}^2)(2L_8 + H_2) \cdot (1 + O(\delta)) \\ &\approx 204\,(2L_8 + H_2) \ . \end{aligned} \tag{3.147}$$

An old computation[95] using QCD sum rules found the value $x = -0.2 \pm 0.1$. In recent years, it has become possible to estimate the value of x using the results of lattice computations, giving [96]

$$x \approx 0.08 \pm 0.16 \ . \tag{3.148}$$

Using the best fit value of $x \approx 0.08$, one finds $\overline{g}_8 \approx 4 \cdot 10^{-4}$. However, due to the large uncertainties, the lattice result is perfectly consistent with $\overline{g}_8 = \overline{h}_8 = 0$. That being said, the estimate in this section crucially depends on the large N_c factorization rules, so that formally subleading contributions can still give a finite contribution to $\overline{g}_8$ even if $x = 0$.

Chapter 4

Lepton Number Violation via Sterile Neutrino Decoherence

4.1 Lepton Number Violation at Colliders

The existence of small but finite masses for the lefthanded SM neutrinos is one of the most undeniable pieces of evidence for new physics. Type-I seesaw models are bottom-up extensions of the SM that seek to explain these masses by supposing the existence of $n \geq 2$ gauge-singlet sterile neutrinos $\overline{\nu}_i$ that mix with the light SM neutrinos ν_a [97, 98, 99, 100, 101, 102]. In addition to the light neutrino masses, this mixing also gives rise to heavy neutrino mass eigenstates $\overline{\upnu}_i$ with masses M_i that are not constrained by SM observations, and a compelling feature of type-I seesaw models is that these heavy neutrinos could help explain a number of open problems in cosmology. For instance, they could serve as potential dark matter candidates [103, 104, 105], or help explain the origin of the primordial baryon asymmetry via the mechanism of "leptogenesis" [106, 107, 108].

In this chapter, I consider the prospect of observing lepton number violation (LNV) in heavy neutrino decays at colliders. In principle, LNV is a generic feature of many SM extensions, and e.g. the ATLAS [109], CMS [110] and LHCb [111] collaborations have all published searches for evidence of sterile neutrinos in same-sign dilepton events at the LHC. Within the context of type-I seesaw models, there has been an ongoing debate as to whether observable LNV at colliders is consistent with loop stability of the small SM neutrino masses [112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128], and during my PhD I investigated aspects of this debate.

In general, the parameter space of type-I seesaw models is characterized by the heavy neutrino masses M_i and by the angles Θ_{ai} that quantify the seesaw mixing between the ν_a and $\overline{\nu}_i$ neutrinos. The difficulties for observing LNV at colliders arise from interplay between the SM neutrino mass matrix m_ν and the sterile neutrino production cross section σ, both of which scale quadratically with the mixing angles,

$$\sigma \propto \sum_a |\Theta_{ai}|^2 \ , \qquad\qquad (m_\nu)_{ab} = -\sum_i \Theta_{ai}\Theta_{bi}M_i \ . \tag{4.1}$$

For collider-accessible masses $M_i \lesssim 1\,\mathrm{TeV}$, a large sterile neutrino production cross section σ can only be consistent with the observed small SM neutrino masses if there are large cancellations between the individual terms in (4.1). In turn, these large cancellations can jeopardize loop stability of the SM neutrino masses, causing them to be fine-tuned in the 't Hooft sense [129, 130]. To protect loop stability of the SM neutrino masses, one has to impose an approximate symmetry that becomes exact in the limit of vanishing neutrino masses. It can be shown that the only suitable candidate for such a symmetry is the so-called "pseudo-Dirac symmetry", which conserves a generalized lepton number $\widetilde{L}$ [131, 130, 132].

In [130], it was pointed out that this approximate $\widetilde{L}$ conservation can suppress LNV in heavy neutrino decays, which seems to force an unhappy choice between one of three options: 1) the neutrino masses M_i are much heavier than $1\,\mathrm{TeV}$, 2) LNV is parametrically suppressed by approximate $\widetilde{L}$ conservation, or 3) the SM neutrino masses are fine-tuned. However, the prospective suppression of LNV decays is the result of destructive interference within a coherently produced superposition heavy neutrino mass eigenstates. Many authors have drawn attention to the fact that these superpositions oscillate on detector length-scales [133, 114, 116, 115, 117, 119, 120, 121, 122, 123]. These oscillations can effective destroy coherence, making destructive interference impossible. Thus, the debate on LNV at colliders turns on whether or not coherently produced superpositions of heavy neutrinos do in fact decohere before they decay.

The results presented in this chapter have been published in [2]. Together with my collaborators, I used two distinct notions of fine-tuning to derive bounds on the suppression of LNV in sterile neutrino decays. For 't Hooft natural type-I seesaw mode, I used the loop-stability requirement for the SM neutrino masses to derive a lower bound on the relative suppression of LNV in sterile neutrino decays. For the more restrictive class type-I seesaw models without large, accidental cancellations, I derived an upper bound on the relative suppression of LNV in sterile neutrino decays. Translating these bounds into conditions for the heavy neutrino masses M_i and mixing angles Θ_{ai}, I have split the available parameter space for collider-accessible type-I seesaw models into three distinct regions:

- At sufficiently small mixing angles, LNV in heavy neutrino decays is unsuppressed for type-I seesaw models without large, accidental cancellations.
- For intermediate mixing angles, both suppressed and unsuppressed LNV is possible for parameter choices that obey both notions of naturalness.
- For sufficiently large mixing angles, LNV in heavy neutrino decays is suppressed for type-I seesaw models without 't Hooft fine tuning.

As a specific example for these generic considerations, I consider the minimal case of a type-I seesaw model with $n = 2$ sterile neutrinos that form a single pseudo-Dirac pair. In this case, the position of the boundaries between these three areas in parameter space is found to depend only on $2\overline{M} \equiv M_1 + M_2$ and $U^2 \equiv \sum_{i,a} |\Theta_{ai}|^2$.

The remainder of this chapter is organized as follows: In section 4.2, I quickly review the type-I seesaw model. In section 4.3, I discuss the sufficient conditions for sterile neutrino decoherence, and derive general criteria for splitting the parameter space, which I then apply to the discussion for $n = 2$ sterile neutrinos. In section 4.4, I study the phenomenological implications for sterile neutrino searches at colliders.

4.2 Introduction to Type-I Seesaw Models

In this section, I give a short introduction to type-I seesaw models with $n \geq 2$ sterile neutrinos $\overline{\nu}_i$, focusing on the limit with pseudo-Dirac symmetry and the loop stability of the light neutrino masses. In two-component notation, the full Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{seesaw}} \ , \tag{4.2}$$

where

$$\mathcal{L}_{\text{seesaw}} = \overline{\nu}_i^\dagger \, \mathrm{i} \, \bar{\partial\!\!\!/} \, \overline{\nu}_i - \frac{1}{2}(M_{ij}\overline{\nu}_i\overline{\nu}_j - F^*_{ai}\overline{\nu}_i l_a \, \widetilde{\phi}^\dagger + \mathrm{h.\,c.}) \ , \tag{4.3}$$

$M_{ij} = M_{ji}$ denotes the sterile neutrino Majorana mass matrix, and F_{ai} is the coupling strength of the sterile neutrino Yukawa interactions. To study LNV at colliders, one also has to account for the interactions of the SM neutrinos ν_a. Working in a basis with diagonalized charged lepton Yukawa and gauge interactions, the relevant part of the SM Lagrangian is

$$\mathcal{L}_{\text{SM},\nu} \equiv \nu_a^\dagger \, \mathrm{i} \, \bar{\partial\!\!\!/} \nu_a - \left(\frac{g}{2c_W} Z_\mu \nu_a^\dagger \overline{\sigma}^\mu \nu_a + \frac{g}{\sqrt{2}} W_\mu^+ \nu_a^\dagger \overline{\sigma}^\mu e_a + \mathrm{h.\,c.} \right) \ . \tag{4.4}$$

After EWSB, one obtains the mass term

$$\mathcal{L}_{\text{seesaw}} \supset \mathcal{L}_{\mathcal{M}} \equiv -\frac{1}{2} \begin{pmatrix} \nu_a \ , & \overline{\nu}_i \end{pmatrix} \begin{pmatrix} 0 & vF^*_{aj} \\ vF^*_{bi} & M_{ij} \end{pmatrix} \begin{pmatrix} \nu_b \\ \overline{\nu}_j \end{pmatrix} + \mathrm{h.\,c.} \ . \tag{4.5}$$

The mass matrix is complex and symmetric, so it can be diagonalized by means of a Takagi factorization,

$$\mathcal{M} \equiv \begin{pmatrix} 0 & vF^* \\ vF^\dagger & M \end{pmatrix} = U^* \begin{pmatrix} D_m & 0 \\ 0 & D_M \end{pmatrix} U^\dagger \ , \tag{4.6}$$

where U is some unitary matrix, and the real, diagonal matrices $D_m \equiv \mathrm{diag}(\hat{m}_1, \hat{m}_2, \hat{m}_3)$, $D_M \equiv (\hat{M}_1, \hat{M}_2, \dots)$ encode the tree-level predictions for the physical masses for the light and heavy neutrino mass eigenstates n and N,

$$\begin{pmatrix} \mathsf{v} \\ \overline{\mathsf{v}} \end{pmatrix} \equiv U^\dagger \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \ , \qquad \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} = U \begin{pmatrix} \mathsf{v} \\ \overline{\mathsf{v}} \end{pmatrix} \ . \tag{4.7}$$

Assuming that the mixing between the ν_a and $\overline{\nu}_i$ is small, the matrix U can be cast as

$$U = \begin{pmatrix} \mathbb{1} - \frac{1}{2}\theta\theta^\dagger & \theta \\ -\theta^\dagger & 1 - \frac{1}{2}\theta^\dagger\theta \end{pmatrix} \begin{pmatrix} U_\nu & 0 \\ 0 & U_N \end{pmatrix} + \mathcal{O}(\theta^3) \ , \tag{4.8}$$

where $vF^\dagger = M\theta^\dagger$. Furthermore, the entries of D_m and D_M are given by the square roots of the eigenvalues of $\mathcal{M}\mathcal{M}^\dagger$. With $\Theta \equiv U_\nu^\dagger \theta U_N$ and $\hat{M} = U_N^T M U_N$, one has

$$D_m = \Theta^* D_M \Theta^\dagger + \mathcal{O}(\Theta^4) \ , \tag{4.9a}$$

$$D_M = \hat{M} + \frac{1}{2}\hat{M}\Theta^\dagger\Theta + \frac{1}{2}\Theta^T\Theta^*\hat{M} + \mathcal{O}(\Theta^3) \ . \tag{4.9b}$$

After the field redefinition (4.7), the full neutrino sector becomes

$$\mathcal{L}_{\mathrm{SM},\nu} + \mathcal{L}_{\mathrm{seesaw}} = \mathcal{L}_{\mathrm{free}} + \mathcal{L}_{\mathrm{int}} \ , \tag{4.10}$$

where

$$\mathcal{L}_{\mathrm{free}} \equiv \mathsf{v}^\dagger \, \mathrm{i}\,\bar{\partial\!\!\!/} \mathsf{v} + \overline{\mathsf{v}}^\dagger \, \mathrm{i}\,\bar{\partial\!\!\!/} \overline{\mathsf{v}} - \frac{1}{2}(\mathsf{v} D_m \mathsf{v} + \overline{\mathsf{v}} D_M \overline{\mathsf{v}} + \mathrm{h.\,c.}) \tag{4.11a}$$

$$\begin{aligned} \mathcal{L}_{\mathrm{int}} \equiv & -\frac{g}{2c_W} Z_\mu \mathsf{v}^\dagger \overline{\sigma}^\mu (\mathsf{v} + \Theta\overline{\mathsf{v}}) - \frac{h}{v}\mathsf{v}\Theta D_M \overline{\mathsf{v}} \\ & - \frac{g}{\sqrt{2}} W_\mu^- e_a^\dagger \overline{\sigma}^\mu U_\nu \, (\mathsf{v} + \Theta\overline{\mathsf{v}}) + \mathrm{h.\,c.} + \mathcal{O}(\Theta^2) \ . \end{aligned} \tag{4.11b}$$

Neglecting loop corrections and higher order contributions in the seesaw expansion in Θ, U_ν has to be identical to the SM PNMS matrix. At this level of accuracy, the only relevant free parameters are encoded in Θ and D_M. Since D_m is real, positive, and diagonal, not all entries of Θ are independent of each other. It is convenient to define the short-hands

$$U_i^2 \equiv \sum_a |\Theta_{ai}|^2 \ , \qquad U_a^2 \equiv \sum_i |\Theta_{ai}|^2 \ , \tag{4.12}$$

so that $U^2 = \sum_i U_i^2 = \sum_a U_a^2$. In this notation, the decay widths Γ_i of the N_i eigenstates can be estimated as [134, 135]

$$\Gamma_i \approx U_i^2 M_i \begin{cases} 0.11 \, G_F M_i^2 & \text{for } M_i \gg M_W \\ 0.04 \, G_F^2 M_i^4 & \text{for } M_i \ll M_W \end{cases} \ , \tag{4.13}$$

with an equal probability of decaying into leptons and antileptons at leading order. Notice that the all-important relative suppression of LNV decays can only occur for decays of coherent superpositions of the $\overline{\nu}_i$ eigenstates. Generically, SM interactions are expected to produce such superpositions, and one the central question investigated in this chapter is for which mixing angles U_i^2 one should expect a coherently produced superposition of sterile neutrinos $\overline{\nu}_i$ to decohere before decaying.

The pseudo-Dirac symmetry is obtained if all of the sterile neutrinos coupling to the standard model can be grouped into $k \leq {}^n\!/2$ pairs of pseudo-Dirac Fermions $(\overline{\nu}_i, \overline{\nu}_{k+i}^\dagger)$ [131, 130]. In this case, D_M and Θ can be cast as [132]

$$D_M = \begin{pmatrix} D_1 & 0 & 0 \\ 0 & D_1 & 0 \\ 0 & 0 & D_2 \end{pmatrix} , \qquad \Theta = \begin{pmatrix} \alpha \ , & \mathrm{i}\,\alpha \ , & 0 \end{pmatrix} , \tag{4.14}$$

where the D_i are real, positive, and diagonal matrices and α is a general complex matrix. As they should, the light neutrino masses vanish in this limit, and any pseudo-Dirac symmetry in nature has to be approximate. Hence, the matrices D_m and Θ can be cast as

$$\Theta = \begin{pmatrix} \alpha - \epsilon_1 \ , & \mathrm{i}(\alpha + \epsilon_1) \ , & \epsilon_2 \end{pmatrix} \tag{4.15a}$$

and

$$D_M = \begin{pmatrix} D_1 - \frac{1}{2}\Delta M & 0 & 0 \\ 0 & D_1 + \frac{1}{2}\Delta M & 0 \\ 0 & 0 & D_2 \end{pmatrix} , \tag{4.15b}$$

where ΔM and the ϵ_i parametrize the deviation from the pseudo-Dirac limit. As they should, the light neutrino masses vanish for $(\Delta M, \epsilon_i) \to 0$. Close to the pseudo-Dirac limit, loop corrections to the small neutrino masses are suppressed by the smallness of the entries of ΔM and ϵ_i. However, if the mixing angles are sufficiently large, their size can compensate even for the smallness of ΔM and ϵ. At 1-loop, the physical light neutrino mass matrix $m_\nu = D_m + \delta m_\nu^{\text{1-loop}} + \dots$ picks up the contribution [136],

$$\delta m_\nu^{\text{1-loop}} = \Theta^* \delta M \Theta^\dagger , \qquad \delta M = \frac{2}{(4\pi v)^2} D_M l(D_M^2) , \tag{4.16}$$

which does not have to be diagonal The loop function l is defined such that it acts on each entry of the diagonal matrix D_M^2 separately. One has

$$l(M_i^2) \equiv \frac{M_i^2}{2}\left(3\frac{\ln(M_i^2/M_Z^2)}{M_i^2/M_Z^2 - 1} + \frac{\ln(M_i^2/M_H^2)}{M_i^2/M_H^2 - 1}\right) \ , \tag{4.17}$$

where M_H and M_Z are the physical masses of the H and Z bosons, and the induced correction to the square sum of the light neutrino masses is

$$\begin{aligned}\sum_a (m_a^2 - \hat{m}_a^2) = 2\sum_a \hat{m}_a (\delta m_\nu^{\text{1-loop}})_{aa} \\ + \sum_{ab} (\delta m_\nu^{\text{1-loop}})^\dagger_{ab} (\delta m_\nu^{\text{1-loop}})_{ba} + \dots\end{aligned} \tag{4.18}$$

Loop stability is jeopardized if the loop corrections on the righthand side of this equation are numerically comparable to the tree-level predictions. Using the physical light neutrino masses as a proxy for the tree-level masses, a sufficient condition for fine tuning is

$$\sum_i m_i^2 \leq \operatorname{tr}\left\{(\delta m_\nu^{\text{1-loop}})^\dagger \delta m_\nu^{\text{1-loop}}\right\} \ . \tag{4.19}$$

In general, δM contains corrections to both D_1 and ΔM. The corrections to D_1 do not break pseudo-Dirac symmetry, and will always remain small compared to the tree-level prediction for the light neutrino masses. Hence, I only keep track of the loop corrections to ΔM. Parametrically, the loop corrections are expected to scale as

$$\operatorname{tr}\left\{(\delta m_\nu^{\text{1-loop}})^\dagger \delta m_\nu^{\text{1-loop}}\right\} \sim (U^2 ||\delta\Delta M||)^2 \ , \tag{4.20}$$

so that eventually U^2 is expected to overcome the smallness of $\delta\Delta M$.

4.3 Criteria for Sterile Neutrino Decoherence

In this section, I combine considerations for sterile neutrino decoherence with fine-tuning arguments to obtain both upper and lower bounds on the relative suppression of LNV in heavy neutrino decays. As a specific example, I consider the minimal model with a single pseudo-Dirac pair of sterile neutrinos.

4.3.1 General Considerations

In general, SM interactions create coherent superposition of the heavy neutrino mass eigenstates N_i. In the common restframe, these states evolves as $|\Psi; t\rangle = \sum_i e^{-\mathrm{i}\, M_i t} \alpha_i |\overline{\nu}_i\rangle$. Close to the Pseudo-Dirac limit, LNV decays are suppressed by destructive interference between the $|\overline{\nu}_i\rangle$, and this interference is captured by the off-diagonal entries of the density matrix $\hat{\rho}_{ij}(t) \equiv \langle\overline{\nu}_i|\Psi; t\rangle\langle\Psi; t|\overline{\nu}_j\rangle$. Decoherence occurs when these off-diagonal entries become negligible, making destructive interference impossible. In general, this is the case for processes that take place on characteristic time-scales much larger than

$$T_{ij} \sim \frac{1}{|M_i - M_j|} \ , \tag{4.21}$$

and this is also true for sterile neutrino decays. The main complication is due to the possibility of the mass eigenstates having wildly different decay widths Γ_i. Without loss of generality, the sterile neutrino masses can be taken to be well ordered, $M_1 < M_2 < \cdots < M_n$. In this case, it is sufficient to compare the decay width Γ_i of each energy eigenstate with the mass splitting between $|\overline{\nu}_i\rangle$ and its adjacent energy eigenstates $|\overline{\nu}_{i-1}\rangle$ and $|\overline{\nu}_{i+1}\rangle$. Up to CP violating corrections, suppression of the LNV decays becomes impossible, if

$$\Gamma_i \ll \min\left(M_i - M_{i-1}, M_{i+1} - M_i\right) \tag{4.22}$$

for all $|\overline{\nu}_i\rangle$. It is important to note that the superposition $|\Psi; t\rangle$ need not contain all of the available sterile neutrino energy eigenstates. The decoherence conditions (4.22) only have to apply to those eigenstates partaking in $|\Psi; t\rangle$. In the language of Feynman diagrams, LNV transitions are mediated by diagrams containing the coherent sum of propagators

$$S_{ab}(p) \approx \mathrm{i} \sum_i \Theta_{ai}\Theta_{bi}\, \mathrm{P}_L \, \frac{M_i}{p^2 - M_i^2 + \mathrm{i}\, M_i \Gamma_i}\, \mathrm{P}_L \ . \tag{4.23}$$

The condition $\Gamma_i \ll \min\left(M_i - M_{i-1}, M_{i+1} - M_i\right)$ implies that the mass-poles in each of these propagators are well separated. In this case, the zero width approximation can be taken for each term individually, giving

$$S_{ab}(p) = \mathrm{i}\, \pi\, \mathrm{P}_L \sum_i \Theta_{ai}\Theta_{bi} M_i\, \delta(p^2 - M_i^2) \ . \tag{4.24}$$

Since the δ-distributions force each four momentum to be on a slightly different mass shell, the individual terms cannot cancel each other, and there is no suppression of LNV amplitudes.

If M_i and Γ_i are treated as free parameters, the above account is exhaustive. However, more information can be extracted by enforcing two separate naturalness conditions:

1. For parameter choices without large, accidental cancellations, relation (4.9a) implies that the physical mass splittings of sterile neutrinos are at least of same order as the light neutrino masses, $\Delta M_{ij} = |M_i - M_j| \gtrsim m_1, m_2$. In this case, LNV decays are unsuppressed if

$$\Gamma_i \ll m_1, m_2 \tag{4.25}$$

   obtains for all $\overline{\nu}_i$. Since $\Gamma_i \propto U_i^2$, this condition is equivalent to a lower bound for U_i^2, below which LNV decays have to be unsuppressed.

2. For parameter choices without 't Hooft fine-tuning, the scaling behaviour (4.20) of the loop corrections to the light neutrino masses implies an upper bound for the sterile neutrino mass splittings ΔM_i. Since decoherence depends on the condition $\Delta M_i > \Gamma_i \propto U_i^2$, this upper bound on ΔM_i can be translated into an upper bound for U_i^2, above which LNV decays have to be suppressed.

Although both of these arguments appeal to notions of naturalness, they are not on equal footing. While large, accidental cancellations may be unsatisfactory on a philosophical level, they do not threaten the internal selfconsistency of theory, and they also pose no major difficulty for predicting experimental observations. On the other hand, the 't Hoof notion of naturalness should be taken somewhat more seriously, since loop stability is a necessary precondition for the validity of perturbative computations. For instance, without loop stability, it becomes effectively impossible to predict the SM neutrino mass matrix with any degree of accuracy, so that the theory loses some of its predictive power.

However, there is no a priori reason to suppose that nature has to be obey either the rule of no-large-cancellations, or 't Hooft naturalness. Here, I only apply these arguments because they are at the center of the debate on the observeability of LNV at colliders. If naturalness is of no concern, then the debate becomes trivial, since there is reason to require the existence of an approximate pseudo-Dirac symmetry. On the other hand, if naturalness considerations are taken seriously, then one has to worry about pseudo-Dirac suppression, and the question of sterile neutrino decoherence becomes relevant. Adopting this stance, one has to accept at least the second argument based on 't Hooft naturalness, and potentially even the weaker argument based on accidental cancellations.

4.3.2 Benchmark Model with $n = 2$

As a specific example, I consider the minimal model with a single pseudo-Dirac pair of sterile neutrinos $\overline{\nu}_1$, $\overline{\nu}_2$. Without loss of generality, D_M and Θ can be parametrized as

$$D_M = \overline{M} \begin{pmatrix} 1-\mu & 0 \\ 0 & 1+\mu \end{pmatrix} , \tag{4.26a}$$

$$\Theta = \frac{v e^{-\mathrm{i}\,\alpha}}{2\overline{M}} \begin{pmatrix} Y_1(2+\mu+\epsilon\, e^{2\,\mathrm{i}\,\alpha}) & \mathrm{i}\, Y_1(2+\mu-\epsilon\, e^{2\,\mathrm{i}\,\alpha}) \\ \mathrm{i}\, Y_2(2+\mu-\epsilon\, e^{2\,\mathrm{i}\,\alpha}) & -Y_2(2+\mu-\epsilon\, e^{2\,\mathrm{i}\,\alpha}) \\ 0 & 0 \end{pmatrix} , \tag{4.26b}$$

where all of the parameters are taken to be real numbers. The explicit breaking of pseudo-Dirac symmetry is quantified in terms of the small parameters $\mu, \epsilon \ll 1$. To leading order in ϵ and μ, one obtains

$$\hat{m}_a = 2\frac{v^2}{\overline{M}} Y_a^2 \epsilon \ , \qquad \frac{1}{2}U^2 = U_i^2 = \frac{v^2}{\overline{M}^2}(|Y_1|^2 + |Y_2|^2) \ , \tag{4.27}$$

so that $\Gamma_1 = \Gamma_2 \equiv \Gamma$. Close to the pseudo-Dirac limit, SM interactions can create and annihilate only the two superpositions $|\Psi_\pm; 0\rangle \equiv |\overline{\nu}_1\rangle \pm \mathrm{i}\,|\overline{\nu}_2\rangle$, and destructive interference between the $|N_i\rangle$ suppresses LNV decays unless the superpositions decohere.

The impact of this decoherence on LNV decays has been studied in [137, 116]. Up to CP violating corrections, it is sufficient to consider decays of $|\Psi_+; 0\rangle$. Using that $|\Psi_+; t\rangle = e^{\mathrm{i}\,M_1 t}|\overline{\nu}_1\rangle + \mathrm{i}\,e^{\mathrm{i}\,M_2 t}|\overline{\nu}_2\rangle$, one finds

$$|\langle\Psi_-; 0|\Psi_+; t\rangle|^2 = \sin^2\left(\frac{1}{2}\Delta M t\right) \ , \qquad \Delta M = 2\overline{M}\mu \ , \tag{4.28}$$

which yields the LNV branching ratio

$$P_{ll} = \Gamma \int_0^\infty \mathrm{d}t \ \sin\left(\frac{1}{2}\Delta M t\right)^2 e^{-\Gamma t} = \frac{1}{2}\frac{\Delta M^2}{\Gamma^2 + \Delta M^2} \ , \tag{4.29}$$

so that $P_{ll} \approx 1/2$ for $\Delta M \gg \Gamma$, as expected. Following [116], I consider the ratio of the LNV and LNC branching ratios,

$$R_{ll} = \frac{P_{ll}}{1 - P_{ll}} = \frac{\Delta M^2}{2\Gamma^2 + \Delta M^2} \ , \tag{4.30}$$

and define LNV decays to be unsuppressed for $R_{ll} > 1/3$. In terms of μ and U^2, this condition is equivalent to

$$4\mu > U^2 \begin{cases} 0.11\, G_F \overline{M}^2 & \text{for } \overline{M} \gg M_W \\ 0.04\, G_F^2 \overline{M}^4 & \text{for } \overline{M} \ll M_W \end{cases} . \tag{4.31}$$

As in the general case, I now proceed by using fine-tuning arguments. In accordance with the first fine-tuning argument in section 4.3.1, I enforce the rule of no-large-cancellations in order to obtain a bound U^2_{LNV}, below of which LNV is unsuppressed. Using relation (4.25), one finds that LNV is expected to be generic for

$$U^2 < U^2_{\mathrm{LNV}} \equiv \frac{m_a}{\overline{M}} \begin{cases} (0.06\, G_F \overline{M}^2)^{-1} & \text{for } \overline{M} \gg M_W \\ (0.02\, G_F^2 \overline{M}^4)^{-1} & \text{for } \overline{M} \ll M_W \end{cases} . \tag{4.32}$$

In accordance with the second fine-tuning argument, I enforce 't Hooft naturalness in order to obtain an upper bound U^2_{stable}, above of which LNV is suppressed. For the loop correction δM, one has

$$\delta M = \delta\overline{M} \cdot \mathbb{1} + \overline{M} \begin{pmatrix} -\delta\mu & 0 \\ 0 & +\delta\mu \end{pmatrix} , \tag{4.33}$$

where

$$\delta\overline{M} = -\frac{2\overline{M} l(\overline{M}^2)}{(4\pi v)^2} \; , \quad \delta\mu = -\mu \frac{4\overline{M}^2 \, l'(\overline{M}^2)}{(4\pi v)^2 - 2l(\overline{M}^2)} \approx -\mu \frac{4\overline{M}^2 l'(\overline{M}^2)}{(4\pi v)^2} \; . \tag{4.34a}$$

The corrections due to $\delta\overline{M}$ do not break pseudo-Dirac symmetry, and the resulting contribution to the light neutrino masses is always small compared to the tree-level prediction. However, the neutrino mass correction due to $\delta\mu$ results in a term

$$\operatorname{tr}\left\{(\delta m_\nu^{\text{1-loop}})^\dagger \delta m_\nu^{\text{1-loop}}\right\} = (U^2\overline{M}\delta\mu)^2 + \dots \tag{4.35}$$

that can violate the loop stability condition (4.19). To prevent this from happening, μ needs to be smaller than

$$\mu_{\text{stable}} \equiv (4\pi v)^2 \frac{(\sum_a m_a^2)^{1/2}}{4U^2\overline{M}^3 \, l'(\overline{M}^2)} \; . \tag{4.36}$$

Since unsuppressed LNV decays can only occur for $\Gamma < 2\overline{M}\mu$, this condition on μ can be translated into an upper bound for the mixing angle U^2,

$$U^4 \leq U^4_{\text{stable}} \equiv (4\pi v)^2 \frac{(\sum_a m_a^2)^{1/2}}{\overline{M}^3 \, l'(\overline{M}^2)} \begin{cases} (0.11\, G_F \overline{M}^2)^{-1} & \text{for } \overline{M} \gg M_W \\ (0.04\, G_F^2 \overline{M}^4)^{-1} & \text{for } \overline{M} \ll M_W \end{cases} , \tag{4.37}$$

where the righthand side depends on $\overline{M}$ only. In summary, the bounds U^2_{LNV} and U^2_{stable} define simple criteria for identifying three distinct regions of parameter space:

- For $U^2 < U^2_{\rm LNV}$, LNV is unsuppressed for parameter choices without large, accidental cancellations.

- For $U^2_{\rm LNV} < U^2 < U^2_{\rm stable}$, LNV can be either suppressed or unsuppressed for parameter choices that fulfill both naturalness conditions in section 4.3.1.

- For $U^2_{\rm stable} < U^2$, LNV is suppressed in 't Hooft natural models.

4.4 LNV in the $n = 2$ Benchmak Model

To conclude this chapter, I compare the bounds $U^2_{\rm LNV}$ and $U^2_{\rm stable}$ obtained for the benchmark model with the currently available parameter space for collider accessible type-I seesaw models.

Figure 4.1 shows the parametric dependence of $U^2_{\rm stable}$ (in green) and $U^2_{\rm LNV}$ (in red). The shading highlights regions of parameter space that are excluded based on the results of fixed target and collider experiments (in grey), and on the basis of the observed values of the light neutrino masses themselves (in blue). The full lines correspond to a normal ordering of the light neutrino masses, while the dashed lines correspond to an inverted ordering of the light neutrino masses. The stark difference between the full and dashed lines for $U^2_{\rm LNV}$ is due to the fact that $m_a^2 = m_1^2 \approx \Delta m^2_{\rm sol}$ is apropriate for an inverted ordering, while $m_a^2 = m_2^2 \approx \Delta m^2_{\rm atm}$ is appropriate for a normal ordering. This can be seen most clearly by using the Casas-Ibarra parametrization, in which the physical sterile neutrino mass splitting takes the form [138]

$$\Delta M^2 = \Delta \hat{M}^2 + M^2_{\theta\theta} + 2\Delta \hat{M} M_{\theta\theta} \cos(2\,\mathrm{Re}\,\omega) \;, \tag{4.38}$$

where $\Delta \hat{M}^2$ is the mass splitting contribution due to the Majorana mass matrix $\hat{M}$, and ω is a general complex valued angle. For normal ordering, one has $M^2_{\theta\theta} = \Delta m^2_{\rm sol}$, and for inverted ordering one has $M^2_{\theta\theta} = \Delta m^2_{\rm atm}$.

In regions that have not yet been excluded, unsuppressed LNV decays are expected to be generic for masses $\overline{M} \ll 10^2$ GeV. For $\overline{M} \gtrsim 10^2$ GeV and $U^2 \lesssim 10^{-6}$, LNV decays may or may not be suppressed, since both $\Delta M > \Gamma$ and $\Delta M < \Gamma$ are possible for generic parameter choices.

Finally, for both $\overline{M} \gtrsim 10^2\,\mathrm{GeV}$ and $U^2 > 10^{-6}$, LNV decays can only be unsuppressed in models with 't Hooft fine tuning.

To summarize, observable LNV should be expected to be a generic feature of type-I seesaw models with sub-TeV scale sterile neutrinos. On the other hand, for models with TeV scale or perhaps even heavier neutrinos, 't Hooft naturalness is inconsistent with LNV for a significant fraction of the parameter space.

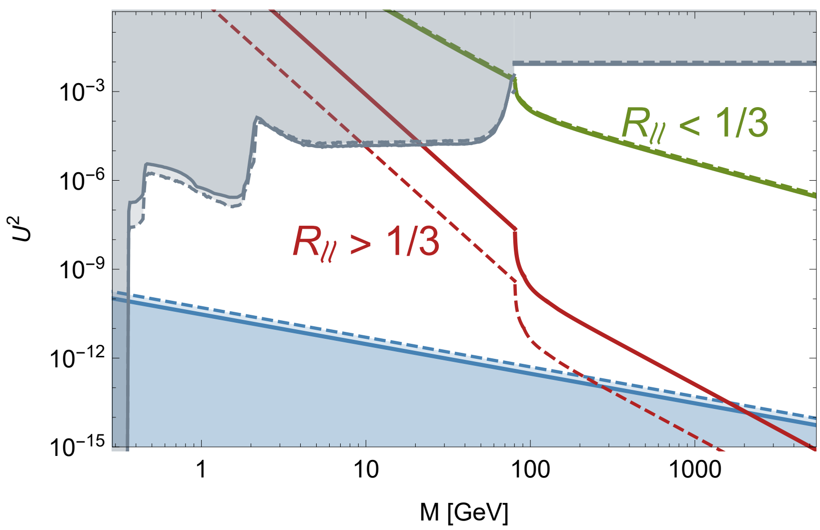

Figure 4.1: Parameter regions with $U^2 > U^2_{\mathrm{stable}}$ (above the green line) and $U^2 < U^2_{\Delta M = m_\nu}$ (below the red line) in the minimal model with $n = 2$. The plot is taken from [2]. The two regimes are well separated by a region, in which both $R_{ll} < 1/3$ and $R_{ll} > 1/3$ are possible for generic parameter choices. The shading highlights areas that are experimentally excluded, and it is based on the global scans in [138, 139]. The red lines are obtained using the analytic estimates for Γ found in [135]. The references [134, 140, 141, 142] contain more rigorous computations. The full lines correspond to normal ordering of the light neutrino masses, and the dashed lines correspond to an inverted ordering of the light neutrino masses.

Chapter 5

Relativistic and Spectator Effects in High-Scale Leptogenesis

5.1 Updating High-Scale Leptogenesis

The origin of the primordial baryon asymmetry in the early universe (BAU) is an unsolved problem of modern cosmology. In addition to explaining the light neutrino masses, a type-I seesaw model can also explain the origin of the BAU via a mechanism called leptogenesis [106]. In leptogenesis, the baryon asymmetry is generated via a two-step process: First, a finite lepton number (L) is generated by CP violating out-of-equilibrium sterile neutrino interactions, and second, nonperturbative SM sphaleron processes in the early universe plasma transform a part of this lepton asymmetry into a finite baryon number (B).

In this chapter, I study the impact of finite temperature effects and partially equilibrated standard model interactions in "high-scale" leptogenesis models [106, 143] in which the mass of the lightest sterile neutrino $\tilde{M}_1$ lies above the Davidson-Ibarra bound, $\tilde{M}_1 \gtrsim 10^9\,\text{GeV}$ [144]. In recent years, much attention has been given to so-called "low scale" leptogenesis models with sterile neutrinos that are much lighter than $10^9\,\text{GeV}$ [107, 108], and I apply methods used to study these models to high-scale leptogenesis.

In general, finite temperature effects are the result of in-medium physics associated with the early universe plasma. They can be important if the lepton asymmetry is generated at least in part when the physical temperature T_{phys} of the early universe plasma is larger than M_1, where the bulk of the sterile neutrinos is relativistic.

Importantly, the timeline of high-scale leptogenesis depends strongly on the washout parameter K, which measures the strength of washout processes in the early universe plasma that can destroy finite lepton asymmetries. In "weak washout" scenarios with $K \ll 1$, these washout processes never equilibrate, so that asymmetries produced at early times, where $T_{\text{phys}} \gg \tilde{M}_1$, can be important. However, in "strong washout" scenarios with $K \gg 1$, washout processes equilibrate at intermediate temperatures $T_{\text{phys}} \gg \tilde{M}_1$, before eventually freezing out at temperatures $T_{\text{phys}} \lesssim \tilde{M}_1$. In this case, the standard lore of high-scale leptogenesis holds that the final asymmetry will be produced entirely after the freeze-out of the washout processes at $T_{\text{phys}} \lesssim M_1$. Following this reasoning, the sterile neutrinos can be treated as nonrelativistic, and finite-temperature effects can be included as small corrections via a formal expansion in $T_{\text{phys}}/\tilde{M}_1$, see e.g. [145, 146, 147, 148, 149, 150].

However, lepton asymmetries produced at early times could survive strong washouts if $B - L$ conserving SM interactions transfer part of the lepton asymmetry into sectors of the SM that do not couple to the sterile neutrinos directly. Provided that these "spectator interactions" remain out of equilibrium until after the washout processes freeze out, they can protect the $B - L$ asymmetry by serving as a bottle neck for the washout. Phenomenologically, models with such "partially equilibrated" spectators are well motivated, since the various SM interactions equilibrate over a wide range of accessible temperatures [151]. In general, the equilibration rates of SM spectator interactions have been studied in [152, 153], and the impact of partially equilibrated spectators in high-scale leptogenesis was studied in [151]. However, the analysis in [151] only applies to strong washout scenarios with initially thermalized sterile neutrinos, where no significant lepton asymmetry can be produced at early time. If the sterile neutrino abundance is initially far from equilibrium, a significant lepton asymmetry can be produced at early times, and it remains to investigate whether partially equilibrated spectator interactions can indeed protect such an asymmetry from strong washouts.

To study the impact of finite temperature and spectator effects in high-scale leptogenesis with arbitrary initial conditions, one has to obtain relativistic interaction rates that describe the CP violating source of lepton asymmetries, the washout processes, and the production of sterile neutrinos in the early universe plasma through the whole transition from the ultrarelativistic regime $T_{\text{phys}} \gg \tilde{M}_1$ to the nonrelativistic regime $T_{\text{phys}} \lesssim \tilde{M}_1$. In recent years, much work has been devoted to understanding these interaction rates, see e.g. the review [154].

Within the context of "low-scale" leptogenesis models, most work has focused on determining the sterile neutrino interaction rates in the ultrarelativistic limit $T_{\text{phys}} \gg \tilde{M}_1$, see e.g. [155, 156, 157, 153]. In this limit, it is important to distinguish processes that either conserve or flip helicity of the SM leptons. Assigning a finite lepton number to sterile neutrinos with definite helicity, helicity conserving processes become lepton-number conserving (LNC), while helicity flip processes become lepton number violating (LNV). The importance of LNV rates for the CP violating source has been pointed out in [158, 159], and the interplay of LNV and LNC rates has been studied e.g. in [160, 161, 120, 162, 163, 164, 165]. As mentioned above, the determination of source and washout rates for arbitray values of $T_{\text{phys}}/\tilde{M}_1$ is important in high-scale leptogenesis scenarios. The interactions rates used in [166, 167, 168] account for time-dilation of fast sterile neutrinos in the eary universe plasma, but not for the associated in-medium effects, and no relativistic corrections are included for the CP violating source. The interaction rates in [169, 170], include thermal effects for general values of $T_{\text{phys}}/\tilde{M}_1$, but without accounting for $2 \leftrightarrow 2$ scattering processes, that are known to generate the dominant contribution for LNC rates in the ultrarelativistic regime $T_{\text{phys}} \gg \tilde{M}_1$. Effective theory approaches have been used in [148, 149, 150] to the interaction rates for small but finite values of $T_{\text{phys}}/\tilde{M}_1$. Finally, the interaction rates in [157, 171] fully include thermal effects for arbitrary values of $T_{\text{phys}}/\tilde{M}_1$, but without distinguishing between LNV and LNC processes, as is necessary for $T_{\text{phys}} \gtrsim \tilde{M}_1$.

The results presented in this chapter have been published in [1].

Focusing on the impact of finite temperature and spectator effects, I consider a simplified model with two strongly hierarchical sterile neutrinos that couple to the same linear combination of SM leptons. Together with my collaborators, I have used methods of non-equilibrium quantum field theory to derive a novel set of momentum-averaged, relativistic fluid equations for this model of high-scale leptogenesis. These equations remain valid throughout the transition from the ultrarelativistic regime $T_{\mathrm{phys}} \gg \tilde{M}_1$ to the nonrelativistic regime $T_{\mathrm{phys}} \lesssim \tilde{M}_1$ and distinguish between sterile neutrinos with positive and negative helicity. Using hard thermal loop (HTL) resummation techniques, I computed the CP violating source term and the helicity dependent interaction rates that appear in the relativistic fluid equations for general values of T_{phys}/M_1 at leading log-accuracy, accounting in particular for $1 \leftrightarrow 2$ decay processes with thermal masses of the SM particles, and $2 \leftrightarrow 2$ scattering processes with t-channel exchanges of SM leptons. In full consistency with prior results, $2 \leftrightarrow 2$ processes dominate LNC rates for $T_{\mathrm{phys}} \gg \tilde{M}_1$, and $1 \leftrightarrow 2$ processes are found to dominate LNV rates. Using these thermal interaction rates, I have found a strong enhancement of the CP violating source term at early times, which can generate large asymmetries. To study the impact of these early asymmetries, I numerically solved the relativistic fluid equations for two specific leptogenesis scenarios:

1. A toy scenario without any spectator interactions, which is useful to isolate the impact of finite temperature corrections. For a vanishing initial abundance, finite temperature corrections to the source term lead to a qualitatively different parametric dependence of the final $B - L$ asymmetry in the weak washout regime. For $K < 0.2$, the sign of the final $B - L$ asymmetry flips, and there is a relative enhancement of the absolute value of the final $B - L$ asymmetry by more than one order of magnitude.

2. A realistic scenario for sterile neutrinos with masses $\sim 10^{13}\,\mathrm{GeV}$, in which b-Yukawa and weak sphaleron interactions are partially equilibrated. In this scenario, finite temperature corrections to the source term lead to another sign flip of the final $B - L$ asymmetry at around $K \approx 30$. For $K \gg 30$, there is a relative enhancement of the absolute value of the final $B - L$ asymmetry by up to two orders of magnitude.

The remainder of this chapter is structured as follows: In section 5.2, I specify the simplified model that I use to study finite temperature and spectator effects in high-scale leptogenesis. In section 5.3, I review a few aspects of finite temperature quantum field theory and the closed time-path (CTP) formalism that are relevant for the remainder of the chaper. In section 5.4, I derive the fully relativistic fluid equations for high-scale leptogenesis and compute the LNC and LNV rates that appear in these equations, with a particular focus on intermediate temperatures. I also compare the relativistic fluid equations with prior equations obtained using the non-relativistic approximation. Finally, in section 5.5, I numerically solve the fluid equations for a setup without spectator interactions, and for a setup with partially equilibrated b-Yukawa and weak sphaleron interactions. In the setup without spectators, I also derive analytic estimates for the final $B - L$ asymmetry in the weak and strong washout regimes $K \ll 1$ and $K \gg 1$.

5.2 Simplified Model for High-Scale Leptogenesis

In this section, I summarize the simplified model I use for studying relativistic and spectator effects in high-scale leptogenesis. In full analogy to chapter 4, I consider a minimal type-I seesaw model with $n = 2$ sterile neutrinos $\overline{\nu}_i$. Computationally, it is convenient to work with the four component spinors

$$N_i \equiv \begin{pmatrix} \overline{\nu}_i \\ \overline{\nu}_i^\dagger \end{pmatrix} , \qquad\qquad \ell_a \equiv \begin{pmatrix} l_a \\ 0 \end{pmatrix} . \tag{5.1}$$

Without loss of generality, the Majorana mass matrix of the sterile neutrinos can be taken to be diagonal, $M = \mathrm{diag}(\tilde{M}_1, \tilde{M}_2)$. Restricting myself to a minimal setup, I assume that both of the sterile neutrino mass eigenstates couple to the same linear combination $\ell_\parallel$ of SM lepton flavours, and that their masses are strongly hierarchical, $\tilde{M}_1 \ll \tilde{M}_2$. Furthermore, I assume that leptogenesis is driven entirely by processes involving the lighter sterile neutrino ν_1. Since $\tilde{M}_1 \ll \tilde{M}_2$, this allows me to simplify the theory by integrating out the heavier sterile neutrino ν_2. The CP violating transitions that source lepton number creation are captured by the two Weinberg-like operators

$$\tilde{O}_L = \tilde{O}_R^\dagger \equiv \frac{F_2^2}{2\tilde{M}_2} (\widetilde{\phi}^\dagger \overline{\ell}_\parallel^c)(\widetilde{\phi}^\dagger \ell_\parallel) , \tag{5.2}$$

which are generated by virtual exchanges of the heavier ν_2. In curved spacetime, this gives the action

$$\begin{aligned} \mathcal{S} = \mathcal{S}_{\mathrm{SM}} + \int \mathrm{d}^4x \sqrt{-g} \Big\{ & \overline{N}_1 \, \mathrm{i} \, \gamma^\alpha e_\alpha^\mu D_\mu N_1 - \tilde{M}_1 \overline{N}_1 N_1 \\ & - [F_1 \overline{N}_1 (\widetilde{\phi}^\dagger \ell_\parallel) + \tilde{O}_L + \mathrm{h.c.}] \Big\} , \end{aligned} \tag{5.3}$$

where D_μ is the covariant derivative, e_α^μ is a vierbein, and $\overline{N}_i = N_i^\dagger \gamma^0$ is now the standard Dirac conjugate spinor. The gamma matrices γ^α are normalized such that they are independent of the metric tensor, $\mathrm{tr}\{\gamma^\alpha \gamma^\beta\} = 2\eta^{\alpha\beta}$.

I assume that leptogenesis takes place during a radiation dominated period in spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology. Working with conformal time t, this gives

$$g_{\mu\nu} = a(t)^2 \eta_{\mu\nu} \ , \qquad\qquad H = \frac{1}{a^2(t)} \frac{d}{dt} a(t) = \frac{t_0}{a_0 t^2} \tag{5.4}$$

where $a(t) = {}^{a_0\,t}\!/\!_{t_0}$ is the scale factor. Notice that the prefactor $\sqrt{-g} = a(t)^4$ breaks canonical normalization of the kinetic terms. Following [172], canonical normalization can be restored via the field redefinitions

$$N_i \to a^{-3/2} N_i \ , \qquad\qquad l_{\parallel} \to a^{-3/2} l_{\parallel} \ , \tag{5.5}$$

which gives

$$\begin{aligned} \mathcal{S} = \mathcal{S}_{\text{SM}} + \int \mathrm{d}^4x \Big\{ & \overline{N}_1 \, \mathrm{i}\, \gamma^\alpha \widetilde{\partial}_\alpha N_1 - \frac{1}{2} M_1 \overline{N}_1 N_1 \\ & - \left[F_1 \overline{N}_1 (\widetilde{\phi}^\dagger \ell_{\parallel}) + O_L + \text{h.c.} \right] \Big\} \ , \end{aligned} \tag{5.6}$$

where $\widetilde{\partial}_\alpha = a(t)\partial_\alpha$ is the partial derivative with respect to locally comoving coordinates, and the effective sterile neutrino mass $M_1 \equiv a\, \tilde{M}_2$ depends on $a(t)$. The operators $O_L = O_R^\dagger$ are identical to the operators $\widetilde{O}_L = \widetilde{O}_R^\dagger$, except with $\tilde{M}_2 \to M_2$, and the momenta k^α associated with $\widetilde{\partial}_\alpha$ are comoving. Neglecting corrections involving gradients of $a(t)$, and using k^α and M_i, one obtains Feynman rules that are formally of the same shape as in Minkowsky space time, which gives the on-shell condition $\eta_{\mu\nu} k^\mu k^\nu = a(t)^2 \tilde{M}_i^2 = M_i^2$.

Finally, I assume that the SM fields remain in kinetic equilibrium throughout leptogenesis, which means that they are well described as thermalized quantum gases with a common physical temperature T_{phys} and individual chemical potentials μ_X. The time evolution of T_{phys} can be inferred from the redshift of the entropy density s_{phys} of the early universe plasma. Assuming that SM degrees of freedom dominate the energy content of the early universe, the Friedman equations imply that s_{phys} redshifts as

$$s_{\text{phys}} = \frac{2\pi^2}{45} g_*(T_{\text{phys}}) T_{\text{phys}}^3 = s \frac{T_0^3 a_0^3}{T^3 a(t)^3} \ , \tag{5.7}$$

where

$$s = \frac{2\pi^2}{45} g_* T^3 \approx 46.8 \cdot T^3 \ , \quad T = \frac{M_\mathrm{p}}{2}\sqrt{\frac{45}{\pi^3 g_*}} \approx 7.1 \cdot 10^{17}\,\mathrm{GeV} \tag{5.8}$$

respectively denote the comoving entropy density and temperature, which remain constant aside from decoupling effects. $M_\mathrm{p} = 1.22 \times 10^{19}$GeV is the Planck mass, T_0 is the temperature of the universe at $t = t_0$, and for $T_\mathrm{phys} \gg v$, the effective number of relativistic SM degrees of freedom is approximately constant, $g_* \equiv g_*(T_\mathrm{phys}) \approx 106.75$. Then, Eq. (5.7) implies that the physical temperature evolves as $T_\mathrm{phys} = {}^{T_0 a_0}/_{a(t)}$[1]. In the remainder of this chapter, I will work in a unit system with $a_0 = 1$, $t_0 = {}^1/_T$, and $T_0 = T$. Furthermore, it is convenient to use the unitless time variable $z = \tilde{M}_1 t = {}^{M_1}/_T$, which is normalized such that $z = 1$ for $T_\mathrm{phys} = \tilde{M}_1$. With these choices, the Hubble rate becomes $H = {}^{\tilde{M}_1^2}/_{Tz^2}$.

For each particle species X, I denote the (anti-)particle number densities in the early universe as $n_X^\pm(t)$, where $\pm$ distinguishes particles and antiparticles. I count each member of a gauge multiplet as a separate particle species, which implies the presence of explicit factors $N_c = 3$ and $g_w = 2$ when summing over the members of the strong SU(3) and weak SU(2) multiplets. Yields are defined by normalizing the net particle number for each species with respect to the comoving entropy density,

$$Y_X \equiv \frac{1}{s}\left(n_X^+ - n_X^-\right) \ . \tag{5.9}$$

The time evolution of the SM yields is determined by the time-evolution of the corresponding chemical potentials, and vice versa. For small chemical potentials and relativistic particle species, one has

$$\mu_X \approx \frac{3 g_s s}{T^2} Y_X \ , \tag{5.10}$$

where g_s counts the spin degrees of freedom for each field. For the sterile neutrinos N_i, the yield (5.9) is not a useful quantity, since the majonara condition implies $n_{N_i}^+ = n_{N_i}^-$. Instead, I define the helicity even and odd yields

$$Y_{N_i\,\mathrm{even/odd}} = \frac{1}{s}(n_{N_i\,+} \pm n_{N_i\,-}), \tag{5.11}$$

[1]See [173] for a review of the equilibrium thermodynamics of the early universe plasma.

where $n_{N_{i\,h}}$ is the number density of sterile neutrinos with a given helicity h. Finally, I quantify the baryon asymmetry in the early universe in terms of the baryon minus lepton number yield $Y_{B-L} \equiv n_{B-L}/s$. Experimentally, it is more convenient to work with the ratio of net baryon and photon number densities

$$\eta_B \approx a_{\rm sph} \frac{Y_{B-L}}{Y_{\gamma,0}} \ , \tag{5.12}$$

where $a_{\rm sph} \approx {}^{28}\!/_{79}$ [174, 175, 176] is the fraction of $B-L$ asymmetry converted into a net baryon number by electroweak sphalerons as they equilibrate, and $Y_{\gamma,0} = 1/7.04$ is the present day photon yield.

Aside from the sterile neutrino Yukawa interactions, I also keep track of $B-L$ conserving SM interactions. Each of these spectator interactions is forced to rapidly approach equilibrium if its equilibration rate Γ is large compared to the Hubble rate, $\Gamma \gg H$. In this case, the net-effect of the spectator interaction is to force the sum of the potentials of the reactants to be equal to the sum of the potentials of the products. On the other hand, if $\Gamma \ll H$, the expansion of the universe dominates, and the spectator interaction is negligible. For $\Gamma \sim H$, the spectator interaction is partially equilibrated, and has to be accounted for by including a corresponding contribution to the equations of motion for each of the SM yields.

The SM equilibration rates depend on the temperature of the early universe plasma, scaling as $\Gamma \sim T_{\rm phys}$. Since the Hubble rate scales as $H \sim T_{\rm phys}^2$, the ratio ${}^{\Gamma}\!/_{H}$ increases over time, causing the SM interactions to gradually equilibrate as the universe cools down. The equilibration temperature $T_{\rm eq}$ for each interaction is defined to be the temperature for which $\Gamma(T_{\rm eq}) = H(T_{\rm eq})$.

For leptogenesis, a spectator interaction can be neglected if the lepton asymmetry is primarily created at temperatures much larger than $T_{\rm eq}$. If the asymmetry is created at temperatures much smaller than $T_{\rm eq}$, the spectator interaction constrains the time-evolution of the SM yields, but it not does not show up in their equations of motion. Finally, if the asymmetry is created at temperatures that are comparable to $T_{\rm eq}$, one has to track the dynamics of the interaction as it equilibrates.

The SM equilibration temperatures impose limits on the validity of the simplified model that I consider in this chapter. An upper bound can be obtained from the fact that the SM fields may be far away from kinetic equilibrium if the expansion of the universe overpowers even the SM gauge interactions. To ensure the validity of the kinetic equilibrium assumption, I restrict myself to leptogenesis scenarios in which the lepton asymmetry is created at temperatures below the equilibration temperature $\sim 10^{16}\,\mathrm{GeV}$ of the electroweak SM gauge interactions.

Similarly, a lower bound can be obtained from the fact that τ lepton Yukawa interactions equilibrate at temperatures around $10^{12}\,\mathrm{GeV}$. Below this temperature, SM Yukawa interactions can transfer lepton asymmetry from $l_{\parallel}$ into other linear combinations of lepton flavours. In this case, one has to track the asymmetry stored in each lepton flavour individually, even if all of the SM lepton Yukawa interactions are either fully equilibrated or negligible [177, 167, 178, 179]. In the temperature ranges with partially equilibrated SM lepton Yukawa interactions, one also has to account for the dynamics of off-diagonal correlations between the different lepton flavours [180, 181]. In order to focus on the impact spectator and relativistic corrections, I further restrict myself to leptogenesis scenarios in which the lepton asymmetry is created at temperatures above the equilibration temperature $\sim 10^{12}\,\mathrm{GeV}$ of τ Yukawa interactions.

5.3 The CTP Formalism

In this section, I present a short review of the closed time path (CTP) formalism. The material collected in this section is well-known and taken from a variety of sources [182, 183, 184, 185, 186, 187].

5.3.1 CTP Correlation Functions

The CTP formalism is designed to compute n-point correlation functions in non-equilibrium quantum field theory. In general, the time-evolution of these n-point functions depends on the initial state of the system at time $t = t_i$, which can be encoded in terms of the von-Neumann density matrix $\hat{\rho}$. This way, one obtains the exact two-point functions

$$\langle \mathcal{O}(u, v) \rangle_{\hat{\rho}} \equiv \mathrm{tr}\left\{ \hat{\rho}\; O(u, v) \right\} \;, \tag{5.13}$$

where $\mathcal{O}(u, v)$ is some composite operator that depends on two field operators evaluated at u and v, respectively. Importantly, $\mathcal{O}(u, v)$ does not have to be time-ordered.

The central idea is to rewrite (5.13) as a path integral along a closed time path that starts at the initial time t_i, moves forward to some final time $t_f \to \infty$, and then returns back to t_i, with the density matrix written as an infinite tower of external current interactions. Just as the conventional path-integral generates time-ordered correlation functions, the CTP path integral generates time-*path* ordered correlation functions. Each field operator picks up an index $a = \pm$, which specifies its position along either the forward ("$+$") or backward ("$-$") branch of the closed time path. Since each field operator can be evaluated on either branch, one obtains four types of two-point functions, denoted as "$++$", "$+-$", "$-+$", and "$--$". In"$++$" (or "T") and "$--$"(or "$\overline{T}$") propagators, both fields are evaluated on the same branch of the time-path, so these propagators correspond to the conventional (anti-)time ordered two-point functions. In "$+-$"(or "$<$") and "$-+$"(or "$>$") propagators, the fields are evaluated on opposed branches of the time-path. These two propagators are also referred to as "Wightman functions".

In preparation for the discussion in the remainder of this section, I focus more closely on the two-point functions involving the SM Higgs boson, the SM leptons, and the sterile neutrinos. For the Higgs boson, one has

$$\mathrm{i}\,\Delta^{++}(u,v) = \langle \mathcal{T}\{\phi(u)\phi^\dagger(v)\}\rangle_{\hat\rho}\ , \quad \mathrm{i}\,\Delta^{<}(u,v) = \langle \phi^\dagger(v)\phi(u)\rangle_{\hat\rho}\ , \tag{5.14a}$$

$$\mathrm{i}\,\Delta^{--}(u,v) = \langle \overline{\mathcal{T}}\{\phi(u)\phi^\dagger(v)\}\rangle_{\hat\rho}\ , \quad \mathrm{i}\,\Delta^{>}(u,v) = \langle \phi(u)\phi^\dagger(v)\rangle_{\hat\rho}\ , \tag{5.14b}$$

where $\mathcal{T}$ and $\overline{\mathcal{T}}$ are (anti-)time-ordering operators. By combining the individual CTP propagators (5.14), one can also obtain advanced and retarded propagators,

$$\mathrm{i}\,\Delta^{a} \equiv \mathrm{i}\,\Delta^{T} - \mathrm{i}\,\Delta^{>} = \mathrm{i}\,\Delta^{\overline{T}} - \mathrm{i}\,\Delta^{<}\ , \tag{5.15a}$$

$$\mathrm{i}\,\Delta^{r} \equiv \mathrm{i}\,\Delta^{T} - \mathrm{i}\,\Delta^{<} = \mathrm{i}\,\Delta^{\overline{T}} - \mathrm{i}\,\Delta^{>}\ , \tag{5.15b}$$

which can be decomposed into hermitian and antihermitian combinations

$$\Delta^{\mathcal{H}} \equiv \frac{1}{2}(\Delta^a + \Delta^r)\ , \qquad \Delta^{\mathcal{A}} \equiv \frac{1}{2\,\mathrm{i}}(\Delta^a - \Delta^r)\ . \tag{5.15c}$$

The antihermitian combination $\Delta^{\mathcal{A}}$ is the spectral function of the Higgs boson. For the SM leptons and the sterile neutrinos, one has

$$\begin{aligned} \mathrm{i}\,S^{++}_{X,\alpha\beta}(u,v) &= \langle \mathcal{T}\{\psi_{X,\alpha}(u)\bar\psi_{X,\beta}(v)\}\rangle\ , \\ \mathrm{i}\,S^{--}_{X,\alpha\beta}(u,v) &= \langle \overline{\mathcal{T}}\{\psi_{X,\alpha}(u)\bar\psi_{X,\beta}(v)\}\rangle\ , \end{aligned} \tag{5.16a}$$

and

$$\begin{aligned} \mathrm{i}\,S^{>}_{X,\alpha\beta}(u,v) &= \langle \psi_{X,\alpha}(u)\bar\psi_{X,\beta}(v)\rangle\ , \\ \mathrm{i}\,S^{<}_{X,\alpha\beta}(u,v) &= -\langle \bar\psi_{X,\beta}(v)\psi_{X,\alpha}(u)\rangle\ . \end{aligned} \tag{5.16b}$$

In full analogy with the scalar case, I also define the advanced and retarded propagators

$$\mathrm{i}\,S^{a}_X \equiv \mathrm{i}\,S^{T}_X - \mathrm{i}\,S^{>}_X = \mathrm{i}\,S^{\overline{T}}_X - \mathrm{i}\,S^{<}_X\ , \tag{5.17a}$$

$$\mathrm{i}\,S^{r}_X \equiv \mathrm{i}\,S^{T}_X - \mathrm{i}\,S^{<}_X = \mathrm{i}\,S^{\overline{T}}_X - \mathrm{i}\,S^{>}_X\ , \tag{5.17b}$$

and further

$$S^{\mathcal{H}}_X \equiv \frac{1}{2}(S^a_X + S^r_X)\ , \quad S^{\mathcal{A}}_X \equiv \frac{1}{2\,\mathrm{i}}(S^a_X - S^r_X) = \frac{\mathrm{i}}{2}(S^{>}_X - S^{<}_X)\ . \tag{5.17c}$$

As before, $S_X^{\mathcal{A}}$ is the spectral function of the fermion in question.

The Wightman function are especially important, since they encode the information necessary to compute the particle number densities $n_X^{\pm}(t)$. This connection can be established most easily by working in Wigner space. In general, the Wigner transform of a generic two-point function $G(u,v)$ is defined as the Fourier transform with respect to the relative coordinate $r = u - v$,

$$G(x,k) \equiv \int \mathrm{d}^4 r \; e^{\mathrm{i}\,kr} G\Big(x + \frac{1}{2}r, x - \frac{1}{2}r\Big) \; . \tag{5.18}$$

Using the Wigner-transformed Wightman functions, it is possible to define generalized momentum distribution functions

$$f_{\ell_\parallel}^{\pm}(t,\boldsymbol{k}) \equiv -\int\limits_0^{\pm\infty} \frac{\mathrm{d}k^0}{2\pi} \,\mathrm{tr}\left[\gamma^0\,\mathrm{i}\,S_{\ell_\parallel}^{<,>}(t,k)\right] \; , \tag{5.19a}$$

$$f_{N_1 h}(t,\boldsymbol{k}) \equiv -\int\limits_0^{\pm\infty} \frac{\mathrm{d}k^0}{2\pi} \,\mathrm{tr}\left[\gamma^0\,\mathrm{P}_h\,\mathrm{i}\,S_{N_1}^{<,>}(t,k)\right] \; , \tag{5.19b}$$

for each fermion species. Since the sterile neutrinos are Majorana particles, both options on the RHS of Eq. (5.19b) yield the same result. Using these distribution functions, fermionic particle number densities $n_{N_1 h}(t)$ and $n_{\ell_\parallel}^{\pm}(t)$ for the sterile neutrinos and Standard Model leptons can be written as [172, 181, 188]

$$\begin{aligned} n(t)_{\ell_\parallel}^{\pm} &\equiv \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} f_{\ell_\parallel}^{\pm}(t,\boldsymbol{k}) \\ &= -\int\limits_0^{\pm\infty} \frac{\mathrm{d}k^0}{2\pi} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \,\mathrm{tr}\left[\gamma^0\,\mathrm{i}\,S_{\ell_\parallel}^{<,>}(t,k)\right] \end{aligned} \tag{5.20a}$$

and

$$\begin{aligned} n(t)_{N_1 h} &\equiv \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} f_{N_1 h}^{\pm}(t,\boldsymbol{k}) \\ &= -\int\limits_0^{\pm\infty} \frac{\mathrm{d}k^0}{2\pi} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \,\mathrm{tr}\left[\gamma^0\,\mathrm{P}_h\,\mathrm{i}\,S_{N_1}^{<,>}(t,k)\right] \end{aligned} \tag{5.20b}$$

$$= -\frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\left[\gamma^0\,\mathrm{P}_h\left(\theta(k_0)\,\mathrm{i}\,S^{<}_{N_1}(t,k) - \theta(\text{-}k_0)\,\mathrm{i}\,S^{>}_{N_1}(t,k)\right)\right] ,$$

where γ^0 is the zeroth gamma matrix, and P_h projects onto an helicity eigenstate with helicity $h = \pm 1$,

$$\mathrm{P}_h \equiv \frac{1}{2}(\mathbb{1} + h\hat{k}^i\gamma_0\gamma_i\gamma_5) \ . \tag{5.21}$$

I have omitted SU(2) indices for the number densities $n(t)^{\pm}_{\ell_\parallel}$, since gauge symmetry of both the initial conditions and the time evolution implies that both components of the SU(2) doublet share the same (anti)particle number densities. For the final expression in Eq. (5.20b), I used the Majorana condition for the sterile neutrino spinors N_i, which implies that particle and antiparticle number densities must be identical.

5.3.2 Constraints from Finite Temperature QFT

Since the SM fields are assumed to remain in kinetic equilibrium throughout leptogenesis, their Wightman functions are constrained by results of finite temperature, rather than non-equilibrium, quantum field theory. In particular, they have to obey a set of generalized Kubo-Martin-Schwinger (KMS) relations. Working in the restframe of the SM plasma, these relations take the shape

$$\mathrm{i}\,\Delta^{<}(k) = e^{-\beta(k_0-\mu_\phi)}\,\mathrm{i}\,\Delta^{>}(k) \ , \tag{5.22a}$$

$$\mathrm{i}\,S^{<}_{\ell_\parallel}(k) = -e^{-\beta(k_0-\mu_{\ell_\parallel})}\,\mathrm{i}\,S^{>}_{\ell_\parallel}(k) \ , \tag{5.22b}$$

where k^0 is the energy and $\beta = {}^1\!/T$ is the comoving temperature of the plasma. Using the KMS relations, it is possible to decompose the Wightman functions as

$$\mathrm{i}\,\Delta^{<}(k) = +2\Delta^{\mathcal{A}}(k)f_\phi(k) \ , \quad \mathrm{i}\,\Delta^{>}(k) = +2\Delta^{\mathcal{A}}(k)(1+f_\phi(k)) \ , \tag{5.23a}$$

$$\mathrm{i}\,S^{<}_{\ell_\parallel}(k) = -2S^{\mathcal{A}}_{\ell_\parallel}(k)f_{\ell_\parallel}(k) \ , \quad \mathrm{i}\,S^{>}_{\ell_\parallel}(k) = +2S^{\mathcal{A}}_{\ell_\parallel}(k)(1-f_{\ell_\parallel}(k)) \ , \tag{5.23b}$$

where the distribution functions

$$f_X(k) = \begin{cases} f_B(k_0-\mu_X) & \text{(bosons)} \\ f_F(k_0-\mu_X) & \text{(fermions)} \end{cases} \tag{5.24}$$

are consistent with the definition in (5.19) and encode the dependence on β and μ_X. f_F and f_B are the standard Fermi-Dirac and Bose-Einstein distributions,

$$f_F(k_0) = \frac{1}{e^{\beta k^0} + 1} \,, \qquad f_B(k_0) = \frac{1}{e^{\beta k^0} - 1} \,. \tag{5.25}$$

The KMS relations are formally exact, so this decomposition also holds when accounting for loop-induced wave-function type corrections of the spectral functions $\Delta^{\mathcal{A}}(k), S^{\mathcal{A}}_{\ell_\parallel}(k)$.

For the sterile neutrinos a parametrization along the lines of Eqs. (5.23) and (5.24) is not necessarily valid, since they may be far away from kinetic equilibrium throughout leptogenesis. For the non-relativistic regime, the kinetic equilibrium ansatz is a good approximation when estimating the final $B-L$ asymmetry [189, 190], but for relativistic sterile neutrinos, one has to allow for the possibility of helicity dependent Wightman functions and distribution functions that can not be characterized in terms of a single chemical potential.

5.3.3 Schwinger-Dyson Equations

Out of equilibrium, it is common to use effective action methods to determine the dynamics of the CTP two-point functions. To do so, the infinite tower of external current interactions generated by the density matrix has to be truncated at some finite order. In order to reproduce finite values for the two-point functions at initial time, one has to retain at least the external current interactions that are quadratic in the dynamical fields of the theory. For such “gaussian initial conditions”, the CTP path integral can be used to define a corresponding two-particle irreducible (2PI) effective action that depends only on the exact CTP one- and two-point functions of the theory. As they should, the density matrix interactions then fix the initial values of these n-point functions.

Varying this 2PI effective action with respect to the exact CTP two point functions (5.13), one obtains a well-known set of formally exact Schwinger-Dyson equations. For the Higgs-boson, one has

$$e^{-\mathrm{i}\diamond}\{k^2 - \Pi^{a,r}\}\{\mathrm{i}\,\Delta^{a,r}\} = \mathrm{i} \,, \tag{5.26a}$$

$$e^{-\mathrm{i}\diamond}\{k^2 - \Pi^{r}\}\{\mathrm{i}\,\Delta^{<,>}\} = e^{-\mathrm{i}\diamond}\{\Pi^{<,>}\}\{\mathrm{i}\,\Delta^{a}\} \,, \tag{5.26b}$$

and for fermions

$$e^{-\,\mathrm{i}\diamond}\{\not{k} - m_X - \Sigma_X^{a,r}\}\{\mathrm{i}\,S_X^{a,r}\} = \mathrm{i}\,\mathrm{P}_X \ , \tag{5.26c}$$

$$e^{-\,\mathrm{i}\diamond}\{\not{k} - m_X - \Sigma_X^{r}\}\{\mathrm{i}\,S_X^{<,>}\} = e^{-\,\mathrm{i}\diamond}\{\Sigma_X^{<,>}\}\{\mathrm{i}\,S_X^{a}\} \ , \tag{5.26d}$$

where $\mathrm{P}_{\ell_\parallel} \equiv \mathrm{P}_R$, $\mathrm{P}_{N_1} \equiv \mathbb{1}$, and $m_{\ell_\parallel} = 0$. The diamond operator $\diamond$ is

$$\diamond\,\{A(x,k)\}\{B(x,k)\} \equiv \frac{1}{2}A(x,k)\Big(\overleftarrow{\partial}_x \cdot \overrightarrow{\partial}_k - \overleftarrow{\partial}_k \cdot \overrightarrow{\partial}_x\Big)B(x,k) \ , \tag{5.27}$$

and the self-energies encode loop corrections to the tree-level propagators. They are defined as functional derivatives of the interacting part of the 2PI effective action,

$$\Pi^{ab}(u,v) \equiv \mathrm{i}\,ab\,\frac{\delta\Gamma^{\mathrm{2PI},\,\geq 2\mathrm{loop}}}{\delta\Delta^{ba}(v,u)} \ , \quad \Sigma_X^{ab}(u,v) \equiv -\,\mathrm{i}\,ab\,\frac{\delta\Gamma^{\mathrm{2PI},\,\geq 2\mathrm{loop}}}{\delta S_X^{ba}(v,u)} \ . \tag{5.28}$$

As for the propagators, one has the combinations

$$\Pi^{a,r} \equiv \Pi^T - \Pi^{<,>} \ , \quad \Pi^{\mathcal{H}} \equiv \frac{\Pi^a + \Pi^r}{2} \ , \quad \Pi^{\mathcal{A}} \equiv \frac{\Pi^a - \Pi^r}{2\,\mathrm{i}} \ , \tag{5.29a}$$

$$\Sigma_X^{a,r} \equiv \Sigma_X^T - \Sigma_X^{<,>} \ , \quad \Sigma_X^{\mathcal{H}} \equiv \frac{\Sigma_X^a + \Sigma_X^r}{2} \ , \quad \Sigma_X^{\mathcal{A}} \equiv \frac{\Sigma_X^a - \Sigma_X^r}{2\,\mathrm{i}} \ . \tag{5.29b}$$

The (anti-)hermitian combinations encode wave-function type corrections that can modify on-shell dispersion relations and induce additional propagating modes. The spectral combinations also determine the equilibration rate of the particle in question.

If there is a meaningful separation of scales between microscopic propagation of particle modes on the one hand, and the macroscopic evolution of the plasma on the other hand, it is possible to expand the Schwinger-Dyson equations (5.26) in the product of gradients $\partial_x \cdot \partial_k$. In general, the size of the gradients ∂_k is given by the typical energy scale of the individual propagating modes, so that in the early universe, one has $\partial_k \sim 1/T$. In contrast, the size of finite gradients ∂_x in conformal time is determined either by the conformal Hubble rate $a(t)H$, which is the typical timescale of the expansion of the universe, or by the typical time-scales of microscopic scattering and decay processes, which are perturbatively suppressed by powers of the SM and BSM couplings. Importantly, these microscopic processes only lead to finite gradients ∂_x while they are out of equilibrium, so that interactions that are continuously in equilibrium throughout leptogenesis cannot invalidate the gradient expansion while ${a(t)H}/{T} = {H}/{T_{\rm phys}} \ll 1$. However, since the gradient expansion at least conceptually relies on the smallness of the perturbative couplings, one has to count $\partial_x \sim \Sigma_X^{\mathcal{H}/\mathcal{A}}, \Pi^{\mathcal{H}/\mathcal{A}}$ in order for the expansion to remain self-consistent. Thus, expanding the Schwinger-Dyson equations to leading order in the gradient expansion, one obtains [172, 181]

$$\left(k^2 - m_X^2 - \Pi^{a,r}\right) \mathrm{i}\,\Delta^{a,r} = \mathrm{i}\ , \tag{5.30a}$$

$$\left(k^2 - m_X^2 - \Pi^{r}\right) \mathrm{i}\,\Delta^{<,>} + \frac{\mathrm{i}}{2} 2k_\mu \partial^\mu (\mathrm{i}\,\Delta^{<,>}) = \Pi^{<,>}\,\mathrm{i}\,\Delta^{a}\ , \tag{5.30b}$$

and

$$\left(\not{k} - m_X - \Sigma_X^{a,r}\right) \mathrm{i}\,S_X^{a,r} = \mathrm{i}\,\mathrm{P}_X\ , \tag{5.30c}$$

$$\left(\not{k} - m_X - \Sigma_X^{r}\right) \mathrm{i}\,S_X^{<,>} + \frac{\mathrm{i}}{2} \not{\partial}_x (\mathrm{i}\,S_X^{<,>}) = \Sigma_X^{<,>}\,\mathrm{i}\,S_X^{a}\ . \tag{5.30d}$$

It is possible to solve Eqs. (5.30a) and (5.30c) exactly, which is equivalent to resumming wave-function type corrections for the spectral functions.

For the sterile neutrinos, the coupling to the SM plasma is perturbatively suppressed by factors of $|F_i|^2$. As a result, it is not necessary to resum these contributions, unless the sterile neutrinos are approximately mass-degenerate. In this case, the wave-function corrections would capture sterile neutrino oscillations that can be the dominant source of the CP violation. Neglecting the selfenergies in Eq. (5.30c), one obtains the tree-level spectral function

$$S_{N_1}^{\mathcal{A}}(k) = \pi\,\mathrm{sign}(k_0)\,\delta(k^2 - M_1^2)\ . \tag{5.31}$$

Since the helicity projection operator (5.21) commutes with the Schwinger-Dyson equations (5.30d), the sterile neutrino Wightman functions can be decomposed into two contributions of definite helicity [191],

$$\mathrm{i}\,S_{N_1}^{<}(k) = -2 \sum_h \mathrm{P}_h\, f_{N_1 h}(k_0)\, S_{N_1}^{\mathcal{A}}(k)\ , \tag{5.32a}$$

$$\mathrm{i}\,S_{N_1}^{>}(k) = 2 \sum_h \mathrm{P}_h (1 - f_{N_1 h}(k_0))\, S_{N_1}^{\mathcal{A}}(k)\ , \tag{5.32b}$$

where the $f_{N_1h}(k_0)$ are defined as in (5.19). In thermal equilibrium, the Wightman functions have to obey the KMS relations (5.22a), so that one obtains $f_{N_1h,\text{eq}}(k_0) = f_F(k_0)$. I define the deviation from equilibrium as

$$\delta f_{N_1h}(k_0) \equiv f_{N_1h}(k_0) - f_{N_1h,\text{eq}}(k_0) \ . \tag{5.33}$$

For the SM leptons and the Higgs boson, the wave function type corrections capture finite temperature physics that modify their dispersion relations and generate a finite width that is related to the damping of propagating modes in medium. Accounting for these effects, one obtains [192, 181]

$$S^{\mathcal{A}}_{\ell_\parallel} = \mathrm{P}_L \, \frac{(\not{p} - \Sigma^{\mathcal{H}}_{\ell_\parallel}) \ \Gamma_{\ell_\parallel} - \Sigma^{\mathcal{A}}_{\ell_\parallel} \ \Omega_{\ell_\parallel}}{\Omega^2_{\ell_\parallel} + \Gamma^2_{\ell_\parallel}} \ \mathrm{P}_R \ , \qquad \Delta^{\mathcal{A}} = \frac{\Gamma_\phi}{\Omega^2_\phi + \Gamma^2_\phi} \ , \tag{5.34}$$

where

$$\begin{aligned} \Gamma_\phi &= \hat{\Pi}^{\mathcal{A}} \ , & \Gamma_{\ell_\parallel} &= 2(k_\mu - \hat{\Sigma}^{\mathcal{H}}_{\ell_\parallel\,\mu})\hat{\Sigma}^{\mathcal{A}\,\mu}_{\ell_\parallel} \ , \\ \Omega_\phi &= k^2 - \hat{\Pi}^{\mathcal{H}} \ , & \Omega_{\ell_\parallel} &= (p_\mu - \hat{\Sigma}^{\mathcal{H}}_{\ell_\parallel\,\mu})^2 - (\hat{\Sigma}^{\mathcal{A}}_{\ell_\parallel\,\mu})^2 \ , \end{aligned} \tag{5.35}$$

and the hat denotes that the selfenergies contain only SM vertices. Since all of the SM particles are massless in the symmetric phase, one has

$$\hat{\Sigma}^{\mathcal{H}/\mathcal{A}}_{\ell_\parallel} = \gamma^\mu \hat{\Sigma}^{\mathcal{H}/\mathcal{A}}_{\ell_\parallel\,\mu} \ , \qquad \hat{\Sigma}^{\mathcal{H}/\mathcal{A}}_{\ell_\parallel,\mu} = \frac{1}{4}\,\mathrm{tr}\left(\gamma_\mu \hat{\Sigma}^{\mathcal{H}/\mathcal{A}}_{\ell_\parallel}\right) \ . \tag{5.36}$$

Accounting for the shape of the spectral functions, the actual time-evolution of the Wightman functions is given by Eqs. (5.30b) and (5.30d). Notice that the spatial gradients vanish since the universe is assumed to be spatially homogeneous and isotropic. Taking the hermitian part of (5.30d), one finds the Kadanoff-Baym equation [181, 193]

$$\begin{aligned} \partial_t \, \mathrm{i}\, \boldsymbol{S}^{<,>}_X = \ &- [\boldsymbol{\mathcal{H}}_{X,\text{eff}}, \boldsymbol{S}^{<,>}_X] + [\boldsymbol{\Sigma}^{<,>}_X, \boldsymbol{S}^{\mathcal{H}}_X] \\ &+ \frac{1}{2}(\{\boldsymbol{\Sigma}^{>}_X, \boldsymbol{S}^{<}_X\} - \{\boldsymbol{\Sigma}^{<}_X, \boldsymbol{S}^{>}_X\}) \end{aligned} \tag{5.37a}$$

where

$$\boldsymbol{S}^{<,>}_X \equiv \mathrm{i}\,\gamma_0 S^{<,>}_X \ , \qquad \boldsymbol{S}^{\mathcal{H}}_X \equiv \mathrm{i}\,\gamma_0 S^{\mathcal{H}}_X \ , \tag{5.37b}$$

$$\boldsymbol{\Sigma}^{<,>}_X \equiv \Sigma^{<,>}_X \gamma_0 \ , \qquad \boldsymbol{\mathcal{H}}_{X,\text{eff}} \equiv (\not{k} - m_X - \Sigma^{\mathcal{H}}_X)\gamma_0 \ . \tag{5.37c}$$

Rewriting the Wightman functions as products of distribution functions $f_X(k)$ and spectral functions, this equation can be used as a starting point for the derivation of Boltzmann equations in kinetic theory.

5.4 Relativistic Fluid Equations for Leptogenesis

In this section, I use the CTP formalism in order to derive a novel set of fully relativistic fluid equations of motion for high-scale leptogenesis. I also compute the dominant contributions to the sterile neutrino interaction rates that appear in these equations for arbitrary $T_{\text{phys}}/\tilde{M}_1 = T/M_1$. Finally, I compare the results obtained in this section with the conventional, nonrelativistic description of high-scale leptogenesis used e.g. in [168, 151].

5.4.1 Derivation of the Fluid Equations

My goal is to derive a fully relativistic set of fluid equations that determine the time-evolution of the yields Y_{B-L} and $Y_{N_1,\text{even/odd}}$. Using relation (5.20a), Y_{B-L} and $Y_{N_1,\text{even/odd}}$ are functionals of the Wightman functions $S_{\ell_\parallel}^{>,<}$ and $S_{N_1 h}^{>,<}$, which obey the Kadanoff-Baym equations (5.37a). Following the approach in [172, 181, 188], I use this fact to rewrite the time derivatives of Y_{B-L} and $Y_{N_1,\text{even/odd}}$ as collision integrals that depend on the Wightman functions and CTP selfenergies (5.28) that account for finite temperature corrections due to the early universe plasma. In principle, evaluating these collision integrals is equivalent to solving the Kadanoff-Baym equations for $S_{\ell_\parallel}^{>,<}$ and $S_{N_1 h}^{>,<}$. To obtain a closed set of fluid equations for $Y_{\ell_\parallel}$ and $Y_{N_1\text{even/odd}}$, I use two simplifying approximations:

First, I linearize the collision integrals by expanding them to leading order in μ_X and $\delta f_{N_1 h}$. This is a standard approximation used in many studies of leptogenesis, see e.g. [172, 181, 194, 195, 196]. It is well justified in strong washout scenarios with thermal initial conditions, since the sterile neutrinos remain approximately thermalized throughout leptogenesis. However, in models with a vanishing initial abundance of sterile neutrinos, a significant lepton asymmetry can be sourced by the sterile neutrinos approaching equilibrium. In this case, the CP source term should be expected to receive order one corrections that can have a significant impact on the baryon asymmetry at recombination. Furthermore, if the sterile neutrinos remain far from equilibrium throughout leptogenesis, order one corrections to the washout and backreaction rates can also impact the final BAU. Investigating the importance of these corrections is an important task, but solving the resulting nonlinear equations is quite challenging. Since my focus is the impact of finite temperature corrections, I follow the standard approach of linearizing the Kadanoff-Baym equations in μ_X and $\delta f_{N_1 h}$.

Second, I neglect the momentum dependence of the sterile neutrino interaction rates that multiply μ_X and $\delta f_{N_1 h}$, replacing them with effective, momentum averaged rates. Using these momentum-averaged rates, the information encoded in $Y_{N_1 \mathrm{even/odd}}$ is sufficient to determine the time-evolution of Y_{B-L}, giving rise to the final fluid equations for high-scale leptogenesis. The momentum-averaging approximation has been shown to be excellent for high-scale leptogenesis with strong washouts and no partially equilibrated spectators [189, 190], However, in studies with with GeV scale sterile neutrinos [197, 162], it has been shown that working with fully momentum dependent rates can yield order one corrections for the final $B-L$ asymmetry. Again, my main focus is the impact of finite temperature and spectator effects, and I neglect these corrections.

The CTP formalism has been used successfully in many leptogenesis studies [198, 199, 200, 201, 202, 172, 181, 203, 204, 194], and in particular within the context of low-scale leptogenesis models [205, 181, 196], where the lepton asymmetry is produced at temperatures much higher than the mass of the sterile neutrinos. However, it is often possible to guess the correct shape of the momentum-averaged fluid equations for high-scale leptogenesis on the basis of physical intuition, using finite-temperature quantum field theory to compute the cross-sections and decay widths that appear in the resulting equations. The semi-rigorous derivation using the CTP formalism has two main advantages:

First, the CTP formalism automatically accounts for finite temperature corrections, the helicity dependence of the sterile neutrinos, and other subtleties that would have to be kept in mind when constructing fluid equations for leptogenesis "by hand".

Second, the first principles derivation yields expressions for the CP violating source term, the sterile neutrino equilibration rates, and the washout rates that connect directly to the underlying description in terms of non-equilibrium quantum field theory. Although the computation of these interaction rates remains technically challenging, the CTP formalism automatically enforces unitarity and CPT invariance, and once again accounts for finite temperature corrections. Computing these rates in the S-Matrix formalism, one would have to enforce unitarity by hand, using a real-intermediate-state subtraction in order to account for the fact that sterile neutrinos are unstable, see e.g. the discussions in [201, 194].

Construction of the Collision Integrals

I begin by constructing the collision interals that govern the time-evolution of the yields $Y_{N_1,\text{even/odd}}$ and Y_{B-L}. First, consider $Y_{N_1,\text{even/odd}}$. Combining expression (5.20a) with the Kadanoff-Baym equations (5.37a), one obtains

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{dz}} Y_{N_1\text{even/odd}} = \frac{\mathrm{d}}{\mathrm{dz}} \frac{1}{s}(n_{N_1+} \pm n_{N_1-}) \qquad (5.38)\\
&= \frac{-1}{2\tilde{M}_1 s} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \operatorname{tr}\Big[\gamma^0 \partial_t \Big(\theta(k_0)\, \mathrm{i}\, S^{<}_{N_1}(t,k) \\
&\qquad - \theta(-k_0)\, \mathrm{i}\, S^{>}_{N_1}(t,k)\Big)(\mathrm{P}_+ \pm \mathrm{P}_-)\Big]\ ,\\
&= \frac{1}{2\tilde{M}_1 s} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \operatorname{sign}(k_0) \operatorname{tr}\Big[\Big(\mathrm{i}\,\Sigma^{>}_{N_1}(k)\, \mathrm{i}\, S^{<}_{N_1}(k) \\
&\qquad - \mathrm{i}\,\Sigma^{<}_{N_1}(k)\, \mathrm{i}\, S^{>}_{N_1}(k)\Big)(\mathrm{P}_+ \pm \mathrm{P}_-)\Big]\ ,
\end{aligned}
$$

where the sterile neutrino selfenergies $\mathrm{i}\,\Sigma^{>,<}_{N_1}$ are defined via (5.28). To leading order in the Yukawa couplings F_1, they receive contributions from the two amputated diagrams shown in Fig. 5.1. One obtains

$$
\mathrm{i}\,\Sigma^{>,<}_{N_1} = g_w\, |F_1|^2 \Big(\mathrm{P}_L\, \gamma_\mu\, \mathrm{i}\, \hat{\Sigma}^{\mu>,<}_{N_1,L} + \mathrm{P}_R\, \gamma_\mu\, \mathrm{i}\, \hat{\Sigma}^{\mu>,<}_{N_1,R}\Big)\ , \qquad (5.39)
$$

where the reduced selfenergies [172]

$$
\mathrm{i}\, \hat{\Sigma}^{\mu>,<}_{N_1,L}(k) = \frac{1}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \operatorname{tr}\left[\gamma^\mu\, \mathrm{i}\, S^{>,<}_{\ell_\parallel}(p)\right] \mathrm{i}\, \Delta^{>,<}(k-p)\ , \qquad (5.40)
$$

$$
\mathrm{i}\, \hat{\Sigma}^{\mu>,<}_{N_1,R}(k) = -\frac{1}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \operatorname{tr}\left[\gamma^\mu\, \mathrm{i}\, S^{<,>}_{\ell_\parallel}(-p)\right] \mathrm{i}\, \Delta^{<,>}(p-k)\ , \qquad (5.41)
$$

capture contributions generated from either only fermions or antifermions propagating in the loops. C is the Dirac charge-conjugation matrix,

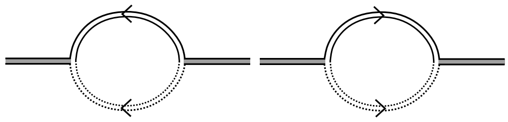

Figure 5.1: 2PI diagrams contributing to the self energy of the sterile neutrino N_1 at leading order. The double lines represent resummed propagators. Scalar lines are dotted, and the shading signifies a sterile neutrino propagator.

while $\Delta^{<,>}$ and $S_{\ell_\|}^{<,>}$ denote the kinetic equilibrium propagators (5.23) with resummed spectral functions (5.76). Notice that these resummed propagators already encode the dominant finite temperature corrections for leptogenesis. Additional vertex type corrections contribute at higher order in the SM gauge couplings, see section 5.4.2 for more details.

Next, consider Y_{B-L}. Since all SM interactions conserve $B-L$, only the sterile neutrino Yukawa interactions contribute to the time derivative of Y_{B-L}. Using the Kadanoff-Baym equations (5.37a) for $S_{\ell_\|}^{>,<}$, one obtains

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{dz}} Y_{B-L} = \frac{\mathrm{d}}{\mathrm{dz}}(Y_B - g_w Y_{\ell_\|}) \qquad (5.42)\\
&= \frac{g_w}{\tilde{M}_1 s} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \operatorname{tr}\left[\gamma^0\Big(\theta(p_0)\,\mathrm{i}\,\partial_t S_{\ell_\|}^{<}(t,p) + \theta(-p_0)\,\mathrm{i}\,\partial_t S_{\ell_\|}^{>}(t,p)\Big)\right] ,\\
&= \frac{-g_w}{\tilde{M}_1 s} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \operatorname{tr}\left[\,\mathrm{i}\,\Sigma_{\ell_\|}^{>}(p)\,\mathrm{i}\,S_{\ell_\|}^{<}(p) - \mathrm{i}\,\Sigma_{\ell_\|}^{<}(p)\,\mathrm{i}\,S_{\ell_\|}^{>}(p)\right] ,
\end{aligned}$$

where the selfenergies $\Sigma_{\ell_\|}^{ab}$ are taken to contain only $B-L$ violating contributions. At order $\mathcal{O}(F^2)$, the only relevant diagram is shown in Fig. 5.2. One obtains

$$\mathrm{i}\,\Sigma_{\ell_\|,LO}^{>,<}(k) = |F_1|^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \mathrm{P}_R\,\mathrm{i}\,S_{N_1}^{>,<}(p)\,\mathrm{P}_L\,\mathrm{i}\,\Delta^{<,>}(p-k) , \qquad (5.43)$$

where as before $\mathrm{i}\,\Delta^{<,>}(p-k)$ denotes the full propagator (5.23). As before, this selfenergy accounts for the dominant finite temperature effects, with further vertex-type corrections contributing at higher order.

In principle, the sterile neutrino Wightman functions $\mathrm{i}\,S_{N_1}^{<,>}$ contain CP-violating contributions associated with sterile neutrino oscillations, but these are encoded in terms of wave-function type corrections for the

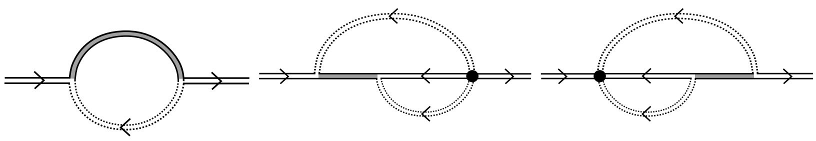

Figure 5.2: 2PI diagrams contributing to the leptonic self-energies. The first diagram is the LO contribution, and the two-loop diagrams generate the CP-violating source term. The conventions for the lines are as in Fig. 5.1. The dotted vertices correspond to the effective interaction of Eq. (5.2). The self energy $\mathrm{i}\,\Sigma_{l_\parallel}$ results when amputating these diagrams.

spectral function (5.31) that only enter into expression (5.43) at order $\mathcal{O}(F^6)$. Instead, the dominant CP violating contributions to the SM lepton selfenergies are generated by the second and third diagram in Fig. 5.1, each of which contains a single insertion of the Weinberg-like operators (5.2). These diagrams give

$$\mathrm{i}\,\Sigma^{>,<}_{\ell_\parallel,S}(k) = |F_1|^2 \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\mathrm{P}_R\,\mathrm{i}\,S^{>,<}_S(p)\,\mathrm{P}_L\,\mathrm{i}\,\Delta^{<,>}(p-k)\ , \qquad (5.44)$$

where the object

$$\begin{aligned}\mathrm{i}\,S^{>,<}_S \equiv\ &\pm\mathrm{i}\,\frac{3F_1^{*2}F_2^2}{M_2\,|F_1|^2}\Big(\mathrm{i}\,S^{>,<}_{N_1}\,\mathrm{i}\,\gamma_\mu\hat{\Sigma}^{\mu\ T,\overline{T}}_{N_1,R} - \mathrm{i}\,S^{\overline{T},T}_{N_1}\,\mathrm{i}\,\gamma_\mu\hat{\Sigma}^{\mu\ >,<}_{N_1,R}\Big) \qquad (5.45)\\ &\mp\mathrm{i}\,\frac{3F_1^{2}F_2^{*2}}{M_2\,|F_1|^2}\Big(\mathrm{i}\,\gamma_\mu\hat{\Sigma}^{\mu\ \overline{T},T}_{N_1,R}\,\mathrm{i}\,S^{>,<}_{N_1} - \mathrm{i}\,\gamma_\mu\hat{\Sigma}^{\mu\ >,<}_{N_1,R}\,\mathrm{i}\,S^{T,\overline{T}}_{N_1}\Big)\end{aligned}$$

captures the CP violation due to the Weinberg-like operators. Using expressions (5.43) and (5.44), it is possible to rewrite the collision integral (5.42) in terms of the lefthanded contribution to the sterile neutrino selfenergy,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{B-L} = \frac{-g_w}{\tilde{M}_1 s}\int\frac{\mathrm{d}^4 k}{(2\pi)^4}\,\mathrm{tr}\,\big[&\big(\mathrm{i}\,S^{>}_{N_1} + \mathrm{i}\,S^{>}_S\big)(k)\,\mathrm{P}_L\,\mathrm{i}\,\Sigma^{<}_{N_1}(k) \qquad (5.46)\\ &-\big(\mathrm{i}\,S^{<}_{N_1} + \mathrm{i}\,S^{<}_S\big)(k)\,\mathrm{P}_L\,\mathrm{i}\,\Sigma^{>}_{N_1}(k)\big]\ .\end{aligned}$$

Small Deviations from Equilibrium

Next, I linearize collision integrals (5.38) and (5.42). For each collision integral, this yields two distinct contributions, the first generated by terms linear in μ_X, and the second generated by terms linear in $\delta f_{N_1 h}$.

First, consider the contributions generated by μ_ℓ and μ_ϕ. Expanding the sterile neutrino self energy (5.39) to linear order in μ_X/T [193], one obtains

$$\mathrm{i}\,\hat{\Sigma}^{\mu>,<}_{N_1,L} = \mathrm{i}\,\hat{\Sigma}^{\mu>,<}_{N_1} - 2\,\frac{\mu_{\ell_\parallel} + \mu_\phi}{T} f_F(k_0)(1 - f_F(k_0))\hat{\Sigma}^{\mu}_{N_1} + \mathcal{O}\big(\mu_X^2/T^2\big) \tag{5.47a}$$

and

$$\mathrm{i}\,\hat{\Sigma}^{\mu>,<}_{N_1,R} = \mathrm{i}\,\hat{\Sigma}^{\mu>,<}_{N_1} + 2\,\frac{\mu_{\ell_\parallel} + \mu_\phi}{T} f_F(k_0)(1 - f_F(k_0))\hat{\Sigma}^{\mu}_{N_1} + \mathcal{O}\big(\mu_X^2/T^2\big) \tag{5.47b}$$

where the reduced self-energies $\hat{\Sigma}^{\mu>,<}_{N_1}$ and $\hat{\Sigma}^{\mu}_{N_1}$ are defined via

$$\hat{\Sigma}^{\mu>,<}_{N_1} \equiv \hat{\Sigma}^{\mu>,<}_{N_1,L}\Big|_{\mu_X=0} = \hat{\Sigma}^{\mu>,<}_{N_1,R}\Big|_{\mu_X=0} \tag{5.48}$$

and

$$\begin{aligned}\hat{\Sigma}^{\mu}_{N_1}(k) &\equiv \frac{\mathrm{i}}{2}\big(\hat{\Sigma}^{\mu>}_{N_1}(k) - \hat{\Sigma}^{\mu<}_{N_1}(k)\big) \\ &= f_F^{-1}(k_0)\int\frac{\mathrm{d}^4p}{(2\pi)^4} f_F(p_0) f_B(k_0 - p_0)\Delta^{\mathcal{A}}(k-p)\,\mathrm{tr}\left[\gamma^\mu S^{\mathcal{A}}_{\ell_\parallel}(p)\right] \,.\end{aligned} \tag{5.49}$$

Taking $\delta f_{N_1 h} \to 0$ and inserting the expansions (5.47a), (5.47b) into Eqs. (5.38) and (5.46), one finds

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{even}}\Big|_{\delta f_{N_1 h}\to 0} = 0\,, \qquad \frac{\mathrm{d}}{\mathrm{dz}}\,g_w\,Y_{l_\parallel}\Big|_{\delta f_{N_1 h}\to 0} = W\,, \tag{5.50a}$$

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{odd}}\Big|_{\delta f_{N_1 h}\to 0} = \widetilde{W}\,, \tag{5.50b}$$

where

$$\widetilde{W} = \frac{-12K}{T^3}\left(Y_{\ell_\parallel} + \frac{1}{2}Y_\phi\right) \tag{5.50c}$$

$$\times \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}(1-f_F(k_0))f_F(k_0)\,\tilde{\gamma}(k)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ ,$$

$$W = \frac{-12K}{T^3}\left(Y_{\ell_\parallel} + \frac{1}{2}Y_\phi\right) \tag{5.50d}$$

$$\times \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}(1-f_F(k_0))f_F(k_0)\,\gamma(k)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ .$$

The constant

$$K \equiv \frac{T}{\tilde{M}_1^2}\Gamma_D(z\to\infty) = \frac{\Gamma_D(z\to\infty)}{H(z=1)} = T\,g_w\frac{|F_1|^2}{16\pi\tilde{M}_1} \tag{5.51}$$

is proportional to the sterile neutrino decay width at zero temperature, and

$$\Gamma_D(z\to\infty) \equiv \Gamma_D(N_1\to l_\parallel\phi) + \Gamma_D(N_1\to \bar{l}_\parallel\phi^\dagger) = g_w\frac{|F_1|^2\tilde{M}_1}{16\pi}\ . \tag{5.52}$$

With this definition, K is identical to the conventional washout parameter as defined in [106, 168, 151]. Finite temperature corrections to this picture are encoded in the dimensionless interaction rates

$$\gamma(k) \equiv \frac{32\pi}{T}\frac{k_\mu\hat{\Sigma}^{\mathcal{A}\mu}_{N_1}(k)}{k_0}\ , \qquad \tilde{\gamma}(k) \equiv \frac{32\pi}{T}\frac{\tilde{k}_\mu\hat{\Sigma}^{\mathcal{A}\mu}_{N_1}(k)}{k_0}\ . \tag{5.53}$$

Finally, the object

$$\tilde{k}^\mu \equiv \frac{1}{2}h\,\mathrm{tr}\Big[\mathrm{P}_h\,\gamma^5\gamma^\mu\not{k}\Big] = (|\boldsymbol{k}|, k_0\hat{\boldsymbol{k}}) \tag{5.54}$$

is orthogonal to k^μ, $k_\mu\tilde{k}^\mu = 0$, and projects the sterile neutrino selfenergy onto an helicity-odd combination of Lorentz indices.

Next, consider the contributions generated by $\delta f_{N_1 h} \neq 0$. Taking $\mu_{l,\phi} \to 0$, one obtains

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{even}}\bigg|_{\mu_X\to 0} = D\ , \qquad \frac{\mathrm{d}}{\mathrm{dz}}\,g_w Y_{l_\parallel}\bigg|_{\mu_X\to 0} = \widetilde{B} + S\ , \tag{5.55a}$$

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{odd}}\bigg|_{\mu_X\to 0} = B\ ,$$

where

$$D = -\frac{K}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\delta f_{N_1,\mathrm{even}}(k_0)\gamma(k)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ , \tag{5.55b}$$

$$B = -\frac{K}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\delta f_{N_1,\mathrm{odd}}(k_0)\gamma(k)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ , \tag{5.55c}$$

$$\widetilde{B} = -\frac{K}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\delta f_{N_1,\mathrm{odd}}(k_0)\tilde{\gamma}(k)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ , \tag{5.55d}$$

$$S = \epsilon_0\frac{K}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\delta f_{N_1,\mathrm{even}}(k_0) \times\left(\frac{(32\pi)^2}{T}\frac{\hat{\Sigma}_{N_1\mu}\hat{\Sigma}^{\mu}_{N_1}(k)}{k_0}\right)\bigg|_{k_0=\sqrt{\boldsymbol{k}^2+M_1^2}}\ , \tag{5.55e}$$

and $f_{N_1,\mathrm{even/odd}} \equiv f_{N_1+} \pm f_{N_1-}$ are helicity even and odd sterile neutrino distribution functions. The constant

$$\epsilon_0 \equiv \frac{\Gamma_D(N_1 \to l_\parallel \phi) - \Gamma_D(N_1 \to \bar{l}_\parallel \phi^\dagger)}{\Gamma_D(N_1 \to l_\parallel \phi) + \Gamma_D(N_1 \to \bar{l}_\parallel \phi^\dagger)} = \frac{1+g_w}{16\pi}\frac{\mathrm{Im}[F_1^{*2}F_2^2]}{|F_1|^2}\frac{\tilde{M}_1}{\tilde{M}_2}\ . \tag{5.56}$$

is defined such that it coincides with the zero-temperature sterile neutrino decay-asymmetry in the strongly hierarchical limit[206, 207]. Notice that the source term (5.55e) is consistent with the calculation in Ref. [206], provided that one evaluates the result obtained there in the limit $M_1/M_2 \to 0$.

Putting everything together, one obtains the evolution equations

$$\frac{\mathrm{d}}{\mathrm{dz}}Y_{N_1\mathrm{even}} = D\ , \qquad \frac{\mathrm{d}}{\mathrm{dz}}g_w Y_{l_\parallel} = \widetilde{B} + S + W\ , \tag{5.57}$$
$$\frac{\mathrm{d}}{\mathrm{dz}}Y_{N_1\mathrm{odd}} = B + \widetilde{W}\ .$$

Working in the restframe of the early universe plasma, it is possible to further simplify the source term S. In this frame, k^i is the only available three momentum that can appear in the reduced sterile neutrino self-energy $\hat{\Sigma}^{\mu}_{N_1}(k)$. Since the four-vectors k^μ and $\tilde{k}^\mu$ are orthogonal, they span the two-dimensional sub-space of four vectors with spatial part proportional to k^i, and $\hat{\Sigma}^\mu_{N_1}(k)$ can be decomposed as

$$\hat{\Sigma}^\mu_{N_1} = \frac{1}{k^2}\left[k^\mu(\hat{\Sigma}^\alpha_{N_1} k_\alpha) - \tilde{k}^\mu(\hat{\Sigma}^\alpha_{N_1}\tilde{k}_\alpha)\right] . \tag{5.58}$$

Combining this relation with the definitions (5.53) of $\gamma(k)$ and $\tilde{\gamma}(k)$, one obtains

$$\frac{(32\pi)^2}{T}\frac{\hat{\Sigma}_{N_1\mu}(k)\hat{\Sigma}^\mu_{N_1}(k)}{k_0} = \beta k_0 \frac{1}{z^2}(\gamma^2(k) - \tilde{\gamma}^2(k)) . \tag{5.59}$$

Momentum Averaged Fluid Equations

To obtain the final fluid equations for leptogensis, I momentum average $\gamma(k)$ and $\tilde{\gamma}(k)$ with respect to the sterile neutrino equilibrium distribution function,

$$\gamma \to \langle\gamma\rangle , \qquad \tilde{\gamma} \to \langle\tilde{\gamma}\rangle . \tag{5.60}$$

For a general function $X(k)$, this average is defined as

$$\langle X\rangle = \frac{2\,s}{Y_{N_1,\mathrm{eq}}} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} X(k) f_F(k)\bigg|_{k^0=\sqrt{\boldsymbol{k}^2+M_1^2}} , \tag{5.61a}$$

where

$$Y_{N_1,\mathrm{eq}} = \frac{T^3}{\pi^2} z^2 \mathcal{I}(z) , \quad \mathcal{I}(z) \equiv \int_z^\infty dy \, \frac{y\sqrt{y^2-z^2}}{e^y+1} \tag{5.61b}$$
$$= z^2 \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n} K_2(n\,z) \approx z^2 K_2(z) .$$

is the sterile neutrino equilibrium yield, and K_n is the n-th modified Bessel function of the second kind. After the replacement (5.60), the momentum integrals in (5.57) can evaluated by using either the even and odd sterile neutrino yields

$$Y_{N_1,\mathrm{even}} - Y_{N_1,\mathrm{eq}} = \frac{1}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\delta f_{N_1,\mathrm{even}}(k_0) , \tag{5.62a}$$

$$Y_{N_1,\mathrm{odd}} = \frac{1}{s}\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\,\delta f_{N_1,\mathrm{odd}}(k_0) , \tag{5.62b}$$

or the dimensionless integrals

$$\mathcal{J}(z) \equiv \int_z^\infty dy \, \frac{y\sqrt{y^2 - z^2}e^y}{(e^y+1)^2} \tag{5.63a}$$
$$= z^2 \sum_{n=1}^{\infty} (-1)^{n-1} K_2\,(nz) \approx z^2 K_2(z) \; ,$$
$$\mathcal{K}(z) \equiv \int_z^\infty dy \, \frac{y^2\sqrt{y^2 - z^2}}{e^y+1} \tag{5.63b}$$
$$= \sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{z^3}{n} K_1(nz) + \frac{3z^2}{n^2} K_2(nz) \right)$$
$$\approx z^3 K_1(z) + 3z^2 K_2(z) \; .$$

Using this notation, the sterile neutrino equilibration terms become

$$B = -\Gamma \, Y_{N_1,\mathrm{odd}} \; , \qquad D = -\Gamma \left(Y_{N_1,\mathrm{even}} - Y_{N_1,\mathrm{eq}} \right) \; , \tag{5.64a}$$
$$\tilde{B} = -\tilde{\Gamma} \, Y_{N_1,\mathrm{odd}} \; , \tag{5.64b}$$

where $\Gamma \equiv K\langle\gamma\rangle$ and $\tilde{\Gamma} \equiv K\langle\tilde{\gamma}\rangle$ are the momentum averaged sterile neutrino equilibration and backreaction rates, respectively. For the washout terms, one finds

$$W = -\eta_{N_1}\Gamma \left(Y_{\ell_\parallel} + \frac{1}{2} Y_\phi \right) \; , \qquad \widetilde{W} = -\eta_{N_1}\tilde{\Gamma} \left(Y_{\ell_\parallel} + \frac{1}{2} Y_\phi \right) \; , \tag{5.65}$$

where the correction factor

$$\eta_{N_1}(z) = \frac{6}{\pi^2} \mathcal{J}(z) \tag{5.66}$$

captures the Boltzmann suppression of the washout terms at late times. Intuitively, the factor η_{N_1} reflects the fact that Γ and $\tilde{\Gamma}$ are defined as averages with respect to the sterile neutrino momentum distribution, rather than the momentum distributions of the SM leptons and Higgs boson. It can be seen to arise by considering the yield corresponding to the difference between the net-numbers of leptons and anti-leptons turned into sterile neutrinos via the Yukawa interactions per given unit z-interval. Since the SM particles are in kinetic equilibrium, this difference is given as

$$W \equiv -\Gamma \frac{2}{s} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \left(\frac{1}{e^{\beta(k_0 - \mu_{l_\parallel} - \mu_H)} + 1} - \frac{1}{e^{\beta(k_0 + \mu_{l_\parallel} + \mu_H)} + 1} \right) \tag{5.67a}$$

$$= \Gamma \frac{12}{T^3} \left(Y_{l_\parallel} + \frac{1}{2} Y_\phi \right) \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{e^{\beta k_0}}{(e^{\beta k_0} + 1)^2} + O\big(\mu_X^2/T^2\big) \ , \tag{5.67b}$$

and therefore

$$\begin{aligned} \eta_{N_1} &\equiv \frac{12}{T^3} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \frac{e^{\beta k_0}}{(e^{\beta k_0} + 1)^2} \\ &= \frac{6}{\pi^2} \int_z^\infty \mathrm{d}y \ y(y^2 - z^2)^{\frac{1}{2}} \frac{e^y}{(e^y + 1)^2} = \frac{6}{\pi^2} \mathcal{J}(z) \ . \end{aligned} \tag{5.68}$$

For the CP-violating source rate (5.59), I average each factor individually. This gives

$$\begin{aligned} \frac{1}{z^2} \beta k_0 (\gamma^2 - \tilde{\gamma}^2) &= \frac{1}{z^2} \beta k_0 (\gamma + \tilde{\gamma})(\gamma - \tilde{\gamma}) \\ &\to \frac{1}{z^2} \langle \beta k_0 \rangle \, \gamma_{\mathrm{LNC}} \, \gamma_{\mathrm{LNV}} \ , \end{aligned} \tag{5.69}$$

where

$$\gamma_{\mathrm{LNC}} = \langle \gamma + \tilde{\gamma} \rangle \ , \qquad \gamma_{\mathrm{LNV}} = \langle \gamma - \tilde{\gamma} \rangle \ , \tag{5.70a}$$

denote LNC and LNV combinations of the dimensionless rates γ and $\tilde{\gamma}$, while

$$\langle \beta k_0 \rangle = \frac{\mathcal{K}(z)}{\mathcal{I}(z)} \tag{5.70b}$$

is the thermal average of the sterile neutrino energy. Putting everything together, one obtains the final, momentum-averaged fluid equations for leptogenesis:

$$\frac{\mathrm{d}}{\mathrm{dz}} Y_{N_1 \mathrm{even}} = -\Gamma \ (Y_{N_1 \mathrm{even}} - Y_{N_1 \mathrm{eq}}) \ , \tag{5.71a}$$

$$\frac{\mathrm{d}}{\mathrm{dz}} Y_{N_1 \mathrm{odd}} = -\Gamma \, Y_{N_1 \mathrm{odd}} - \eta_{N_1} \tilde{\Gamma} \left(Y_{\ell_\parallel} + \frac{1}{2} Y_\phi \right) \ , \tag{5.71b}$$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dz}} Y_{B-L} = &- \epsilon_{\mathrm{eff}} \, \Gamma \, (Y_{N_1 \mathrm{even}} - Y_{N_1 \mathrm{eq}}) \\ &+ \tilde{\Gamma} \, Y_{N_1 \mathrm{odd}} + \eta_{N_1} \, \Gamma \left(Y_{\ell_\parallel} + \frac{1}{2} Y_\phi \right) \ , \end{aligned} \tag{5.71c}$$

where

$$\epsilon_{\text{eff}} = \epsilon_0 \langle \beta k_0 \rangle \frac{2\, \gamma_{\text{LNC}}\, \gamma_{\text{LNV}}}{z^2(\gamma_{\text{LNC}} + \gamma_{\text{LNV}})} \,. \tag{5.72}$$

is the effective finite temperature sterile neutrino decay asymmetry at finite temperatures. Using these equations, it is easy to verify that γ_{LNC} conserves the generalized lepton number $\tilde{L} = L - n_{N_1\text{odd}}$. One obtains

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dz}} Y_{B-\tilde{L}} &= \frac{\mathrm{d}}{\mathrm{dz}} \left(Y_{B-L} + Y_{N_1\text{odd}} \right) \qquad (5.73) \\ &= K \gamma_{\text{LNV}} \left[\eta_{N_1} \left(Y_{\ell_\parallel} + \frac{1}{2} Y_\phi \right) - Y_{N_1\text{odd}} \right] + (\text{source term}) \,, \end{aligned}$$

where γ_{LNV} acts as the only source of $\widetilde{L}$ violation.

5.4.2 Computation of the rates γ_{LNC} and γ_{LNV}

It remains to compute the dimensionless interaction rates γ_{LNC} and γ_{LNV}, which have been defined in Eqs. (5.70a) and (5.53). These damping coefficients depend on the reduced sterile neutrino selfenergy $\hat{\Sigma}^\mu_{N_1}(k)$, which encodes finite temperature effects in the early universe plasma as higher order loop corrections.

In the nonrelativistic limit $z \to \infty$, one may neglect these finite temperature corrections, and the computation of the damping coefficients is relatively simple. One finds [156]

$$\hat{\Sigma}^\mu_{N_1, T=0}(k) = \frac{k^\mu}{32\pi} \,, \tag{5.74a}$$

and therefore

$$\gamma_{\text{LNC}} = \gamma_{\text{LNV}} = z \left\langle \frac{M_1}{k_0} \right\rangle = z\, \frac{1}{\mathcal{I}(z)} \int_z^\infty dy\, \frac{\sqrt{y^2 - z^2}}{e^y + 1} \approx z \frac{K_1(z)}{K_2(z)} \,. \tag{5.74b}$$

The computation at arbitrary z is more involved, since finite temperature effects can cause a breakdown of naive perturbation theory. Specifically, the selfenergies of the SM leptons and the Higgs boson contain so-called "hard thermal loop" (HTL) contributions that scale as $\kappa^2 T^2 / m^2$ [208], where m is a relevant mass or momentum scale, and κ is either a gauge coupling, a Yukawa coupling, or the square root of a scalar quartic coupling. In the ultrarelativistic limit $z \to \infty$, the temperature enhancement T^2/m^2 easily compensates for the perturbative suppression κ, and one has to resum the HTL contributions, which yields effective HTL vertices and propagators for the SM leptons and the Higgs boson. Within the CTP formalism, this HTL resummation is equivalent to using the full spectral functions (5.76) for the Higgs boson and the SM leptons, with the selfenergies (5.36) evaluated at $k^2 \to 0$.

The equilibration rates associated with $\gamma_{\rm LNC}$ are well known in the ultrarelativistic limit $z \to 0$, see e.g. [155, 156, 153, 171, 209]. There, the HTL resummation regulates an IR divergence in $2 \leftrightarrow 2$ scattering processes involving a t-channel exchange of SM leptons, and the divergent $2 \leftrightarrow 2$ contributions are found to be logarithmically enhanced, scaling as $g^2 \ln g^{-2}$, where g symbolically denotes either the electroweak SU(2) gauge coupling g_2 or the U(1) hypercharge gauge coupling g_1. Further vertex and wavefunction type corrections to $\hat{\Sigma}^{\mu}_{N_1}$ scale as g^2, and are therefore subdominant.

For the purpose of studying thermal corrections in high-scale leptogenesis, I need to evaluate $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$ for general $z \neq 0$. In general, this is equivalent to computing the sterile neutrino reduced selfenergy (5.49). To remain consistent with the well-known results for $\gamma_{\rm LNC}$ in the ultrarelativistic limit, I have to include thermal corrections that contribute at order $g^2 \ln g^{-2}$ or less. Working at this level of accuracy, the only relevant contribution is

$$\hat{\Sigma}^{\mu}_{N_1}(k) = f_F^{-1}(k_0) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} f_F(p_0) f_B(k_0 - p_0) \times \Delta^{\mathcal{A}}(k-p) \, \mathrm{tr}\left[\gamma^\mu S^{\mathcal{A}}_{\ell_\parallel}(p)\right] , \tag{5.75}$$

where the HTL resummed spectral functions $S^{\mathcal{A}}_{\ell}$ and $\Delta^{\mathcal{A}}$ regulate the IR divergences that result in the logarithmic enhancement of this diagram. Explicitly, the spectral functions are given as

$$S^{\mathcal{A}}_{\ell_\parallel} = \mathrm{P}_L \frac{\left(\not{p} - \Sigma^{\mathcal{H}}_{\ell_\parallel}\right) \Gamma_{\ell_\parallel} - \Sigma^{\mathcal{A}}_{\ell_\parallel} \Omega_{\ell_\parallel}}{\Omega^2_{\ell_\parallel} + \Gamma^2_{\ell_\parallel}} \mathrm{P}_R , \qquad \Delta^{\mathcal{A}} = \frac{\Gamma_\phi}{\Omega^2_\phi + \Gamma^2_\phi} , \tag{5.76}$$

where

$$\Gamma_\phi\,(q) = \Pi^{\mathcal{A}}\ , \qquad \Gamma_\ell\,(p) = 2\left(p_\mu - \Sigma^{\mathcal{H}}_{l,\mu}\right)\cdot\Sigma^{\mathcal{A},\mu}_l\ ,$$
$$\Omega_\phi\,(q) = q^2 - \Pi^{\mathcal{H}}\ , \quad \Omega_\ell\,(p) = \left(p_\mu - \Sigma^{\mathcal{H}}_{l,\mu}\right)^2 - \left(\Sigma^{\mathcal{A}}_{l,\mu}\right)^2\ . \tag{5.77}$$

Since the sterile neutrino reduced selfenergy (5.49) is defined in terms of the sterile neutrino selfenergy in thermal equilibrium, the SM selfenergies Π^{ab} and Σ^{ab}_ℓ have to be evaluated for vanishing chemical potentials $\mu_X = 0$. Keeping only the HTL contributions [210, 211], one obtains

$$\Pi^{\mathcal{H},\mathrm{HTL}} = m_\phi^2\ , \quad \Sigma^{\mathcal{H},\mathrm{HTL}}_l(p) = \frac{m_l^2}{4}\frac{\tilde{\not p}}{|\boldsymbol{p}|^2}\ln\left|\frac{p_0+|\boldsymbol{p}|}{p_0-|\boldsymbol{p}|}\right| - \frac{m_l^2}{2}\frac{\hat{\not p}}{|\boldsymbol{p}|^2}\ ,$$
$$\Pi^{\mathcal{A},\mathrm{HTL}} = 0\ , \qquad \Sigma^{\mathcal{A},\mathrm{HTL}}_l(p) = \frac{m_l^2}{4}\frac{\tilde{\not p}}{|\boldsymbol{p}|^2}2\pi\,\theta(-p^2)\ , \tag{5.78}$$

where now $\tilde{p}^\mu \equiv (|\boldsymbol{p}|, p_0\frac{\boldsymbol{p}}{|\boldsymbol{p}|})$ and $\hat{p}^\mu \equiv (0, \boldsymbol{p})$. The thermal masses m_ϕ^2 and m_l^2

$$m_l^2 = \frac{1}{16}(3g_2^2 + g_1^2)T^2\ , \quad m_\phi^2 = \frac{1}{16}(3g_2^2 + g_1^2 + 4h_t^2 + 8\lambda_\phi)T^2 \tag{5.79}$$

regulate the all-important infrared divergence in the $2 \leftrightarrow 2$ scattering processes with a t-channel exchange of SM leptons, while h_t is the top-quark Yukawa-coupling and λ_ϕ the quartic self-coupling of the Higgs boson.

To evaluate the reduced selfenergy (5.75), I split the momentum integral into a region with time-like lepton momenta $p^2 > 0$, and a region with space-like lepton momenta $p^2 < 0$.

First, consider the region with $p^2 > 0$, where $\Gamma_\ell, \Gamma_\phi = 0$. Hence, the resummed in-medium spectral functions (5.76) can be written as

$$p^2 > 0: \quad \begin{aligned} S^{\mathcal{A},\mathrm{HTL}}_\ell(p) &= \mathrm{P}_L\left[\left(\not p - \not\Sigma^{\mathcal{H},\mathrm{HTL}}_\ell\right)\pi\,\mathrm{sign}(p^0)\,\delta\left(\Omega_\ell\right)\right]\mathrm{P}_R \\ \Delta^{\mathcal{A},\mathrm{HTL}}(p) &= \pi\,\mathrm{sign}(p^0)\,\delta(p^2 - m_\phi^2)\ . \end{aligned} \tag{5.80}$$

Physically, these δ distributions select on-shell modes that describe the propagation of dressed particle modes in the early universe plasma, and

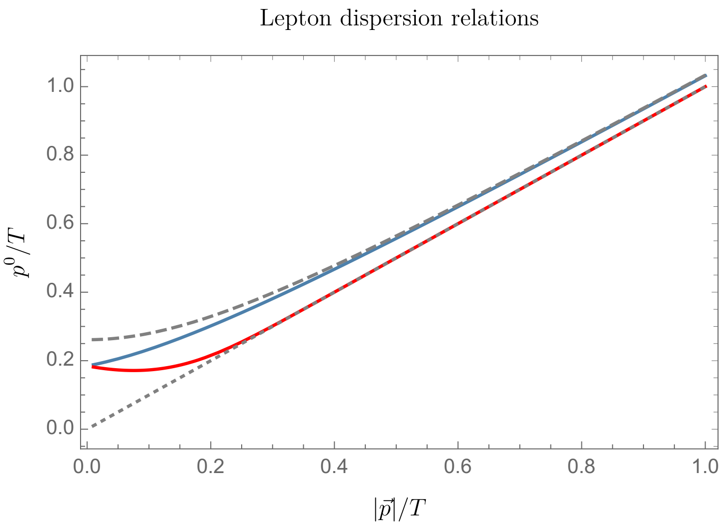

Figure 5.3: The HTL dispersion relations for leptonic pseudoparticles (solid blue) and holes (solid red), plotted against the dispersion relation of a massless particle (dotted gray) and a particle with mass m_l (dashed gray).

the resulting contribution to $\hat{\Sigma}^{\mu}_{N_1}$ captures $1 \leftrightarrow 2$ decay and inverse decay processes involving one dressed Higgs particle, one dressed SM lepton, and one sterile neutrino N_1. There is an important complication: It is well known that the on-shell condition $\Omega_\ell(p) = 0$ actually permits two solutions $p_0 = \omega_{p,h}(\boldsymbol{p})$ for each three-momentum $\boldsymbol{p}$, and as a result, the SM lepton field can create two distinct types of dressed particle modes. Typically, ω_p branch solutions are called "pseudo-particle" modes, while the ω_h branch solutions are called "collective excitations" or "hole" modes.

The momentum dependence of both branches is shown in Fig. 5.3. For $|\boldsymbol{p}| \gtrsim T$, the pseudo-particle and hole modes fulfill simplified dispersion relations,

$$\omega_p(\boldsymbol{p}) \approx \boldsymbol{p}^2 + m_\ell^2 \ , \qquad\qquad \omega_h(\boldsymbol{p}) \approx \boldsymbol{p}^2 \ . \tag{5.81}$$

Furthermore, the hole mode contributions become exponentially suppressed due to the Jacobian $\Delta_h(p)$ that arises from rewriting

$$\delta(\Omega_\ell) = \sum_{p,h} \Delta_{b,h} \delta(p_0^2 - \omega_{p,h}^2) \ . \tag{5.82}$$

For small $|\boldsymbol{p}|$, the integrand scales as $|\boldsymbol{p}|^2\Delta_{p,h} \sim |\boldsymbol{p}|$, so that this regime is dynamically suppressed. Focusing on the contribution from hard momenta $|\boldsymbol{p}| \gtrsim T$, one can approximate the integral (5.75) by neglecting hole modes and using the simplified dispersion relation (5.81) for the pseudo-particle modes. This way, one obtains the SM lepton spectral function

$$p^2 > 0: \qquad S_\ell^{\mathcal{A},\mathrm{HTL},} \approx \mathrm{P}_L \left[\not{p}\, \pi \,\mathrm{sign}(p_0)\, \delta(p^2 - m_l^2)\right] \mathrm{P}_R \ . \tag{5.83}$$

For the reduced selfenergy (5.75), using this simplified spectral function gives the $1 \leftrightarrow 2$ pseudo-particle contribution

$$\begin{aligned}\hat{\Sigma}^\mu_{N_1,1\leftrightarrow 2}(k) \approx \frac{T}{32\pi|\boldsymbol{k}|} \Bigg[&\frac{|A|^2}{M_1^2} I_0\Big(\frac{A^2 k_0}{M_1^2 T}, \frac{B^2|\boldsymbol{k}|}{M_1^2 T}, \frac{k_0}{T}\Big)\, k^\mu \\ &+ \frac{\mathrm{sgn}(A^2)T}{|\boldsymbol{k}|} I_1\Big(\frac{A^2 k_0}{M_1^2 T}, \frac{B^2|\boldsymbol{k}|}{M_1^2 T}, \frac{k_0}{T}\Big)\, \tilde{k}^\mu \Bigg] ,\end{aligned} \tag{5.84}$$

where $\tilde{k}$ is defined as in Eq. (5.53), and the quantities A, B are

$$A^2 \equiv M_1^2 + m_l^2 - m_\phi^2, \qquad B^2 \equiv \sqrt{A^4 - 4m_l^2 M_1^2} \ . \tag{5.85}$$

The dimensionless integrals $I_{0,1}(\alpha,\beta,y)$ are given as

$$I_0(\alpha,\beta,y) = \beta + \ln\left|\frac{1+e^{-\frac{1}{2}(\alpha+\beta)}}{1+e^{-\frac{1}{2}(\alpha-\beta)}}\right| + \ln\left|\frac{1-e^{\frac{1}{2}(\alpha-\beta)-y}}{1-e^{\frac{1}{2}(\alpha+\beta)-y}}\right| , \tag{5.86a}$$

$$\begin{aligned}I_1(\alpha,\beta,y) = \beta\Big[&\ln\big|\big(1+e^{-\frac{1}{2}(\alpha+\beta)}\big)\big(1+e^{-\frac{1}{2}(\alpha-\beta)}\big)\big| \\ &- \ln\big|\big(1-e^{\frac{1}{2}(\alpha-\beta)-y}\big)\big(1-e^{\frac{1}{2}(\alpha+\beta)-y}\big)\big|\Big] \\ - 2\Big[&\mathrm{Li}_2(-e^{-\frac{1}{2}(\alpha-\beta)}) - \mathrm{Li}_2(-e^{-\frac{1}{2}(\alpha+\beta)}) \\ &+ \mathrm{Li}_2(e^{\frac{1}{2}(\alpha-\beta)-y}) - \mathrm{Li}_2(e^{\frac{1}{2}(\alpha+\beta)-y})\Big] ,\end{aligned} \tag{5.86b}$$

where $\mathrm{Li}_2(x)$ denotes the polylogarithmic function of second order. Finally, one obtains $1 \leftrightarrow 2$ contributions to the damping coefficients $\gamma^{1\leftrightarrow 2}_{\mathrm{LNC/LNV}}$ by inserting the result (5.84) into the momentum averages defined in Eq. (5.70a), and then evaluating the resulting one-dimensional integral numerically.

Next, consider the region with space-like momenta $p^2 < 0$. In this regime, the Higgs boson spectral function is exactly the same as the $p^2 > 0$ expression (5.80), but the SM lepton spectral function (5.76) picks up a finite width $\Gamma_\ell \neq 0$. Since the Higgs boson spectral function continues to select on-shell modes that describe dressed Higgs particles in the early plasma, the $p^2 < 0$ contributions to $\hat{\Sigma}^\mu_{N_1}$ can be interpreted as arising from scattering processes with t-channel exchanges of the SM leptons. In the ultrarelativistic limit $z \to 0$, this region contains the logarithmically enhanced $2 \leftrightarrow 2$ processes that have been found in prior computations [155, 156, 153, 209]. Working at leading log accuracy, only these enhanced $2 \leftrightarrow 2$ processes contribute to $\hat{\Sigma}^\mu_{N_1}$ in the $p^2 < 0$ region.

The analytic result (5.84) is useful, but to test the validity of the approximation(5.83), one has to compare it with a numerical computation that retains collective excitations and uses the full dispersion relations $\omega_{p,h}$ for both the pseudo-particle and hole modes. Furthermore, an analytic computation of the $2 \leftrightarrow 2$ contributions for general z is extremely involved, so that these have to be computed numerically as well. Together with my collaborators in [1], I have performed the a numerical computation of the individual $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ contributions $\gamma^{1\leftrightarrow 2}_{\text{LNC/LNV}}$ and $\gamma^{2\leftrightarrow 2}_{\text{LNC/LNV}}$ to the LNC and LNV rates that retains the full, momentum dependent HTL selfenergies (5.78).

Specifically, we used an adaptive Monte-Carlo method, and evaluated the integral for 100 values of z with even logarithmic spacing that lie between $z_i = 10^{-2}$ and $z_f = 10^2$. The values of the SM coupling constants are determined by solving two-loop renormalization group equations of the Standard Model [212], where the initial conditions chosen such that the SM couplings accurately predict collider measurements. This matching to experimental results includes two-loop threshold corrections for the Higgs parameters [213], as well as one-loop electroweak [214] and three-loop QCD corrections [215, 216] for the determination of the top Yukawa in terms of the top pole mass. The Higgs and top masses are taken from recent ATLAS measurements, giving $m_\phi = 124.97$ GeV [217] and $m_t = 172.69$ GeV [218], and the remaining parameters are fixed via PDG data [55]. Using the renormalization scale $\mu = T_{\text{phys}} = 10^{12}$ GeV, this yields

$$\frac{1}{2}g_1^2 + \frac{3}{2}g_2^2 = 0.546\ , \qquad h_t = 0.485\ , \qquad \lambda_\phi = -0.00187\ . \tag{5.87}$$

It is convenient to interpolate the results obtained for $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$ by using a phenomenological fit function. For this function, I distinguish between contributions from $1 \leftrightarrow 2$ processes with pseudo-particle lepton modes, $1 \leftrightarrow 2$ processes with collective excitations, and $2 \leftrightarrow 2$ processes with t-channel exchanges of SM leptons. Defining $L \equiv \log_{10} z$, this gives

$$\gamma_{\rm LNC/LNV} = \gamma^{1\leftrightarrow 2}_{\rm LNC/LNV} + \gamma^{1\leftrightarrow 2,h}_{\rm LNC/LNV} + \gamma^{2\leftrightarrow 2}_{\rm LNC/LNV}\,, \tag{5.88}$$

where the hole mode contributions are negative and

$$\begin{aligned}
&-\log_{10}|\gamma^{1\leftrightarrow 2}_{\rm LNC}| \\
&\approx \begin{cases} \Big(1 + 0.125 e^{-73.8(L+0.994)^2}\Big)\Big(\frac{1}{|L|^{6.51}} + 1.79\Big) & z \le 0.3\,, \\ \Big(1 + 139 e^{-27.9(L+0.653)^2}\Big)\Big(\frac{35.2}{|L+1.97|^{5.66}} - 0.994L\Big) & z > 0.3\,, \end{cases} \\
&-\log_{10}|\gamma^{1\leftrightarrow 2,h}_{\rm LNC}| \\
&\approx \begin{cases} \Big(1 + 1.69 e^{-1.01(L-1.31)^2}\Big)\Big(\frac{0.002}{|L|^{9.53}} + 2.29\Big) & z \le 0.36\,, \\ \Big(1 + 1.16 e^{-5.59(L-0.607)^2}\Big)\Big(\frac{0.586}{|L-1.42|^{5.25}} + 1.98\Big) & z > 0.80\,, \\ \Big(1 + 0.733 e^{15.5(L+0.186)^2}\Big)\Big(\frac{0.631}{|L-0.578|^{-50.2}} + 1.20\Big) & \text{other } z\,, \end{cases} \\
&-\log_{10}|\gamma^{2\leftrightarrow 2}_{\rm LNC}| \\
&\approx 0.987\Big(0.417L + \sqrt{(0.174L - 0.161)L + 0.0571} + 0.615\Big) \\
&\qquad\qquad\qquad \times \Big(1 + 0.180 e^{-5.00|0.412-L|^{2.2}}\Big)\,,
\end{aligned}$$

$$\begin{aligned}
&-\log_{10}|\gamma^{1\leftrightarrow 2}_{\rm LNV}| \\
&\approx \begin{cases} \Big(1 - 0.887 e^{-11.1(L+0.753)^2}\Big)\Big(\frac{0.584}{|L+0.179|^{9.06}} - 1.90L - 0.256\Big) & z \le 0.55\,, \\ \Big(1 + 19.0 e^{-213(L+0.360)^2}\Big)\Big(\frac{0.117}{|L+0.399|^{1.39}} - 1.023L\Big) & z > 0.55\,, \end{cases} \\
&-\log_{10}|\gamma^{1\leftrightarrow 2,h}_{\rm LNV}|
\end{aligned}$$

$$\approx \begin{cases} \left(1 + 23.7e^{-10.6(L-0.011)^2}\right)\left(\frac{4.44\cdot 10^{-7}}{|L+4.70|^{-8.63}} - 2.00L + 2.52\right) & z \leq 0.36\,, \\ \left(1 + 0.302e^{0.218(L-0.971)^2}\right)\left(\frac{0.400}{|L+0.931|^{-5.41}} + 2.30\right) & z > 0.65\,, \\ \left(1 - 0.947e^{-1.85(L+0.191)^2}\right)\left(\frac{1.97}{|L-0.613|^{-60.1}} + 63.2\right) & \text{other } z\,, \end{cases}$$

$$\log_{10}|\gamma_{\rm LNV}^{2\leftrightarrow 2}| \approx \left(0.648L - \sqrt{L(1.83L - 0.704) + 0.195} - 1.03\right) \times \left(1 + 0.291e^{-4.46(L-0.297)^2}\right)\,.$$

Figure 5.4 shows the final results for the numerical computation of $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$, comparing them to the phenomenological fit functions (5.88). For most values of z, the relative accuracy of these fit functions is better than 5%, but the accuracy of the $\gamma_{\rm LNV}$ fit functions drops to around 10-30% for a small interval close to the kinematic threshold of $1 \leftrightarrow 2$ processes at $z \approx 0.1$.

In the ultrarelativistic limit, $\gamma_{\rm LNC}$ is dominated by contributions from $2 \leftrightarrow 2$ scattering, while $\gamma_{\rm LNV}$ is dominated by $1 \leftrightarrow 2$ processes. Since $\epsilon_{\rm eff}\,\Gamma \propto \gamma_{\rm LNV}\,\gamma_{\rm LNC}$, this implies that the CP violating source could be underestimated by up to two orders of magnitude if $1 \leftrightarrow 2$ processes are neglected the ultrarelativistic regime. This result could be important for low-scale leptogenesis scenarios with "light" GeV-scale sterile neutrinos, such as in e.g. [159, 158, 160, 161, 120, 162, 163, 164, 165]. For both $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$, the $1 \leftrightarrow 2$ contribution becomes negative between $z \approx 0.1$ and $z \approx 0.6 - 0.7$, and it vanishes completely for values around $z \approx 0.3 - 0.4$. This structure is mainly the result of kinematic blocking due to the thermal masses m_ϕ and m_ℓ in the early universe plasma:

- For $z \lesssim 0.1$, one has $m_\phi > m_\ell + M_1$, so that the thermal masses m_ϕ^2 and m_ℓ^2 are large enough to allow for $\phi \leftrightarrow \ell N_1$ decay and inverse decay processes involving both on-shell pseudo-particle and hole-mode SM leptons. Since the Jacobian Δ_h suppresses the propagation of hole modes, the contribution from pseudo-particle modes dominates.

- For $0.1 \lesssim z \lesssim 0.4$, one has $m_\ell + M_1 > m_\phi > M_1$, so that $\phi \leftrightarrow \ell N_1$ processes can only create and annihilate on-shell hole-mode SM leptons. The associated sign flip for $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$ results from the hole modes carry negative lepton number.

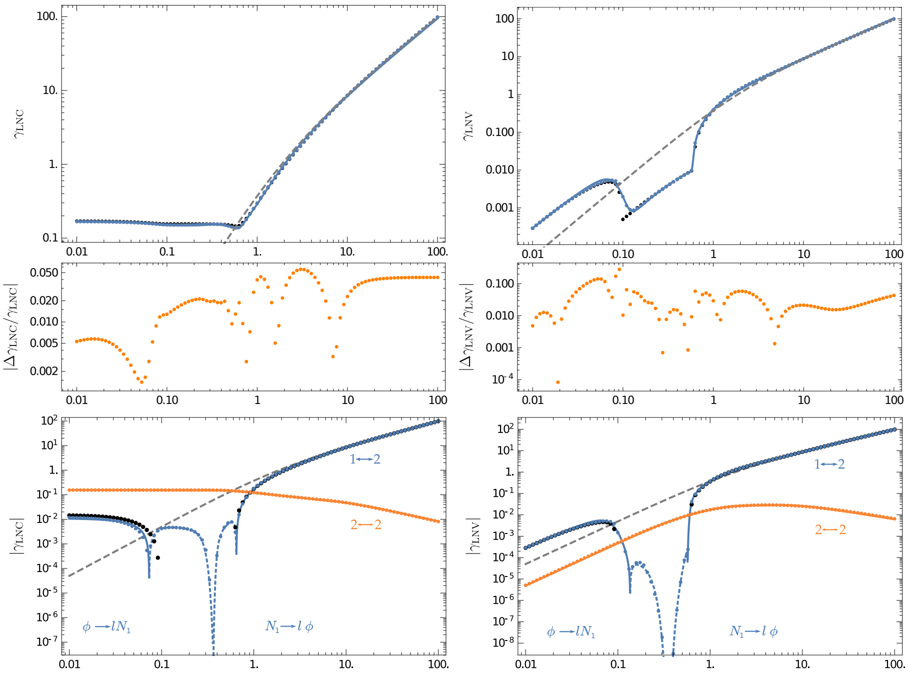

Figure 5.4: First line: Numerical results for $\gamma_{\rm LNC/LNV}(z)$ (blue dots) compared to the fit functions (5.88) (solid blue lines) and the result with simplified lepton propagators for the $1 \leftrightarrow 2$ processes (black dots). Second line: The relative deviations between the full numerical results and the approximate fit functions (orange dots). Third line: Absolute values of the individual $1 \leftrightarrow 2$ contributions (blue), $1 \leftrightarrow 2$ contributions with simplified lepton propagators (black), and $2 \leftrightarrow 2$ contributions (orange) for $\gamma_{\rm LNC/LNV}(z)$. Numerical results are shown as dots, while the fit functions are shown as solid lines (for positive contributions) or dashed lines (for negative contributions, generated by hole contributions). In all plots, dashed gray lines show the nonrelativistic $1 \leftrightarrow 2$ result $\gamma_{\rm LNC/LNV} \approx z K_1(z)/K_2(z)$.

- For $z \approx 0.4$, one has $m_\phi \approx M_1$, so that both $N_1 \leftrightarrow \ell\phi$ and $\phi \leftrightarrow \ell N_1$ processes are kinematically forbidden, and neither can contribute to $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$.

- For $0.4 \lesssim z \lesssim 0.7$, one has $m_\ell + m_\phi > M_1 > m_\phi$, so that $N_1 \leftrightarrow \ell\phi$ processes with hole-mode SM leptons are now allowed, while Higgs decays are completely forbidden.

- Finally, for $0.7 \lesssim z$, one has $M_1 > m_\ell + m_\phi$, so that $N_1 \leftrightarrow \ell\phi$

decay processes with both on-shell pseudo-particle and hole-mode SM leptons are allowed. At the same time, the hole modes start to decouple, since they would be created with hard momenta $|\boldsymbol{p}| \sim M_1 \gtrsim T$.

Of course, this is only an approximate picture. Using the full dispersion relations $\omega_{p,h}$ instead of the simplified relations (5.81), one obtains significant corrections in regions close to the kinematic thresholds at $z \approx 0.1$, $z \approx 0.4$, and $z \approx 0.7$. For instance, at $z = 0.1$, the kinetimatic blocking of pseudo-particle processes is softened, with some contributions bleeding over into the kinematically forbidden regime.

Generically, one may expect the hole-mode $1 \leftrightarrow 2$ contributions to become important in the regime from $z \approx 0.4$ to $z \approx 0.7$, where pseudo-particle $1 \leftrightarrow 2$ contributions are blocked. However, in this regime, $2 \leftrightarrow 2$ scattering contributions dominate both γ_{LNC} and γ_{LNV}, so that hole-modes are negligible for both γ_{LNC} and γ_{LNV}.

5.4.3 Comparison with the Nonrelativistic Approximation

As a consistency check, I compare the results obtained in this section with the nonrelativistic fluid equations used in [168, 151], which are expected to be valid for leptogenesis scenarios with strong washouts and negligible spectator effects, where the final $B-L$ asymmetry is generated at late times.

In this setup, the sterile neutrino Yukawa interactions are helicity blind, so that there is only a single independent sterile neutrino yield $Y_{N_1} = Y_{N_1+} + Y_{N_1-}$. The time-evolution of this yield is driven by a single equilibration rate Γ^{NR}, and there is no counterpart for the helicity odd rate $\tilde{\Gamma}$. Finally, the CP violating parameter ϵ_0 receives no finite temperature corrections.

To reproduce this nonrelativistic description, I neglect spectator interactions and assume $K \gg 1$. Since the washout rapidly equilibrates at early times, this enforces the initial conditions $Y_{N_1\mathrm{odd}} = Y_{B-L} = 0$ until washout processes start to freeze out at around $z \gtrsim 1$. In this limit, the interaction rates $\gamma_{\mathrm{LNC/LNV}}$ are determined by expression (5.74b),

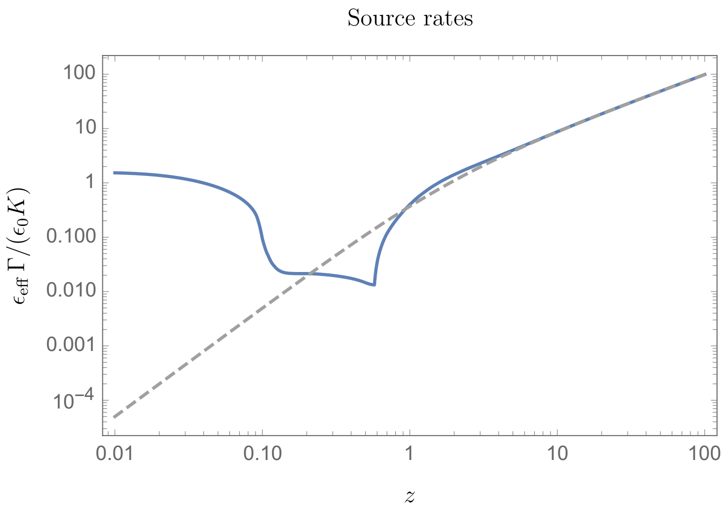

Figure 5.5: Comparison between the finite temperature source rate and its nonrelativistic approximation. Noitce the large enhancement for $z < 0.1$.

which gives

$$\Gamma^{\rm NR} = z\frac{K_1(z)}{K_2(z)} \approx z\left(1 - \frac{3}{2z}\right) \;, \qquad\qquad \tilde{\Gamma}^{\rm NR} = 0 \;. \tag{5.89}$$

As it should, $\tilde{\Gamma}^{\rm NR}$ vanishes, and the expression obtained for $\Gamma^{\rm NR}$ is identical to the one used in [168]. However, for the washout rate $\eta_{N_1}\Gamma^{\rm NR}$, one has

$$\eta_{N_1}\Gamma^{\rm NR} \approx K\frac{6}{\pi^2}z^3 K_1(z) \approx K\frac{6}{\pi^2}z^2\sqrt{\frac{\pi}{2}z}\,e^{-z} \;. \tag{5.90}$$

Comparing this expression with the result in [168], one finds a relative prefactor of ${}^{12}\!/\!{}_{\pi^2}$. However, using the second expression in (5.90), I recover the expression used for the washout term in [151]. The derivation of the washout rate in [168] seems to be based on the invalid assumption that the washout rate $\eta_{N_1}\Gamma$ is related to the equilibration rate Γ via the ratio of sterile neutrino and SM lepton number densities in the early universe, giving $\eta_{N_1} = {n_\ell^\pm}/{n_{N_1}^\pm}$. This relation is invalid, since the washout is not proportional to the SM lepton charge asymmetry $(n_\ell^+ - n_\ell^-)$ rather than the total SM lepton number density $(n_\ell^+ + n_\ell^-)$.

Putting everything together, one finds the nonrelativistic fluid equations

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{even}} = -\Gamma^{\mathrm{NR}}\big(Y_{N_1\mathrm{even}} - Y_{N_1\mathrm{eq}}\big)\ , \tag{5.91a}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{B-L} = -\epsilon_{\mathrm{eff}}^{\mathrm{NR}}\Gamma^{\mathrm{NR}}\big(Y_{N_1\mathrm{even}} - Y_{N_1\mathrm{eq}}\big) \\ + \eta_{N_1}\Gamma^{\mathrm{NR}}\big(Y_{l_\parallel} + \frac{1}{2}Y_\phi\big)\ .\end{aligned} \tag{5.91b}$$

Finally, consider the effective finite temperature decay asymmetry ϵ_{eff}. Using the nonrelativisitic expressions (5.74b), one has

$$\epsilon_{\mathrm{eff}}^{\mathrm{NR}}(z) \approx \epsilon_0 \frac{K_1(z)(zK_1(z) + 3K_2(z))}{zK_2(z)^2} \approx \epsilon_0\left(1 + \frac{3}{2z^2}\right)\ . \tag{5.92}$$

Figure 5.5 compares the full expression $\epsilon_{\mathrm{eff}}\Gamma$ with the nonrelativistic approximation $\epsilon_{\mathrm{eff}}^{\mathrm{NR}}\Gamma^{\mathrm{NR}}$. As mentioned in the previous section, $\phi \leftrightarrow \ell N_1$ Higgs decays strongly enhance the source term for $z < 0.1$. As will be shown in the next section, this enhancement is of critical importance if a significant fraction of the final $B - L$ asymmetry is produced at early times.

5.5 Implications for the Final $B-L$ Asymmetry

In this section, I use the fully relativistic fluid equations (5.71) in order to perform a parameter scan of the final $B-L$ asymmetry $Y_{B-L}(z \to \infty)$. To study the impact of finite temperature and spectator effects, I consider two specific setups:

1. I isolate the impact of finite temperature effects by considering a toy scenario without spectator interactions. Assuming that all SM yields vanish initially, this implies $Y_{B-L} = -g_w Y_{\ell_\parallel}$ and $Y_\phi = 0$, so that the fluid equations (5.71) form a closed system that depends only on the two free parameters ϵ_0 and K. With this setup, I compare the final $B-L$ asymmetry obtained using the fully relativistic fluid equations (5.71) with the final $B-L$ asymmetry obtained using the nonrelativistic equations (5.91).

2. I consider a realistic scenario for sterile neutrinos with a mass $\tilde{M}_1 \sim 10^{13}$ GeV, where b-Yukawa and weak sphaleron interactions have to be treated as partially equilibrated [151]. In this scenario, the final $B-L$ asymmetry depends on ϵ_0, K, and also $\tilde{M}_1$. In order to study the impact of properly account for the partial equilibration of spectator interaction, I compare the $B-L$ asymmetry obtained in this realistic setup with the $B-L$ asymmetry obtained in a simplified setup, where the b-Yukawa and weak sphaleron interactions are approximated as being fully equilibrated.

In both cases, I restrict myself to scenarios in which both Y_{B-L} and $Y_{N_1\text{odd}}$ vanish at initial time z_0, $Y_{B-L}(z_0) = Y_{N_1\text{odd}}(z_0) = 0$.

5.5.1 Scenario without Spectators

First, I consider the toy scenario without spectator effects. In this case, finite temperature effects are expected to be important in the weak washout regime $K \ll 1$ with a vanishing initial abundance of sterile neutrinos, where a significant fraction of the final $B-L$ asymmetry can be produced at early times. In full consistency with the standard lore of high-scale leptogenesis, finite temperature corrections are expected to be irrelevant in the strong washout regime $K \gg 1$, since there are no spectators that could protect an initial $B-L$ asymmetry from being washed out. That being said, a proper accounting of the missing factor of $^{12}/_{\pi^2}$ for the washout rate in the minimal description of [168] is expected to yield order one corrections to the final $B-L$ asymmetry in the strong washout regime.

Since the fluid equations (5.71) are linear, they can be solved using a time-ordered matrix exponential. Using $Y_{N_1\text{odd}}(z_0) = Y_{B-L}(z_0) = 0$, one obtains the general solution

$$\begin{aligned} Y_{N_1\text{even}}(z) &= e^{-I(z,z_0)} Y_{N_1\text{even}}(z_0) \\ &\qquad + \int_{z_0}^{z} \mathrm{d}z' e^{-I(z,z')} \Gamma(z') Y_{N_1\text{eq}}(z') \ , \end{aligned} \tag{5.93a}$$

$$\begin{aligned} Y(z) = \int\limits_{z_0}^{z} \mathrm{d}z' \mathcal{T} e^{\int_{z'}^{z} \mathrm{d}z'' W(z'')} \\ \times S(z') \big(Y_{N_1\text{even}}(z') - Y_{N_1,\text{eq}}(z') \big) \ , \end{aligned} \tag{5.93b}$$

where the exponential $\mathcal{T} e^{\int \mathrm{d}z'' W(z'')}$ captures the washout processes, and

$$Y \equiv \begin{pmatrix} Y_{N_1\text{odd}} \\ Y_{B-L} \end{pmatrix} \ , \quad S \equiv \begin{pmatrix} 0 \\ -\epsilon_{\text{eff}} \Gamma \end{pmatrix} \ , \quad W \equiv \begin{pmatrix} -\Gamma & \frac{\eta_{N_1}}{2} \tilde{\Gamma} \\ \tilde{\Gamma} & -\frac{\eta_{N_1}}{2} \Gamma \end{pmatrix} \ . \tag{5.94a}$$

The remaining integral factor is

$$I(z, z') \equiv \int_{z'}^{z} \mathrm{d}z'' \Gamma(z'') \ . \tag{5.94b}$$

An analogous solution to the non-relativistic fluid equations (5.91a)–(5.91b) is obtained by taking $\Gamma \to \Gamma^{\text{NR}}$, $\tilde{\Gamma} \to 0$, and $\epsilon_{\text{eff}} \to \epsilon_0$. Since the sterile neutrino yield has to be helicity-symmetric in the nonrelativistic limit, one also has $Y_{N_1\text{odd}} = 0$ and $Y_{N_1\text{even}} = Y_{N_1+} = Y_{N_1-} \equiv Y_{N_1}$.

The final $B-L$ asymmetry $Y_{B-L}(z \to \infty)$ depends on the two free parameters ϵ_0 and K. Following [219], I use the parametrization

$$Y_{B-L}(z \to \infty) = -\epsilon_0 \ Y_{N_1,\text{eq}}(z_0) \cdot \kappa_f \ , \qquad \kappa_f \equiv \kappa(z \to \infty) \ . \tag{5.95}$$

The efficiency factor

$$\kappa(z) \equiv -\int\limits_{z_0}^{z} \mathrm{d}z' \left[\mathcal{T} e^{\int_{z'}^{z} \mathrm{d}z'' W(z'')} \right]_{22} \frac{\epsilon_{\text{eff}}(z')}{\epsilon_0} \frac{\mathrm{d}}{\mathrm{d}z'} \frac{Y_{N_1\text{even}}(z')}{Y_{N_1,\text{eq}}(z_0)} \tag{5.96}$$

captures the impact of finite washouts $K \neq 0$ and general initial conditions $Y_{N_1,\text{even}}(z_0) \neq Y_{N_1,\text{eq}}(z_0)$. It is independent of ϵ_0 and normalized such that one obtains $\kappa_f = 1 + \mathcal{O}(K)$ for initially equilibrated sterile neutrinos in the nonrelativistic approximation. Explicitly,

$$\kappa^{\text{nr}}(z) \equiv -\int\limits_{z_0}^{z} \mathrm{d}z' e^{-1/2\int_{z'}^{z}\mathrm{d}z''\eta_{N_1}(z'')\Gamma(z'')} \frac{\mathrm{d}}{\mathrm{d}z'} \frac{Y_{N_1}(z')}{Y_{N_1,\text{eq}}(z_0)} \ . \tag{5.97}$$

Analytic Approximations

In the two limiting cases $K \ll 1$ and $K \gg 1$, it is possible to find analytic approximations for both κ_f^{nr} and κ_f.

First, I re-derive the standard semianalytical approximations used for the nonrelativistic efficiency factor κ_f^{nr}. Accounting for the missing prefactor $^{12}/_{\pi^2}$ in the washout rate, this is expected to yield order one corrections compared to the estimates obtained in [168]. Although κ_f^{nr} is only accurate for $K \gg 1$, I also re-derive the $K \ll 1$ expression in order to compare the κ_f with a completely accurate estimate of the nonrelativistic predictions.

Following [168], I decompose κ_f^{nr} into separate contributions generated from sterile neutrino production and decays,

$$\kappa_f^{\text{nr}} = \kappa_+^{\text{nr}} + \kappa_-^{\text{nr}} \ , \qquad\qquad \kappa_-^{\text{nr}} = \kappa_f^{\text{nr}}(z_{\text{eq}}) \ , \tag{5.98}$$

where z_{eq} is defined to be the value of z at which the sterile neutrinos become overabundant for vanishing initial conditions, $Y_{N_1}(z_{\text{eq}}) = Y_{N_1\text{eq}}(z_{\text{eq}})$. Using the nonrelativistic rate Γ^{NR}, this condition becomes

$$(z_{\text{eq}})^2 K_2(z_{\text{eq}}) \stackrel{!}{=} K \int_0^{z_{\text{eq}}} dz' z'^3 K_1(z'). \tag{5.99}$$

In the strong washout regime, the size equilibration rate $\Gamma \propto K$ ensures that equilibration occurs at very early times, so that one may set $z_{\text{eq}} \to 0$. Using that $Y_{N_1} \approx Y_{N_1,\text{eq}}$, the nonrelativistic efficiency factor (5.97) becomes

$$\begin{aligned} \kappa^{\text{nr}}(z) &\approx \frac{1}{2}\int_0^z dz' z'^2 K_1(z') e^{-3K/\pi^2 \int_{z'}^{z} dz'' z''^3 K_1(z'')} \\ &\equiv \frac{1}{2}\int_0^z dz' e^{-\mathcal{I}(z')} \ . \end{aligned} \tag{5.100}$$

As in [168], this integral can be evaluated using a saddle point approximation. This gives

$$\kappa^{\rm nr}(z) \approx \frac{\pi^2}{6K\bar{z}}\left(1 - e^{-3K/\pi^2 \int_{z'}^{z} dz'' z''^3 K_1(z'')}\right) , \tag{5.101}$$

where the saddle point $\bar{z}$ has to satisfy the condition [168]

$$\frac{3}{\pi^2}\bar{z}^3 K_1(\bar{z}) + \frac{3}{\bar{z}} - \frac{K_2(\bar{z})}{K_1(\bar{z})} = 0 . \tag{5.102}$$

At the end of the day, one obtains the final efficiency factor

$$\kappa_f^{\rm nr} \approx \kappa_+^{\rm nr} \approx \frac{\pi^2}{6K\bar{z}} \qquad (K \gg 1) . \tag{5.103}$$

In the weak washout regime, one has to distinguish between nonvanishing and vanishing initial conditions for Y_{N_1}. For nonvanishing initial conditions, the dominant contribution to $\kappa_f^{\rm nr}$ is

$$\kappa_f^{\rm nr} \approx \frac{Y_{N_1 \rm even}(z_0)}{Y_{N_1,\rm eq}(z_0)} + O(K) . \tag{5.104}$$

For vanishing initial conditions $Y_{N_1 \rm even}(z_0) = 0$, the estimate is more involved. Following [168], I first consider the contribution $\kappa_-^{\rm nr}$. Since sterile neutrino production is suppressed by factors of K, the small but finite sterile neutrino yield on the right-hand side of the fluid equation (5.91a) is negligible compared to the contribution generated by the equilibrium yield $Y_{N_1 \rm eq}$. Using expression (5.61b) for $Y_{N_1 \rm eq}$, one finds

$$\frac{Y_{N_1}(z)}{Y_{N_1 \rm eq}(z_0)} \approx \frac{1}{2}\int_0^z dz' \Gamma(z') z'^2 K_2(z'), \quad z < z_{\rm eq} . \tag{5.105}$$

Combining this expression with expression (5.66) for η_{N_1}, one obtains

$$\kappa_-^{0,\rm nr} \approx -\frac{1}{2}\int_0^{z_{\rm eq}} \Gamma(z') z'^2 K_2(z') e^{-3/\pi^2 \int_{z'}^{z} dz'' \Gamma(z'') z''^2 K_2(z'')} \tag{5.106a}$$

$$\approx -\frac{\pi^2}{6}\left(1 - e^{-6/\pi^2 Y_{N_1}(z_{\rm eq})/Y_{N_1 \rm eq}(z_0)}\right) . \tag{5.106b}$$

Since the equilibration time is large, $z_{\rm eq} \gg 1$, that the washout rate can be ignored when estimating $\kappa_+^{\rm nr}$. Hence,

$$\kappa_+^{0,\rm nr}(z) \approx \frac{1}{Y_{N_1 \rm eq}(z_0)} (Y_{N_1}(z) - Y_{N_1}(z_{\rm eq})) \qquad (z > z_{\rm eq}) \ , \tag{5.107}$$

Thus, the final efficiency factor becomes

$$\begin{aligned} \kappa_f^{0,\rm nr} &\approx \frac{Y_{N_1}(z_{\rm eq})}{Y_{N_1 \rm eq}(z_0)} - \frac{\pi^2}{6}\left(1 - e^{-6/\pi^2 Y_{N_1}(z_{\rm eq})/Y_{N_1 \rm eq}(z_0)}\right) \\ &\approx \frac{3}{\pi^2}\left(\frac{Y_{N_1}(z_{\rm eq})}{Y_{N_1 \rm eq}(z_0)}\right)^2 \approx 1.65\, K^2 \ , \end{aligned} \tag{5.108}$$

where $z_{\rm eq}$ has been determined numerically, giving $Y_{N_1}(z_{\rm eq})/Y_{N_1 \rm eq}(z_0) \approx 2.33K$ for $K \ll 1$.

Now, I derive a semianalytical approximation of the fully relativistic efficiency factor κ_f in the weak washout regime $K \ll 1$. I do not consider the strong washout regime, since finite temperature corrections are expected to be negligible for $K \gg 1$.

First, consider the formal limit $K \to 0$. For a finite initial abundance $Y_{N_1 \rm even}(z_0)$, one obtains the same prediction for κ_f as in the nonrelativistic case,

$$\kappa_f \approx \frac{Y_{N_1 \rm even}(z_0)}{Y_{N_1,\rm eq}(z_0)} + O(K) \ . \tag{5.109}$$

To obtain a finite $B - L$ asymmetry $Y_{N_1 \rm even} = 0$, one has to consider washout parameters $K \neq 0$. Neglecting the washout matrix W, one obtains

$$\kappa(z) \approx -\frac{Y_{N_1 \rm even}(z)}{Y_{N_1,\rm eq}(z_0)} - \int_{z_0}^{z} \mathrm{d}z' \left(\frac{\epsilon_{\rm eff}(z')}{\epsilon_0} - 1\right) \frac{\mathrm{d}}{\mathrm{d}z'} \frac{Y_{N_1 \rm even}(z')}{Y_{N_1,\rm eq}(z_0)} \ . \tag{5.110}$$

The integrand in the second term is dominated by values of $z' \lesssim 1$, since $\epsilon_{\rm eff} \to \epsilon_0$ as finite temperature corrections become negligible for $z' \gtrsim 1$. For such small values of z, ${}^{\mathrm{d}Y_{N_1 \rm even}(z')}/{}_{\mathrm{d}z'}$ can be approximated using a formal expansion in powers of $\Gamma(z')$. To do so, I use the fluid equation (**??**) and reinsert the formal solution (5.93a) for $Y_{N_1\mathrm{even}}(z')$ on the right-hand side. Expanding around $\Gamma(z')=0$, this gives

$$\frac{\mathrm{d}}{\mathrm{d}z'}Y_{N_1\mathrm{even}}(z') \approx \Gamma(z')Y_{N_1\mathrm{eq}}(z') + \mathcal{O}(\Gamma^2)\ . \tag{5.111}$$

Inserting this expression into Eq. (5.110), one finds

$$\kappa(z) \approx -\frac{Y_{N_1\mathrm{even}}(z)}{Y_{N_1,\mathrm{eq}}(z_0)} - \int\limits_{z_0}^{z}\mathrm{d}z' \left(\frac{\epsilon_{\mathrm{eff}}(z')}{\epsilon_0} - 1\right)\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)}\ . \tag{5.112}$$

Taking the limit $z\to\infty$, the first term vanished, and this is expression becomes strictly proportional to K. The constant of proportionality can be estimated using the numerical results I have obtained for γ_{LNC}, γ_{LNV}, and $Y_{N_1\mathrm{eq}}(z')$. Choosing $z_0 = 0.01$, one has

$$\begin{aligned}\kappa_f^0 &\equiv \kappa_f|_{Y_{N_1}(z_0)=0}\\ &\approx -\int\limits_{z_0}^{\infty}\mathrm{d}z\ \Gamma(z)\frac{Y_{N_1,\mathrm{eq}}(z)}{Y_{N_1,\mathrm{eq}}(z_0)}\left(\frac{\epsilon_{\mathrm{eff}}(z)}{\epsilon_0} - 1\right) \approx -0.32\,K.\end{aligned} \tag{5.113}$$

This choice for z_0 coincides with the initial value of z that I use for the numerical integration of the fluid equations. That being said, the dependence of the integral on z_0 is very mild. For instance, choosing $z_0 = 0$, the constant of proportionality increases by about 4%.

The result (5.113) is qualitatively different from its nonrelativistic estimate (5.108). In the nonrelativistic fluid equations, the sterile neutrino decay asymmetry ϵ_0 is temperature independent. For vanishing initial conditions, the $B-L$ asymmetry created by the production of sterile neutrinos at early times exactly equal to the opposite-sign asymmetry created by the subsequent sterile neutrino decays. Neglecting the washout, one would have obtained a vanishing efficiency factor $\kappa_f^{\mathrm{nr}} = 0$, and with a small but finite washout, one obtains a small but positive $\kappa_f^{\mathrm{nr}} > 0$. This final efficiency factor scales as K^2, with one factor of K resulting from $\mathrm{d}Y_{N_{\mathrm{even}}}/\mathrm{d}z \propto K$ and another factor of K associated with the all-important washout processes.

In contrast, the fully relativistic efficiency factor (5.113) is not the result of washout processes. Rather, the finite temperature corrections for $\epsilon_{\text{eff}} > \epsilon_0$ cause the asymmetry sourced by the initial production of sterile neutrinos to be much larger than the opposite-sign asymmetry generated by their decay. This yields a final efficiency factor of negative sign, $\kappa_f < 0$. Since $\epsilon_{\text{eff}}/\epsilon_0$ is independent of K, the final efficiency factor now scales as $\kappa_f \propto K$, where the sole factor of K is associated with the sterile neutrino equilibration rate $\mathrm{d}Y_{N_{\text{even}}}/\mathrm{d}z \propto K$.

Parameter Scan

Finally, I evaluate κ_f for general $K \sim \mathcal{O}(1)$ by integrating the fluid equations (5.71) numerically. In practice, I evaluate $\kappa(z)$ for some $z > z_f$, where $T_f = M_1/z_f$ is the is the so-called "freeze-out" temperature, below which the $B - L$ asymmetry becomes approximately constant. In the strong washout regime, I use the nonrelativistic result [168]

$$z_f \approx 1 + \frac{1}{2}\log\left(1 + \frac{\pi K^2}{1024}\log^5\left(\frac{3125\pi K^2}{1024}\right)\right) \lesssim 10^2 \ . \tag{5.114}$$

In the weak washout regime, freeze-out occurs for $z_f \gg 1$ due to the long sterile neutrino lifetime $\tau \sim 1/T{\cdot}K$, the estimate (5.114) is no longer valid. Working with a chosen relative accuracy $r \ll 1$, I define the freeze-out time z_f to be the smallest z that fulfills the condition

$$|\delta(z_f)| < r \ , \qquad \delta(z) \equiv \frac{\kappa_f - \kappa(z)}{\kappa_f} \ . \tag{5.115}$$

My goal is to find an upper bound for the order of magnitude of z_f in the weak washout regime. Neglecting the washout exponential in expression (5.96), one obtains

$$\delta(z) \approx \frac{1}{\kappa_f}\left[\frac{Y_{N_1\text{even}}(z)}{Y_{N_1,\text{eq}}(z_0)} - \int_z^\infty \mathrm{d}z' \left(\frac{\epsilon_{\text{eff}}(z')}{\epsilon_0} - 1\right)\frac{\mathrm{d}}{\mathrm{d}z'}\frac{Y_{N_1\text{even}}(z')}{Y_{N_1,\text{eq}}(z_0)}\right] \ . \tag{5.116}$$

Inserting solution (5.93a) for $Y_{N_1\mathrm{even}}(z)$ and expanding the resulting expression to first order in K, this gives

$$\begin{aligned}\delta(z) \approx \frac{1}{\kappa_f}\Bigg[& e^{-I(z,z_0)}\frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} + \int_{z_0}^{z} \mathrm{d}z' e^{-I(z,z')}\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} \\ & - \int_{z}^{\infty} \mathrm{d}z' \Big(\frac{\epsilon_{\mathrm{eff}}(z')}{\epsilon_0} - 1\Big)\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} \\ & + \int_{z}^{\infty} \mathrm{d}z' \Big(\frac{\epsilon_{\mathrm{eff}}(z')}{\epsilon_0} - 1\Big)\Gamma(z') e^{-I(z',z_0)}\frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)}\Bigg] \ .\end{aligned} \tag{5.117}$$

In this expression, the exponential suppression factors $I(z, z_0)$ have to be included since they regulate the z' integrals for $z \to \infty$. Using that $z_f \gg 1$ in the weak washout regime, I approximate the suppression factor as

$$I(z, z_0) = \int_{z_0}^{z} \mathrm{d}z'\Gamma(z') = \frac{1}{2}Kz^2\Big(1 + \mathcal{O}(1/z)\Big) \ . \tag{5.118}$$

To proceed, I evaluate each term on the right-hand side of Eq. (5.117) individually. The first term is

$$\frac{1}{\kappa_f}e^{-I(z,z_0)}\frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} \approx \frac{1}{\kappa_f}e^{-\frac{1}{2}Kz^2}\frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} \ , \tag{5.119}$$

so that for nonvanishing initial conditions for the sterile neutrinos, one obtains $z_f \gtrsim 1/\sqrt{K} \gg 1$. To find an upper bound on z_f, it is sufficient to evaluate the remaining terms in this regime. Taking $z_f \gtrsim 1/\sqrt{K}$ in the second term, I split the corresponding integral into two regions with $z' < \tilde{z}$ and $\tilde{z} < z'$, with $\tilde{z}$ chosen such that $1 \ll \tilde{z} \ll 1/\sqrt{K}$. In the first region, one obtains the bound

$$\begin{aligned}\int_{z_0}^{\tilde{z}} \mathrm{d}z' e^{-I(z,z')}\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} &\lesssim e^{-\frac{1}{2}Kz^2}\int_{z_0}^{\tilde{z}} \mathrm{d}z'\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} \\ &\lesssim 2.65\, K\, e^{-\frac{1}{2}Kz^2} \ .\end{aligned} \tag{5.120}$$

Since $\tilde{z} \gg 1$, I use the nonrelativistic approximation to evaluate the integrand in the second region. With

$$\Big(\frac{\epsilon_{\mathrm{eff}}(z)}{\epsilon_0} - 1\Big)\Gamma(z) \approx \frac{3}{2z}K \ , \qquad \frac{Y_{N_1\mathrm{eq}}(z)}{Y_{N_1\mathrm{eq}}(0)} \approx \frac{1}{2}\sqrt{\frac{\pi}{2}} z^{3/2} e^{-z} \ , \tag{5.121}$$

I obtain

$$\begin{aligned} \int\limits_{\tilde{z}}^{z} \mathrm{d}z' e^{-I(z,z')}\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} &\approx \sqrt{\frac{\pi}{8}}K \int\limits_{\tilde{z}}^{z} \mathrm{d}z' \sqrt{z'} e^{-\frac{1}{2}K(z^2 - z'^2) - z'} \\ &\lesssim K\sqrt{\frac{\pi z}{8}} e^{-\frac{1}{2}Kz^2} \ . \end{aligned} \tag{5.122}$$

I also use the approximations (5.118) and (5.121) to estimate the integrals in the third and fourth term. This gives

$$\begin{aligned} &\int\limits_{z}^{\infty} \mathrm{d}z' \ \Big(\frac{\epsilon_{\mathrm{eff}}(z')}{\epsilon_0} - 1\Big)\Gamma(z')\frac{Y_{N_1\mathrm{eq}}(z')}{Y_{N_1,\mathrm{eq}}(z_0)} \\ \approx\ & \frac{3}{4}K\sqrt{\frac{\pi}{2}} \int\limits_{z}^{\infty} \mathrm{d}z' \ \sqrt{z'} e^{-z'} \approx \frac{3}{4}K\sqrt{\frac{\pi z}{2}} e^{-z} \end{aligned} \tag{5.123a}$$

and

$$\begin{aligned} &\int\limits_{z}^{\infty} \mathrm{d}z' \ \Big(\frac{\epsilon_{\mathrm{eff}}(z')}{\epsilon_0} - 1\Big)\Gamma(z') e^{-I(z',z_0)} \frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} \\ \approx\ & \frac{3}{2}K \int\limits_{z}^{\infty} \mathrm{d}z' \ \frac{1}{z'} e^{-\frac{1}{2}Kz'^2} \frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} \\ \approx\ & \frac{3e^{-\frac{1}{2}Kz^2}}{2z^2} \frac{Y_{N_1\mathrm{even}}(z_0)}{Y_{N_1,\mathrm{eq}}(z_0)} \ . \end{aligned} \tag{5.123b}$$

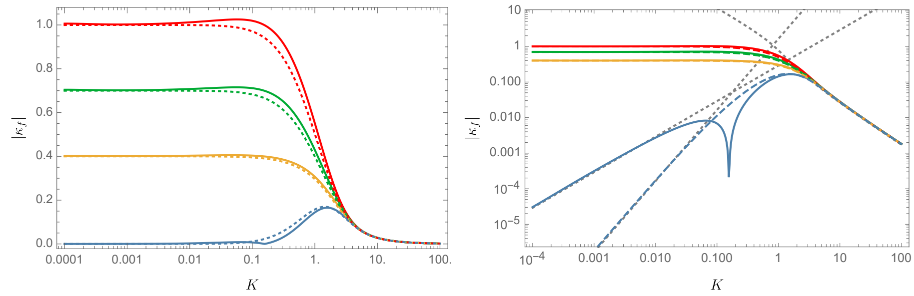

Figure 5.6: Numerical scan of κ_f vs K for various initial conditions, comparing the nonrelativistic approximation (coloured dashed lines) with the fully relativistic result (solid lines). Both plots show the same data, but either with single logarithmic axes (left) or double logarithmic axes (right). The initial conditions at $z = 0.01$ are $Y_{N_1}/Y_{N_1\text{eq}} = 1$ (red), 0.7 (green), 0.4 (orange) and 0 (blue). On the right, the dashed gray lines correspond to the weak washout estimate in Eqs. (5.113), (5.108) and the strong washout estimate of Eqs. (5.103), (5.102).

Substituting the estimates (5.119), (5.120) , (5.122), (5.123b) and (5.123a) into expression (5.117) for $\delta(z)$, one finds

$$|\delta(z)| \lesssim \frac{1}{\kappa_f}\left[\frac{Y_{N_1\text{even}}(z_0)}{Y_{N_1,\text{eq}}(z_0)} e^{-\frac{1}{2}Kz^2}\left(1+\frac{3}{2}\frac{1}{z^2}\right) + Ke^{-\frac{1}{2}Kz^2}\left(\sqrt{\frac{\pi z}{8}}+2.65\right)+\frac{3}{4}K\sqrt{\frac{\pi z}{2}}e^{-z}\right] \tag{5.124a}$$

$$\approx \frac{1}{\kappa_f}\left[\frac{Y_{N_1\text{even}}(z_0)}{Y_{N_1,\text{eq}}(z_0)} e^{-\frac{1}{2}Kz^2} + \frac{1}{2}K\sqrt{\frac{\pi z}{2}}e^{-\frac{1}{2}Kz^2}\right], \tag{5.124b}$$

For nonvanishing initial conditions and $r \approx 10^{-2}$, only the first term is relevant, and one obtains $z_f^2 \lesssim {}^{10}/_{K}$. For vanishing initial conditions and $r \approx 10^{-2}$, one has $z_f \lesssim {}^{1}/_{K}$.

Figure 5.6 shows the final $B - L$ asymmetry obtained for various initial conditions and values of K between 10^{-4} and 10, with the fluid equations integrated up to $z \approx 400$. Evaluating $\delta(z)$ at the lower bound $K \approx 10^{-4}$, one finds that numerical deviations associated with the finite value of z are expected to be less than $|\delta(z = 400)| \lesssim 3 \cdot 10^{-4}$ for nonvanishing initial conditions and $|\delta(z = 400)| \lesssim 1.3 \cdot 10^{-2}$ for vanishing initial conditions.

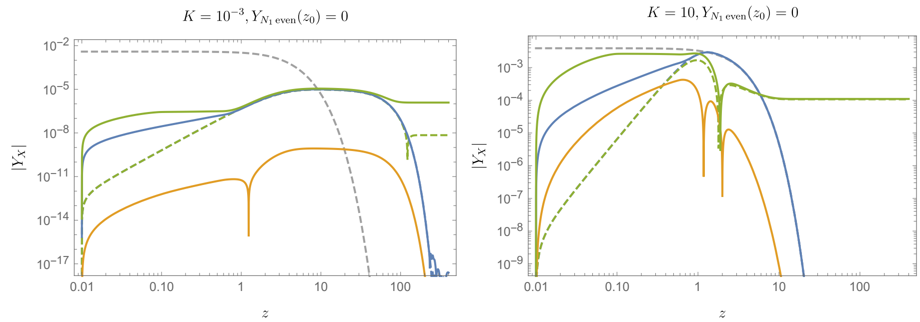

Figure 5.7: Time evolution of appropriately rescaled Yields $|Y_{N_1\mathrm{even}}(z)|$ (blue), $|Y_{B-L}(z)/\epsilon_0|$ (green), and $|Y_{N_1\mathrm{odd}}(z)/\epsilon_0|$ (orange) for vanishing initial conditions and washout parameter $K = 0.001$ (left) and $K = 10$ (right). Solid lines show fully relativistic solutions, and dashed lines show solutions obtained using the nonrelativistic fluid equations. The dashed gray line denotes the equilibrium yield $Y_{N_1\mathrm{eq}}$.

First, consider the weak washout regime $K \ll 1$. As expected, relativistic corrections are irrelevant for nonvanishing initial conditions $Y_{N_1\mathrm{even}}(z_0) \neq 0$, and the nonrelativistic estimate (5.109) accurately predicts the final $B - L$ asymmetry. Relativistic corrections become important for vanishing initial conditions $Y_{N_1\mathrm{even}}(z_0) = 0$, causing a sign flip for washouts below $K \approx 0.2$. Furthermore, the semianalytic estimate (5.113) accurately predicts the final $B - L$ asymmetry for $K < 0.01$. In the intermediate regime $0.1 < K < 1$, relativistic corrections mildly enhance the final $B - L$ asymmetry by a factor of less than 10%. Finally, relativistic corrections are negligible in the strong washout regime $1 < K$, with the final $B - L$ asymmetry being entirely consistent with the nonrelativistic estimate (**??**).

Figure 5.7 illustrates the enhancement of the final $B - L$ asymmetry in the weak washout regime, showing the time-evolution of the appropriately rescaled yields Y_{B-L}, $Y_{N_1\mathrm{even}}$, and $Y_{N_1\mathrm{odd}}$ for vanishing initial conditions. With such initial conditions, the thermally enhanced source term always $\epsilon_{\mathrm{eff}}\Gamma$ produces large initial asymmetries. For small values of K (left plot), this large initial asymmetry survives throughout leptogenesis, and it easily exceeds the opposite-sign asymmetry produced after the sterile neutrino become overabundant. In contrast, the initial asymmetry is washed out for larger values of K (right plot), so that the opposite-sign late time asymmetry dominates the final $B - L$ yield.

5.5.2 Scenario with Partially Equilibrated Spectators

Next, consider the realistic scenario for sterile neutrinos with masses $\tilde{M}_1 \sim 10^{13}\,\mathrm{GeV}$. In the critical temperature range $T \lesssim M_1$, SM b-Yukawa and weak sphaleron interactions are partially equilibrated [151]. SM gauge, t-Yukawa, and strong sphaleron interactions are fully equilibrated, while the remaining SM Yukawa interactions are negligible.

Fluid Equations with Partially Equilibrated Spectators

To account for both fully and partially equilibrated spectator interactions, one has to keep track of the individual SM quark and lepton yields. In the electroweak sector, the lefthanded flavour combination $\ell_\parallel$ coupling to the sterile neutrinos is augmented by two lefthanded perpendicular flavour combinations $\ell_{\perp i}$ that do not couple to the sterile neutrinos directly. Since the SM lepton Yukawa interactions are negligible, both of these flavour combinations share the same yield $Y_{\ell_\perp} \equiv Y_{\ell_\perp 1} = Y_{\ell_\perp 2}$. In the strong sector, one has to keep track of the three lefthanded quark doubles Q_i as well as six righthanded quark singlets u_i and d_i with $i = 1, 2, 3$. Since first and second generation SM quark Yukawa interactions are negligible, the associated quark flavours share common yields $Y_Q \equiv Y_{Q_i}$ and $Y_d \equiv Y_{u_i} = Y_{d_i}$ with $i = 1, 2$. I denote the distinct yields for third quark generation as Y_{Q_3}, $Y_t = Y_{u_3}$, and $Y_b = Y_{d_3}$. For the description of partially equilibrated b-Yukawa interactions, it is also useful to consider the difference $Y_{\Delta\mathrm{down}} \equiv Y_b - Y_d$.

Partially equilibrated spectators promote some of the SM yields to independent dynamical variables. To capture the time-evolution of these yields, one has to augment the fluid equations (5.71) by adding one equation for each interaction that is partially equilibrated. In the case of partially equilibrated b-Yukawa and weak sphaleron interactions, I follow the approach used in [151], adding equations that determined the time-evolution of $Y_{\ell_\perp}$ and Y_{Δ_down}. This way, one obtains the complete set of fluid equations [151]

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{even}} = -\Gamma\left(Y_{N_1\mathrm{even}} - Y_{N_1\mathrm{eq}}\right)\,, \tag{5.125a}$$

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{N_1\mathrm{odd}} = -\Gamma\,Y_{N_1\mathrm{odd}} - \eta_{N_1}\tilde{\Gamma}\left(Y_{l_\parallel} + \frac{1}{2}Y_\phi\right)\,, \tag{5.125b}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{B-L} = \tilde{\Gamma}\,Y_{N_1\mathrm{odd}} - \epsilon_{\mathrm{eff}}\Gamma\left(Y_{N_1\mathrm{even}} - Y_{N_1\mathrm{eq}}\right)&\\ + \eta_{N_1}\Gamma\left(Y_{l_\parallel} + \frac{1}{2}Y_\phi\right)\,,&\end{aligned} \tag{5.125c}$$

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{\Delta_{\mathrm{down}}} = -\Gamma_b\big(Y_b - Y_{Q_3} + \frac{1}{2}Y_\phi\big)\,, \tag{5.125d}$$

$$\frac{\mathrm{d}}{\mathrm{dz}}\,Y_{l_\perp} = -\Gamma_{\mathrm{ws}}\big(9Y_{Q_3} + 18Y_Q + 3Y_{l_\parallel} + 6Y_{l_\perp}\big)\,. \tag{5.125e}$$

Here, the b-Yukawa and weak sphaleron equilibration rates are [220, 153, 151]

$$\Gamma_b \approx 1.0\cdot 10^{-2}\,\frac{h_b^2 T}{\tilde{M}_1}\,, \tag{5.126}$$

$$\Gamma_{\mathrm{ws}} \approx (8.24\pm 0.10)\left(\log\left(\frac{m_D}{g_2^2 T}\right) + 3.041\right)\frac{g_2^2 T^3}{2{m_D}^2\tilde{M}_1}\,\alpha_2^5\,, \tag{5.127}$$

where h_b is the bottom-Yukawa coupling, $\alpha_2 \equiv g_2^2/4\pi$ is the coupling strength of the electroweak $\mathrm{SU}(2)_L$ gauge interaction, and $m_D^2 \approx \frac{11}{6}g_2^2T^2$ is the Debye mass of the $\mathrm{SU}(2)_L$ gauge bosons.

By themselves, the fluid equations (5.125) do not form a closed system. Rather, one has to relate the five SM yields $Y_{l_\parallel}$, Y_{Q_3}, Y_b, Y_Q and Y_ϕ that appear on the right-hand side to the three SM yields Y_{B-L}, $Y_{\ell_\perp}$ and $Y_{\Delta_{\mathrm{down}}}$ that appear on the left-hand side. Following [151], I establish this relation using five constraints obtained from weak hypercharge conservation, full equilibration of t-Yukawa and strong sphaleron interactions, and the flavour-blind nature of both weak and strong sphaleron interactions. At the end of the day, this gives [151]

$$\begin{pmatrix} Y_{l_\parallel}\\ Y_{Q_3}\\ Y_b\\ Y_Q\\ Y_\phi \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 1 & 0\\ \frac{1}{23} & \frac{1}{2} & -\frac{10}{23}\\ \frac{1}{46} & \frac{1}{2} & \frac{18}{23}\\ -\frac{1}{46} & \frac{1}{2} & \frac{5}{23}\\ -\frac{7}{23} & 0 & \frac{24}{23} \end{pmatrix} \begin{pmatrix} Y_{B-L}\\ Y_{l_\perp}\\ Y_{\Delta_{\mathrm{down}}} \end{pmatrix}\,. \tag{5.128}$$

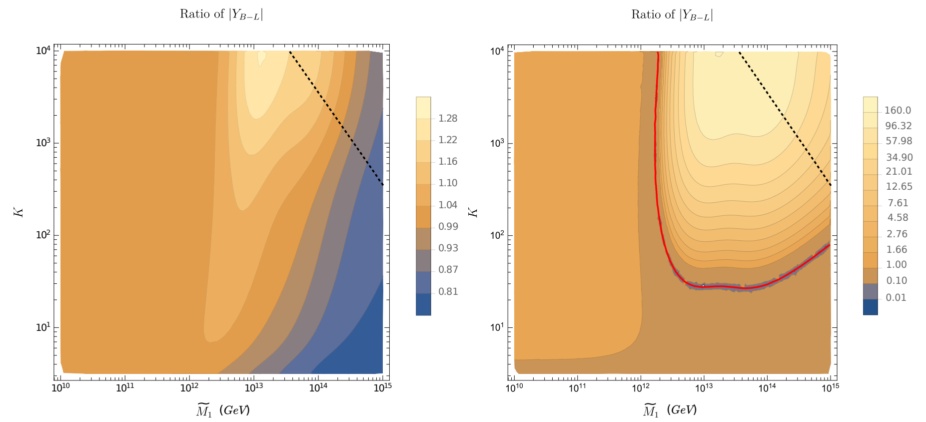

Figure 5.8: Ratios of final $B-L$ asymmetry obtained with and without including partially equilibrated bottom-Yukawa and weak sphalerons interactions. On the left: initially equilibrated sterile neutrinos; on the right: vanishing initial abundance of sterile neutrinos. The bright red line highlights the sign change of the final $B-L$ asymmetry and the dotted black lines indicates the region in which $|F_1|^2 > 4\pi$.

Finally, scenarios with fully equilibrated b-Yukawa and weak sphaleron interactions can be recovered by replacing the two SM fluid equations (5.125d) and (5.125e) with analogous chemical equilibrium constraints. This gives [151]

$$Y_{\ell_\parallel} = -\frac{13}{30} Y_{B-L} \ , \qquad\qquad Y_\phi = -\frac{1}{5} Y_{B-L} \ . \tag{5.129}$$

Using these relations, the BSM fluid equations (5.71) can be transformed into a closed system without having to keep track of any further SM yields.

Numerical Solutions

To estimate the impact of properly accounting for the partially equilibration of b-Yukawa and weak sphaleron interactions, I compare the final $B-L$ yield obtained by solving the full set of fluid equations (5.125) with the same yield obtained using relation (5.129).

Fig. 5.8 shows the ratio of these two $B-L$ asymmetries in dependence of the two free parameters $\tilde{M}_1$ and K, and for both $Y_{N_1\text{even}}(0) = Y_{N_1\text{eq}}(0)$ and $Y_{N_1\text{even}}(0) = 0$ initial conditions. Focusing on strong washouts, the

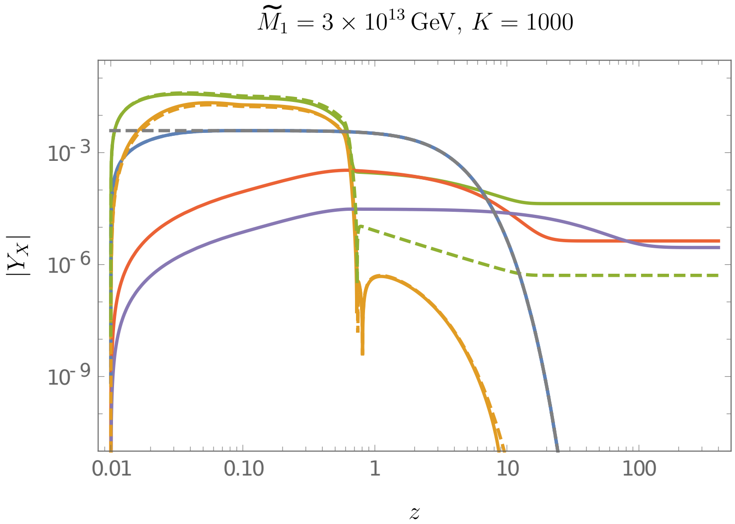

Figure 5.9: Time evolution of the yields obtained by solving the fluid equations with partially (solid lines) or fully equilibrated (dashed lines) spectators, for $\tilde{M}_1 = 3 \times 10^{13}$ GeV and $K = 1000$. The particular yields are $|Y_{N_1\text{even}}(z)|$ (blue), $|Y_{B-L}(z)/\epsilon_0|$ (green), $|Y_{N_1\text{odd}}(z)/\epsilon_0|$ (orange), $|Y_{\Delta_\text{down}}(z)/\epsilon_0|$ (red) and $|Y_{\Delta_{l_\perp}}(z)/\epsilon_0|$ (purple). The dashed gray line shows the equilibrium yield for Y_{N_1}.

parameter scan is restricted to washout parameters $K > 3$. In this regime, the final $B-L$ asymmetry without partially equilibrated spectators becomes initial condition independent, so that both plots in Fig. 5.8 are normalized to the same baseline asymmetry. Finally, sterile neutrino Yukawa interactions are nonperturbative in regions of parameter space above the dotted black line.

As is to be expected, there is no discernible difference between the treatments with either partially of fully equilibrated spectators for $\tilde{M}_1 \ll 10^{12}\,\text{GeV}$. In this regime, b-Yukawa and weak sphaleron interactions equilibrate well before the washout processes freeze-out, and any asymmetry produced at early times is inevitably washed out. In the regime with $\tilde{M}_1 \gtrsim 10^{12}\,\text{GeV}$, the inclusion partially equilibrated spectators lead to a strong initial condition dependence of the final $B-L$ asymmetry:

- For thermalized initial conditions $Y_{N_1\text{even}}(0) = Y_{N_1\text{eq}}(0)$, one obtains order one corrections to $Y_{B-L}(z \to \infty)$, but the proper accounting of partial equilibration does not change the overall picture. This result is consistent with the findings obtained in the nonrelativistic description of [151].

- For a vanishing initial conditions $Y_{N_1\text{even}}(0) = 0$, the proper accounting of partial equilibration leads to more more drastic alterations: First, the final $B - L$ asymmetry changes sign for large K, starting at $K \gtrsim 30$ for $\tilde{M}_1$ near $\sim 10^{13}\,\text{GeV}$. In the plot on the righthand side of Fig. 5.8, this sign flip is indicated by a red contour line. Second, the final $B - L$ asymmetry in the flipped regime is strongly enhanced, increasing by up to two orders of magnitude.

Figure 5.9 illustrates the mechanism for this enhancement, showing the time-evolution of appropriately rescaled yields for $\tilde{M}_1 = 3 \times 10^{13}$ GeV and $K = 10, 1000$. At early times, the finite temperature enhanced source rate $\epsilon_{\text{eff}}\Gamma$ produces an initial asymmetry that is far in excess of the eventual asymmetry at recombination. Although the spectator interactions are relatively inefficient, they can transfer a small but sufficient fraction of this initial asymmetry into spectator fields that do not couple directly to the washout. Once the washout has become active, these inefficient spectator interactions function as a bottleneck, protecting the transferred asymmetries from being washed out. For smaller values of K, this mechanism becomes less and less important because the initial asymmetry transferred to the spectator fields decreases with $\epsilon_{\text{eff}}\Gamma \sim K$, while the width of the protective bottleneck remains the same.

Chapter 6

Summary and Outlook

To conclude this dissertation, I give a short summary of the results presented in each chapter, and provide an outlook towards potentially interesting future lines of research.

6.1 The Portal Effective Theory Framework

In chapter 2, I have constructed generic electroweak and GeV scale PET Lagrangians that capture SM extensions in which the SM couples to hidden sectors via a single hidden mediator with spin $S = 0$, $S = 1/2$, or $S = 1$ and a mass at or below the electroweak scale. Using effective field theory methods to construct a minimal basis of portal operators for each mediator type, I assumed that the resulting portal interactions are parametrically suppressed by a generic small parameter $\epsilon \ll 1$. Since higher dimensional portal operators have to be suppressed by a generic new physics scale $1/\Lambda \equiv \epsilon/v$, only operators with dimension $d \leq 5$ can contribute at leading order in ϵ.

For the electroweak scale PET Lagrangians, I have constructed a minimal basis of portal operators that contribute at leading order in ϵ. For spin $S = 0$ and $S = 1$ mediators, the PET Lagrangians conserve both baryon and lepton number, while the spin $S = 1/2$ PET conserves only baryon number. After electroweak symmetry breaking, all three PET Lagrangians exhibit mass mixing between SM and hidden degrees of freedom. In general, this mixing cannot be eliminated without making assumptions about the internal structure of the hidden sector.

For the GeV scale PET Lagrangians, I have constructed a minimal basis of portal operators with dimension $d \leq 7$. In this case, the hidden mediator is assumed to be active at the GeV scale, with a mass $\ll 10\,\mathrm{GeV}$. As with the electroweak scale PETs, I have included only operators that are suppressed by at most a single power of ϵ, with the higher dimensional $d = 6, 7$ operators being generated by virtual exchanges of the heavy SM bosons. Accordingly, they have to be counted as ϵG_F rather than ϵ^3, and the spin $S = 1$ PET only contains operators with dimension $d \leq 6$. Focusing on quark flavour changing $\Delta F \neq 0$ transitions induced by virtual $W^\pm$ exchanges, I have also identified a minimal basis of $d = 6, 7$ operators that contribute to $\Delta F \neq 0$ transitions at leading order in the (4π) power counting rules of naive dimensional analysis. In particular, the resulting GeV scale PET captures charged Kaon decays such as $K^+ \to \pi^+ S_\text{hidden}$, which can be constrained using observations at fixed target experiments such as NA62.

In general, the PET framework makes no assumption about the internal structure of the hidden sector. For this reason, it can be used to constrain hidden sectors in a way that remains largely model independent. In principle, one could even use observations at low energy fixed target experiments such as NA62 in order to constrain hidden sector models at or above the electroweak scale. For this, one has to match the electroweak and GeV scale portal Lagrangians, which could be an interesting future project. It could also be interesting to derive specific PETs for various SM effective theories. In this work, I have already obtained a specific PET for χPT. In principle, it is straightforward to derive analogous PETs for heavy quark effective theory (HQET), which could be used to study portal induced B and D meson decays, or pNRQED (potential nonrelativistic QED), which could be used for hidden sector searches in QED precision tests.

6.2 PET Chiral Perturbation Theory

In chapter 3, I have constructed a PET χPT Lagrangian that couples the light pseudoscalar mesons of χPT to generic hidden sector models captured by the PET framework developed in chapter 2. To be specific, I considered hidden sectors that couple to the SM via a single light hidden mediator with spin ≤ 1 and a mass below $M_K \approx 493\,\mathrm{MeV}$.

Working at order $\mathcal{O}(\epsilon)$, I used an external current approach to construct the resulting PET χPT Lagrangian. To do so, I rewrote both the $d \leq 5$ and $d = 6, 7$ portal operators as interactions between QCD gauge singlets and 10 generic, external currents l_μ, r_μ, χ, a, ϑ, ξ, $t_{\mu\nu}$, A, B, and C. The contributions to χPT generated by the external currents l_μ, r_μ, χ, ϑ, and $t_{\mu\nu}$ are well known. To capture the impact of ϑ, it is appropriate to work in the U(3) version of χPT, which entails a simultaneous expansion in $1/N_c$ and small momenta p^2. Using this power counting, I extended the external current picture to account for the leading order contributions generated by generic currents a, A, B, C, and ξ. The final PET χPT Lagrangian contains a number of new low energy constants (=LECs) associated with operators containing the external currents a, A, B, C, and ξ. I combined large N_c factorization rules with the well-known low-energy realizations of QCD gauge singlet quark bilinears to estimate the new LECs associated with A, B, and C. Finally, I exploited the conformal anomaly of QCD in order to estimate the new LECs associated with a.

The leading order PET χPT Lagrangian is suitable to study general portal induced transitions at fixed targed experiments such as NA62, DUNE, or SHiP. In particular, I have accounted for the WZW action, which is necessary to capture neutral Pion decays $\pi^0 \to \gamma\gamma_{\mathrm{dark}}$ involving dark photons, and $W^\pm$ boson induced octet and 27-plet contributions, which are necessary to capture charged Kaon decays, such as $K^+ \to \pi^+ S$ involving hidden (pseudo-)scalars and Axion-like particles. It is worth emphasizing that the PET χPT Lagrangian makes no assumption on the internal structure of the hidden sector. It can be used to compute general BSM transition amplitudes involving spin $S = 0$, $S = 1/2$ and $S = 1$ mediators in a way that remains equally applicable to a wide range of popular SM extensions.

To conclude, it should be noted that contributions of higher order p^2 or $1/N_c$ may be important for certain portal induced transitions. Naively, next-to-leading order contributions are expected to yield corrections of order $M_K^2/\Lambda_\chi^2 \sim 1/N_c \sim 30\%$, giving a comparable theoretical uncertainty for the leading order predictions. That being said, the available parameter space for many popular SM extension spans many orders of magnitude, and even an order of magnitude prediction can be used to exclude large sections of this parameter space.

6.3 Lepton Number Violation at Colliders

In chapter 4, I studied lepton number violation (LNV) at colliders within the context of type-I seesaw models without fine-tuning. In these models, LNV can occur in sterile neutrino decays, and the question of fine-tuning arises from tension between the large sterile neutrino production cross sections required for *observable* LNV and the smallness of the SM neutrino masses generated via mixing with the sterile neutrinos.

Studying this tension, I have applied two separate notions of naturalness: First, naturalness in the 't Hoof sense, which requires that the SM neutrino masses should be stable under loop corrections, and second, the more restrictive notion that the theory should contain no large, accidental cancellations. For 't Hooft natural models, I have obtained a lower bound for the relative suppression of LNV sterile neutrino decays compared to LNC sterile neutrino decays, and for models without large, accidental cancellations I have obtained an additional upper bound for the suppression of LNV sterile neutrino decays. Using these bounds, it is possible to split the available parameter space for type-I seesaw models into three distinct regions:

- For sufficiently small mixing angles, LNV sterile neutrino decays are unsuppressed in models without large, accidental cancellations.
- For intermediate mixing angles, LNV sterile neutrino decays may or may not be suppressed in models without either 't Hooft fine-tuning or large, accidental cancellations.
- For sufficiently large mixing angles, LNV sterile neutrino decays are suppressed in models without 't Hooft fine-tuning.

In general, the boundaries between each of these regions depend on the sterile neutrino masses.

As a specific example, I have computed the position of the boundaries between these three regions for a minimal benchmark model with $n = 2$

sterile neutrinos. In this model LNV is unsuppressed for mixing angles $U^2 < U^2_{\rm LNV}$, and suppressed for mixing angles $U^2 > U^2_{\rm stable}$, where both $U^2_{\rm LNV}$ and $U^2_{\rm stable}$ depend only on $\overline{M}$.

Roughly, these bounds imply that LNV is unsuppressed if the heavy neutrino mass eigenstates are lighter than $M_W \sim 80\,\mathrm{GeV}$. On the other hand, an observable LNV at the LHC can be inconsistent with 't Hooft natural SM neutrino masses for heavy neutrino mass eigenstates that are heavier than $\sim 80\,\mathrm{GeV}$.

6.4 Relativistic and Spectator Effects in High-Scale Leptogenesis

In chapter 5, I studied the impact of relativistic and spectator effects in leptogenesis with heavy sterile neutrinos. To focus on these effects, I considered a minimal model for high-scale leptogenesis, in which two sterile neutrinos N_i with strongly hierarchical masses $M_1 \ll M_2$ couple to the same linear combination $\ell_\parallel$ of SM lepton flavours. Using the CTP formalism of nonequilibrium quantum field theory, I have derived a set of momentum-averaged fluid equations for high-scale leptogenesis that remains valid throughout the whole transition from the ultrarelativisitc regime $z \ll 1$ to the nonrelativistic regime $z \gg 1$. These fluid equations account for finite temperature corrections, helicity dependent sterile neutrino interactions, and quantum statistical factors associated with the Fermi-Dirac distributions.

Using hard thermal loop (HTL) resummation techniques, I computed the dimensionless LNC and LNV rates $\gamma_{\rm LNC}$ and $\gamma_{\rm LNV}$ that appear in these equations. The rates $\gamma_{\rm LNC/LNV}$ govern sterile neutrino equilibration, $B-L$ washout, and the CP violating sourcing of $B-L$ asymmetry in the early universe plasma. At leading-log accuracy, they receive contributions from $1 \leftrightarrow 2$ processes involving SM lepton pseudo-particle modes, $1 \leftrightarrow 2$ processes involving SM lepton collective excitations, and logarithmically enhanced $2 \leftrightarrow 2$ scattering contributions. For $\gamma_{\rm LNV}$, $1 \leftrightarrow 2$ pseudo-particle contributions are found to dominate in both the ultrarelativistic and nonrelativistic regimes, while $2 \leftrightarrow 2$ contributions dominate in the intermediate regime, for $0.1 \lesssim z \lesssim 0.8$. As is expected from prior results, $2 \leftrightarrow 2$ scattering contributions dominate $\gamma_{\rm LNC}$ for both ultrarelativistic and intermediate regimes, while $1 \leftrightarrow 2$ pseudoparticle contributions dominate γ_{LNC} at lower temperatures. In both cases, collective excitations are subdominant.

One important result is that the effective finite temperature sterile neutrino decay asymmetry ϵ_{eff} is strongly enhanced at early times. In weak washout scenarios with a vanishing initial abundance of sterile neutrinos, this strong enhancement of ϵ_{eff} corrects the sign and scaling behaviour of the final efficiency factor κ_f. Explicitly, one obtains the efficiency factor $\kappa_f \approx -0.32\,K$, compared to $\kappa_f^{\mathrm{NR}} \approx 1.65\,K^2$ in the nonrelativistic approximation used in [168], with the κ_f sign flip occurring at $K \approx 0.2$. In strong washout scenarios with a vanishing initial abundance, interplay between the early time enhancement of ϵ_{eff} an the partial equilibration of spectator interactions leads to another sign flip κ_f, starting at $K \approx 30$ for $M \sim 10^{13}\,\mathrm{GeV}$. In the flipped regime, for $K \gg 30$, one finds that the final efficiency factor is strongly enhanced by up to two orders of magnitude.

As the present work has shown, the final $B - L$ asymmetry in high-scale leptogenesis scenarios with a vanishing initial abundance of sterile neutrinos is quite sensitive to the effective sterile neutrino decay asymmetry ϵ_{eff}. In principle, ϵ_{eff} receives further corrections from nonlinear terms proportional $\delta f_{N_1 h}^2$, and it remains to study the impact of such corrections.

Additionally, it may be interesting to study the impact of partially equilibrated spectators for lower sterile neutrino masses $M_1 \ll 10^{12}\,\mathrm{GeV}$. In that regime, one has to account for lepton flavour oscillations, so that it is necessary to track the asymmetry stored in each lepton flavour separately. In principle, it is well known how to account for such lepton flavour effects, and combining this description with partially equilibrated spectator interactions should pose no major difficulties.

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